ME-504: Final Exam
Spring 2012

1. For the convection diffusion equation

$$-\frac{d^2\phi}{dx^2}+50\frac{d\phi}{dx}=0\ ,\ \phi(0)=0\ ,\ \left.\frac{d\phi}{dx}\right|_{(x=2)}=1$$

use 10 elements to find a numerical solution with weighting functions

$w_i = N_i + \frac{\alpha h}{2}\frac{dN_i}{dx}$ and

a) The Galerkin method $\alpha = 0$.
b) The Petrov Galerkin method $\alpha = \coth(\gamma/2) - 2/\gamma$.
c) The upwind method $\alpha = 1$.

Compare your results with the exact solution.

2. Find the element equations for the convection diffusion equation $\frac{\partial\phi}{\partial t} - D\frac{d^2\phi}{dx^2} + u\frac{d\phi}{dx} = S$ using the bilinear space-time functions given in Eq.(9.3) and the quadratic in space weighting functions of Eq. (9.32).

3. Consider the equation $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + u = 0$.

a) Using the Galerkin formulation combined with the $\theta -$method with $\theta = 1$, find the amplification factor $|\xi|$ and the phase error Θ. Plot the results for $c = 0.5$ and $c = 1$ as a function of L/h.
b) Find the stability characteristics of the algorithm when $c = 1$ and $\theta = 0$.

4. Find the element equations corresponding to Eq. (10.18) for the bilinear velocity, constant pressure element $0 \le x \le h$, $0 \le y \le k$. Show the special case $h = k$.

5. Problem 10.6 in the text (Note: the figure is 10.6, not 10.5)