

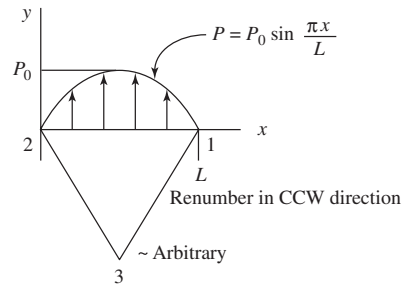
Then

$$f_{s_{14}} = \frac{1}{6} L p_1^{\text{avg}} + \frac{1}{3} L p_2 = \frac{1}{3} L \frac{p_0}{4} = \frac{p_0 L}{12} \rightarrow \text{Like 6.11 b with } f_{sx} = \frac{pL}{12}$$

$$f_{s_{34}} = \frac{1}{6} L p_3 + \frac{1}{3} L p_2 = \frac{3}{12} L p_0 = \frac{p_0 L}{4} \Rightarrow f_{3x} \text{ in 6.11b}$$

(These answers match P6.11 for special case)

(b)



$$\{f_s\} = \int_s \int [N_s]^T [T_s] ds$$

$[T_s]$ = Surface tractions

$$= \begin{Bmatrix} T_{sx} \\ T_{sy} \end{Bmatrix} = \begin{Bmatrix} 0 \\ P_0 \sin \pi \frac{x}{L} \end{Bmatrix}$$

$[N_s]$ = Shape function matrix evaluated along edge 1-2

$$= \begin{bmatrix} N_i & 0 & N_j & 0 & N_m & 0 \\ 0 & N_i & 0 & N_j & 0 & N_m \end{bmatrix}$$

Let $i = 1$

$j = 2$

$m = 3$

$$N_i - N_1 = \frac{1}{2A} (\alpha_i + \beta_i x + \gamma_i y)$$

$$N_1(y=0) = \frac{1}{2A} (\alpha_i + \beta_i x)$$

$$\begin{aligned} \alpha_i &= x_j y_m - y_j x_m \\ &= 0(y_m) - 0(x_m) = 0 \end{aligned}$$

$$\begin{aligned} \beta_i &= y_j - y_m \\ &= 0 - y_m = -y_m \end{aligned}$$

$$N_1(y=0) = \frac{1}{2A} (0 - y_m x) = \frac{-y_m x}{2A}$$

$$N_j = N_2 = \frac{1}{2A} (\alpha_j + \beta_j x + \gamma_j y)$$

$$N_2(y=0) = \frac{1}{2A} (\alpha_i + \beta_i x)$$

$$\begin{aligned} \alpha_j &= y_j x_m - x_j y_m \\ &= 0(x_m) - Ly_m = -Ly_m \end{aligned}$$

$$\begin{aligned} \beta_j &= y_m - y_i \\ &= y_m - 0 = y_m \end{aligned}$$

$$N_2(y=0) = \frac{1}{2A} [-Ly_m + y_m x] = \frac{y_m}{2A} [x-L]$$

$$N_m = N_3 = \frac{1}{2A} [\alpha_m + \beta_m x + \gamma_m y]$$

$$N_3(y=0) = \frac{1}{2A} [\alpha_m + \beta_m x]$$

$$\alpha_m = x_i y_j - y_i x_j = L(0) - 0(0) = 0$$

$$\beta_m = y_i - y_j = 0$$

$\therefore N_m(y=0) = 0$ As expected

$$\begin{aligned} \{f_s\} &= \int_0^{x=L} \int_0^{z=t} \begin{bmatrix} N_1 & 0 \\ 0 & N_1 \\ N_2 & 0 \\ 0 & N_2 \\ N_3 & 0 \\ 0 & N_3 \end{bmatrix} \left\{ \begin{array}{c} 0 \\ P_0 \sin \frac{\pi x}{L} \end{array} \right\} dz dx \\ &= t \int_0^{x=L} \begin{bmatrix} 0 \\ N_1 P_0 \sin \frac{\pi x}{L} \\ 0 \\ N_2 P_0 \sin \frac{\pi x}{L} \\ 0 \\ N_3 P_0 \sin \frac{\pi x}{L} \end{bmatrix} dx = t \int_0^{x=L} \begin{bmatrix} 0 \\ -\frac{y_m x}{2A} P_0 \sin \left(\frac{\pi x}{L}\right) \\ 0 \\ \frac{y_m}{2A} (x-L) P_0 \sin \frac{\pi x}{L} \\ 0 \\ 0 \end{bmatrix} dx \end{aligned}$$

2nd term in (A) ($y_m = y_3$)

$$\begin{aligned} f_{s1y} &= \frac{-t y_3 P_0}{2A} \int_0^{x=L} x \sin \left(\frac{\pi x}{L}\right) dx & \int u dv = u - \int v du \\ & & u = x \quad du = dx \\ & & dv = \sin \frac{\pi x}{L} dx \quad v = -\cos \frac{\pi x}{L} \end{aligned}$$

$$= \frac{-t y_3 P_0}{2A} \left[-\frac{xL}{\pi} \cos \pi \frac{x}{L} + \int_0^{x=L} \frac{L}{\pi} \cos \left(\pi \frac{x}{L}\right) dx \right]$$

$$= \frac{-t y_3 P_0}{2A} \left[-\frac{xL}{\pi} \cos \left(\pi \frac{x}{L}\right) + \frac{L^2}{\pi^2} \sin \left(\pi \frac{x}{L}\right) \right] \Big|_0^L$$

$$= \frac{-t y_3 P_0}{2A} \left[\frac{-L^2}{\pi} (-1) + 0 - 0 - 0 \right]$$

$$f_{s1y} = \frac{-t y_3 P_0}{2A} \left(\frac{L^2}{\pi} \right) = \frac{t P_0 L}{\pi}$$

$$A = \frac{-1}{2} L y_3$$

4th term in (A)

$$f_{s2y} = t \int_0^L \left[\frac{y_3}{2A} P_0 x \sin \frac{\pi x}{L} - \frac{y_3}{2A} P_0 L \sin \left(\frac{x\pi}{L} \right) \right] dx$$

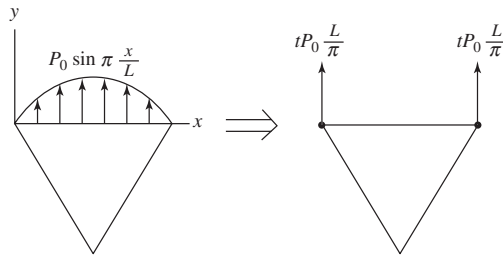
$$= \frac{t y_3 P_0}{2A} \underbrace{\int_0^L x \sin \frac{\pi x}{L} dx}_{\text{DONE IN 2nd TERM}} - \frac{t y_3 P_0 L}{2A} \int_0^L \sin \frac{\pi x}{L} dx$$

$$= \frac{t y_3 P_0}{2A} \left[\frac{L^2}{\pi} \right] + \frac{t y_3 P_0 L}{2A} \left[\frac{L}{\pi} \cos \pi \frac{x}{L} \right]_0^L$$

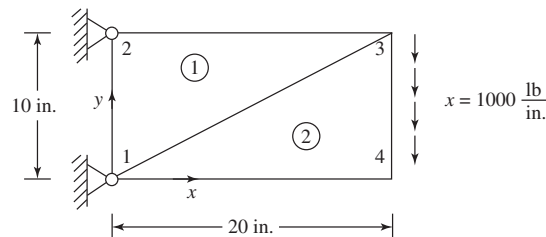
$$= \frac{t y_3 P_0}{2A} \frac{L^2}{\pi} + \frac{t y_3 P_0 L}{2A} \frac{L}{\pi} [-1 + 1]$$

$$f_{s2y} = \frac{t y_3 P_0 L^2}{2A \pi} = \frac{t \cancel{y_3} L^2 P_0}{\cancel{\left[\frac{1}{2} \cancel{L} \cancel{y_3} \right]} \pi} = \frac{t L P_0}{\pi}$$

$$\therefore \vec{f}_s = \begin{Bmatrix} f_{s1x} \\ f_{s1y} \\ f_{s2x} \\ f_{s2y} \\ f_{s3x} \\ f_{s3y} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{t P_0 L}{\pi} \\ 0 \\ \frac{t P_0 L}{\pi} \\ 0 \\ 0 \end{Bmatrix}$$



6.13



Equation (6.3.6)

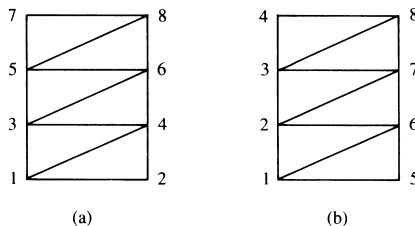
$$\begin{Bmatrix} f_{B1x}^{(1)} \\ f_{B1y}^{(1)} \\ f_{B2x}^{(1)} \\ f_{B2y}^{(1)} \\ f_{B5x}^{(1)} \\ f_{B5y}^{(1)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 77.1 \\ 0 \\ 77.1 \\ 0 \\ 77.1 \end{Bmatrix} \frac{(0.4\text{ m})(0.2\text{ m})(0.005\text{ m})}{(2)(3)} = \begin{Bmatrix} 0 \\ 5.14 \\ 0 \\ 5.14 \\ 0 \\ 5.14 \end{Bmatrix} 10^{-3} \text{ kN}$$

All body force matrices for each element identical to above
 Adding the 4 element body force matrices

$$\{F_B\} = \begin{Bmatrix} f_{B1x} \\ f_{B1y} \\ f_{B2x} \\ f_{B2y} \\ f_{B3x} \\ f_{B3y} \\ f_{B4x} \\ f_{B4y} \\ f_{B5x} \\ f_{B5y} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 10.28 \\ 0 \\ 10.28 \\ 0 \\ 10.28 \\ 0 \\ 10.28 \\ 0 \\ 20.56 \end{Bmatrix} \text{ N}$$

- 6.16** The triangular element is called a constant strain triangle (CST) because the strain is constant throughout the element.
- 6.17** The stresses are also constant as the strains are constant.
- 6.18**
- No, bending in the plane takes place
 - Yes, loads in-plane of the wall
 - Yes, a plane stress problem
 - Yes, if loads in-plane of the bar
 - Yes, a plane strain problem
 - Yes, a plane stress problem
 - No, loads out of the plane of the wrench
 - Yes, as loads in the plane
 - No, bending in the plane takes place
- 6.19** We must connect the beam element to two or more nodes of a plane stress element. The beam must be along the edge of the plane stress element.

6.20



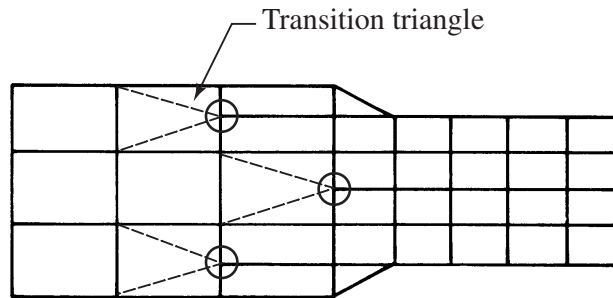
$$n_b = n_0 (m + 1)$$

(a) $n_b = 2(3 + 1) = 8$

(b) $n_b = 2(5 + 1) = 12$

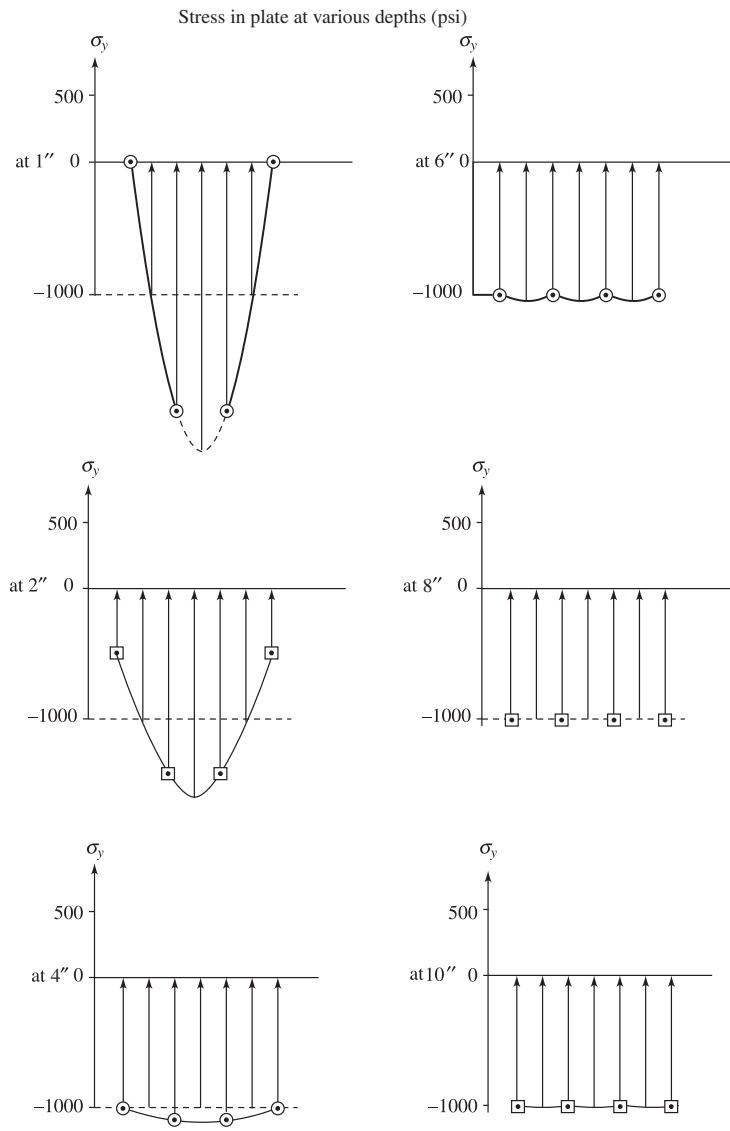
Chapter 7

- 7.1** For the simple 4 noded elements it is a violation of displacement compatibility to have a mid-side node. Some of the elements have mid-side nodes in this model. Use 'transition' triangle to go from smaller to larger rectangular elements.



○ = Mid-side nodes

- 7.2** The mesh sizing is not fine enough in the reentrant corner region at C . We need smaller elements near point C and small radius at C .
- 7.3** Based on the formulation used here we can not have $\mu = 0.5$ for the plane strain case as the denominator in the material property matrices $[D]$ (see Equation (6.1.10) and $[K]$ (see Equation (6.4.3) becomes zero. A penalty formulation see Reference [7] can be used to avoid this problem.
- 7.4** The structure is plane strain if this section represents a cross section of a long structure in which the loads do not vary in the z direction.
The structure is a plane stress problem if this section is a thin plate type structure with loads in the plane of the structure only.
Also see Section 6.1 for descriptions of plane stress and plane strain and examples of each.
- 7.5** When abrupt changes in thickness at E 's occur from element to element.
- 7.6** Unit thickness/7.7 (a) best aspect ratio.
- 7.9** (a) No, as replacing a portion of the patch by a different material with different mechanical properties will in general produce non-uniform strain under constant state of applied stress. For rigid body mode tests, however, different mechanical property materials still result in rigid body displacement.
- (b) Yes, the patch can be arbitrary in shape. If we apply a test displacement field of $u_x = 1$, $u_y = 0$ at the external nodes of a patch of say 4 elements and set the internal nodal force to zero, then solve for the displacement components at internal node i , these displacement components should agree with the value of the displacement function at that node. Also the strain function or field should vanish identically at any point over each element.
- (c) Yes, we can mix triangular and quadrilateral elements in a 2-d patch test as long as the material properties are the same.
- (d) No. Mixing bars with plane elements would alter the constant strain states as the plane element and bar are of different structural types.
- (e) The patch test should be applied when developing new finite elements, to determine if the element can represent rigid body motion as well as states of constant strain when these conditions occur.



7.15

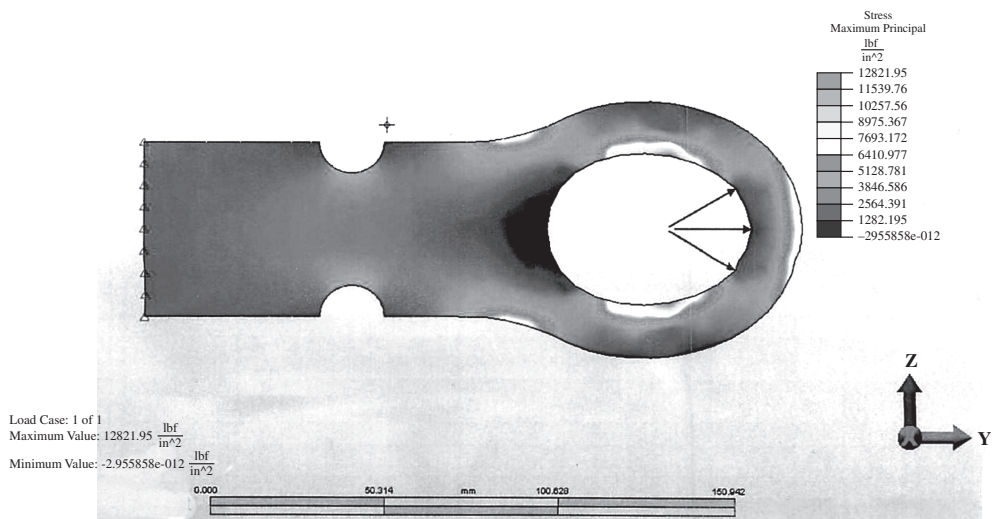
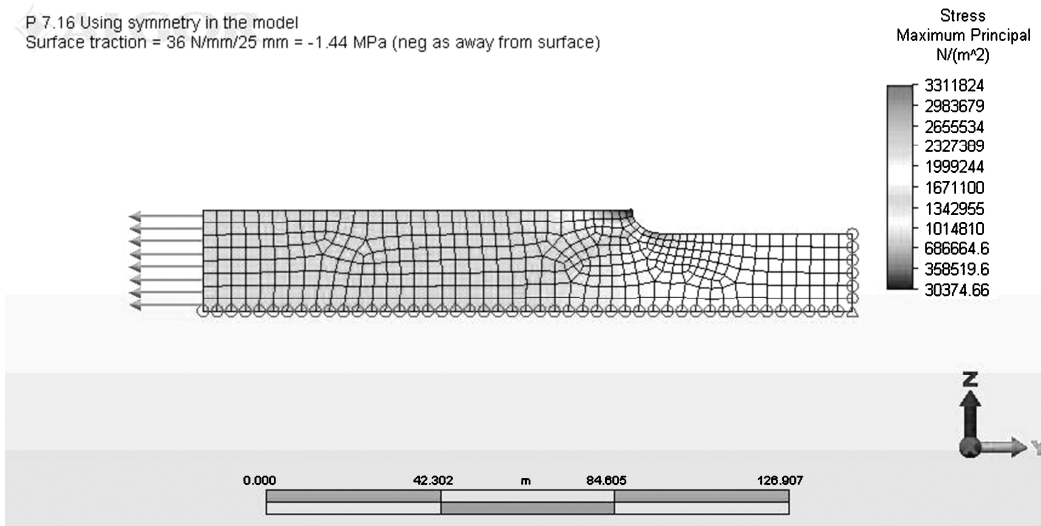


Figure. 8

The maximum principal stresses are at the top and bottom of the circular opening of the connecting rod. The maximum stresses can be seen in Figure 8 above. Using a mesh density of only 400 (Figure 1- Figure 4) the precision in the area of interest (i. e., where the maximum stress occurs); see Figure 1 and Figure 4, the precision was about 0.28 further refinement was required. The mesh was refined to 800 and 1200 then finally to 1600. With the mesh density being 1600 the precision was less than 0.1 in. the place of interest and can be assumed correct. The maximum principal stress of about $12822 \frac{\text{lb}}{\text{in}^2}$ was determined.

7.16

P 7.16 Using symmetry in the model
 Surface traction = $36 \text{ N/mm}/25 \text{ mm} = -1.44 \text{ MPa}$ (neg as away from surface)



7.17

