

ME 360L: Solution to Sample Problems 2

1. a)

$$N_1(x, y) = (1 - x/3)(1 - y/2) \quad , \quad N_2(x, y) = (x/3)(1 - y/2) \quad , \quad N_3(x, y) = (x/3)(y/2) \quad , \\ N_4(x, y) = (1 - x/3)(y/2)$$

$$\text{b) } \mathbf{B} = \frac{1}{6} \begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -2 & 0 & -1 & 0 & 1 & 0 & 2 \\ -2 & -1 & -1 & 1 & 1 & 1 & 2 & -1 \end{bmatrix}$$

$$\text{c) } \boldsymbol{\varepsilon} = -10^{-3} \begin{bmatrix} 1.667 \\ 3.33 \\ 3.33 \end{bmatrix}$$

$$\text{d) } \boldsymbol{\sigma} = - \begin{bmatrix} 88 \\ 126.5 \\ 38.5 \end{bmatrix} \text{ kN} / \text{m}^2$$

$$\text{2. a) } \quad x = 5.5 \quad y = 6.0$$

$$\text{b) } \boldsymbol{\varepsilon} = -10^{-5} \begin{bmatrix} 0 \\ 10 \\ 10 \end{bmatrix} \quad \boldsymbol{\sigma} = 10^5 \begin{bmatrix} 23.1 \\ 76.9 \\ 26.9 \end{bmatrix}$$

$$\text{3. a) } \quad N_1(x, y) = y/5 \quad , \quad N_2(x, y) = 1 - x/10 - y/5 \quad , \quad N_3(x, y) = x/10$$

$$\text{b) } \mathbf{f} = 8,333 \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{c) } \mathbf{B} = \frac{1}{10} \begin{bmatrix} 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & -2 & 0 & 0 \\ 2 & 0 & -2 & -1 & 0 & 1 \end{bmatrix}$$

d) $u = 0.0108 \quad v = 0.0$

e)
$$\boldsymbol{\sigma} = \begin{bmatrix} 0 \\ 0 \\ 1.7 \times 10^3 \end{bmatrix}$$

4)
$$\mathbf{f} = 714.3 \begin{bmatrix} 0 \\ 1 \\ -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} lb$$

5. a) $u_2 = -u_5 = -\frac{L\alpha\beta\theta}{4 + \beta} \quad u_3 = -u_4 = -\frac{2L\alpha\beta\theta}{4 + \beta}$

b) $\sigma_{1-2} = \sigma_{2-3} = \sigma_{3-4} = \sigma_{4-5} = \sigma_{5-6} = -\frac{E\alpha\beta\theta}{4 + \beta}$

6. a) At the top node $u = 0$, $v = 20.95\alpha\theta$

b) $\sigma_{left\ bar} = 0.2514E\alpha\theta$, $\sigma_{center\ bar} = -0.302E\alpha\theta$, $\sigma_{right\ bar} = 0.2514E\alpha\theta$

7. $\theta = \frac{3\delta}{\alpha L}$