## You are allowed to bring a two page sheet containing equations and a calculator

I. Answer the following multiple choice questions ( 5 pts each), choose one answer only!

1. Which of the following phenomena is not a quantum effect?
A. Zero-point energy.
B. Quantized energy.
C. Quantized angular momentum.
D. Tunneling.
E. Energy conservation.
2. What is the spin multiplicity of the ground electronic state of a Na atom?
A. 0 .
B. 1 .
C. 2 .
D. 3 .
E. 4 .
3. For an electron restricted to a two-dimensional rectangular well, how many quantum numbers does it have?
A. 0 .
B. 1 .
C. 2 .
D. 3 .
E. 4 .
4. The term "interstate crossing" is used for
A. a transition from an excited singlet state to another singlet state.
B. a transition from the ground singlet state to an excited singlet state.
C. a transition from an excited singlet state to an excited triplet state.
D. a transition from an excited triplet state to an excited singlet state.
E. a transition from an excited triplet state to the ground singlet state.
5. Which of the following conditions is appropriate for a symmetric rotor?
A. $I_{x}=I_{y}=I_{z}$.
B. $I_{x}=I_{y} \neq I_{z}$.
C. $I_{x} \neq I_{y} \neq I_{z}$.
D. $I_{x}=I_{y}, I_{z}=0$.
E. none of the above.
6. The probability for finding a particle at a point $(x, y, z)$ in a three-dimensional space is given by
A. $\Psi(x, y, z)$
B. $|\Psi(x, y, z)|^{2}$
C. $\Psi(x, y, z) d x d y d z$
D. $|\Psi(x, y, z)|^{2} d x d y d z$
E. $\int|\Psi(x, y, z)|^{2} d x d y d z$
7. Which of the following approximations assumes the separation of electronic and nuclear motions?
A. The Born-Oppenheimer approximation.
B. The Hund's rule.
C. Heisenberg's uncertainty principle.
D. The Hartree-Fock approximation.
E. The Hückel approximation.
8. How many vibrational quantum numbers does the molecule $\mathrm{CH}_{4}$ have?
A. 5 .
B. 9 .
C. 10 .
D. 12 .
E. 15 .
9. What point group does the molecule $\mathrm{CO}_{2}$ belong to?
A. $\mathrm{C}_{\mathrm{i}}$.
B. $\mathrm{C}_{\mathrm{s}}$.
C. $\mathrm{D}_{\infty h}$.
D. $\mathrm{C}_{\mathrm{ov}}$.
E. $\mathrm{C}_{2 \mathrm{v}}$.
10. Which of the following molecules has no rotational spectrum?
A. $\mathrm{NH}_{3}$.
B. HCl .
C. $\mathrm{N}_{2}$.
D. $\mathrm{H}_{2} \mathrm{O}$.
E. NO.
II. Solve the following simple problems
11. ( 10 pts ) An electron is trapped in a one-dimensional well, which has a length of 1.5 nm . Calculate the photon wavelength needed to excite the electron from the ground state to the first excited state.
12. ( 10 pts ) An atom has an electronic configuration corresponding to ${ }^{4} \mathrm{D}$. Determine all possible $J$ values and write the all the term symbols.
13. ( 10 pts ) Electrons ejected from a metal by a 346 nm light are detected to have a velocity of $1.3 \times 10^{5} \mathrm{~m} / \mathrm{s}$. Calculate the work function of the metal in eV .
14. ( 10 pts ) An electron is measured to have a speed of $1.5 \times 10^{5} \mathrm{~m} / \mathrm{s}$ with an uncertainty of $0.5 \times 10^{5} \mathrm{~m} / \mathrm{s}$. Calculate the uncertainty of its position.
15. (10pts) A proton is accelerated to a speed of $2.3 \times 10^{3} \mathrm{~m} / \mathrm{s}$. Calculate its de Broglie wavelength in nm .
16. (10 pts) Prove that the linear combinations $p_{x} \pm i p_{y}$ are eigenfunctions of the angular momentum operator $\hat{L}_{z}=-i \hbar \frac{\partial}{\partial \phi}$ and calculate the corresponding eigenvalues.
$\left(p_{x}=f(r) \sin \theta \cos \phi, p_{y}=f(r) \sin \theta \sin \phi\right)$

## III. Solve the following more complicated problems

17. ( 15 pts ) Determine if the following integral of $\mathrm{H}_{2} \mathrm{O}$ is zero based on the symmetry of the molecule. The character table is attached in the back of the test.

$$
I=\int \Psi\left(a_{1}\right) x \Psi\left(b_{2}\right) d \tau
$$

in which $\Psi\left(a_{1}\right)$ and $\Psi\left(b_{2}\right)$ are molecular orbitals of $\mathrm{H}_{2} \mathrm{O}$ belonging to the $\mathrm{A}_{1}$ and $\mathrm{B}_{2}$ irreducible representations of the $\mathrm{C}_{2 \mathrm{v}}$ group.

Character table for $\mathbf{C}_{\mathbf{2 v}}$ point group

|  | $\mathbf{E}$ | $\mathbf{C}_{\mathbf{2}}(\mathrm{z})$ | $\boldsymbol{\sigma}_{\mathrm{v}}(\mathbf{x z})$ | $\boldsymbol{\sigma}_{\mathrm{v}}(\mathbf{y z})$ | linear, <br> rotations | quadratic |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{A}_{\mathbf{1}}$ | 1 | 1 | 1 | 1 | z | $\mathrm{x}^{2}, \mathrm{y}^{2}, \mathrm{z}^{2}$ |
| $\mathbf{A}_{\mathbf{2}}$ | 1 | 1 | -1 | -1 | $\mathrm{R}_{\mathrm{z}}$ | xy |
| $\mathbf{B}_{\mathbf{1}}$ | 1 | -1 | 1 | -1 | $\mathrm{x}, \mathrm{R}_{\mathrm{y}}$ | xz |
| $\mathbf{B}_{2}$ | 1 | -1 | -1 | 1 | $\mathrm{y}, \mathrm{R}_{\mathrm{x}}$ | yz |

18. ( 15 pts ) Draw the molecular orbital energy level diagram for $\mathrm{O}_{2}$. Calculate the bond order and the spin multiplicity of the molecule. Sketch the shapes of the HOMO and LUMO. Derive the electronic configuration for $\mathrm{O}_{2}{ }^{-}$and $\mathrm{O}_{2}{ }^{+}$and compare their relative stability.
19. ( 20 pts ) The IR spectrum of HBr is dominated by a strong line at $2649 \mathrm{~cm}^{-1}$. Determine the force constant and zero-point energy of this molecule, assuming it can be approximated by a harmonic oscillator. If the hydrogen is replaced by deuterium, what will be the transition frequency in $\mathrm{cm}^{-1}$ for DBr ?
20. (20 pts) The microwave spectrum of $\mathrm{H}^{35} \mathrm{Cl}$ consists of equally spaced lines separated by $21.2 \mathrm{~cm}^{-1}$. Calculate the rotational constant and the moment of inertia of the molecule and determine its bond length. Explain the quantization of the rotational angular momentum by drawing the vector model for the $J=2$ state.
21. ( 20 pts ) A spectral line in the Balmer $(2 \rightarrow n)$ series of the hydrogen atom was detected at $15233 \mathrm{~cm}^{-1}$. Find out the principle quantum number ( $n$ ) of the state into which electron made the transition. If the electron is further excited to the vacuum by a 256 nm photon, what will be the kinetic energy of the free electron? $\left(R_{H}=109677 \mathrm{~cm}^{-1}\right)$
