Chapter 1. Introduction

I. Classical Physics

Classical Mechanics (Newton): It predicts the motion of classical particles with elegance and accuracy.

$$F = ma = \frac{dp}{dt}, \quad p = mv = m\frac{dq}{dt}$$

F: force
a: acceleration
p: momentum
q: position

Particle travels in a trajectory $q(t)$, $p(t)$ with well-defined energy:

$$E = \frac{p^2}{2m} + V(q), \qquad F = -\frac{dV(q)}{dq}.$$

Translation

Zero force ($F = 0$), initial momentum p_0

$$\frac{dp}{dt} = 0, \quad p = p_0.$$

so

$$E = \frac{p_0^2}{2m}$$

Rotation

Angular momentum

$$J = I\omega$$

ω: angular velocity
$I = mr^2$: moment of inertia

$$\frac{dJ}{dt} = F_T$$

F_T: torque force. For constant torque,

$$J(t) = J_0 + F_T t\,,$$
$$E = \frac{J^2}{2I} = \frac{(J_0 + F_T t)^2}{2I}\,.$$

Vibration (Harmonic oscillator)

Hook's law:

$$F = -kq,$$

k: force constant
q: displacement

Potential energy:

$$V(q) = -\int F dq = -\int (-kq) dq = \frac{1}{2} kq^2$$

Newton's equation

$$m\frac{d^2q}{dt^2} = F = -kq$$

Solutions:

$q = A sin\omega t,$ $\qquad\qquad$ $\omega = (k/m)^{1/2}$

$$p = m\frac{dq}{dt} = m\omega A\cos\omega t$$

$$\begin{aligned}
E &= \frac{p^2}{2m} + V \\
&= \frac{(m\omega A\cos\omega t)^2}{2m} + \frac{1}{2}k(A\sin\omega t)^2 \\
&= \frac{m\omega^2 A^2}{2}\left[(\cos\omega t)^2 + (\sin\omega t)^2\right] \\
&= \frac{m\omega^2 A^2}{2} = \frac{kA^2}{2}
\end{aligned}$$

When $\omega t = 0$, $\qquad q = 0$ $\qquad$ and $\quad p = p_{max} = m\omega A$

When $\omega t = \pi/2$, $\qquad p = 0$ $\qquad$ and $\quad q = q_{max} = A$

Classical Electrodynamics (Maxwell): The four equations specify properties of electromagnetic wave.

Light is an electromagnetic wave with a speed of light with oscillating electric and magnetic fields.

$$\lambda\nu = c$$

λ: wavelength (cm)
ν: frequency (Hz = s^{-1})
c: speed of light (3.0×10^{10} cm/s)

γ-ray x-ray UV visible IR radio wave

λ

ν

II. Failures of classical physics

So in classical world, there is either wave or particle. The two never mix. But this was about the change. Several experimental observations around 1900 provided critical tests of the classical theory and gave birth to the new quantum mechanics.

Blackbody radiation:

Blackbody is an object that absorbs and emits at all frequencies.
A good example is a cavity with a pinhole.
A practical example is a heated metal or the sun.

Observations:

- independent of material
- radiation has continuous spectrum and a peak
- spectral peak changes with temperature

Wien's displacement law (empirical):

$$\lambda_{\max} = \frac{c_2}{5T}, \qquad c_2 = 1.44\ cm\ K$$

Example: Calc. T of the sun ($\lambda_{\max}$ = 480 nm)

$$T = \frac{c_2}{5\lambda_{\max}} = \frac{1.44 \times 10^7\, nm\ K}{5 \times 480 nm} = 6000K$$

Rayleigh-Jeans law (classical electrodynamics theory)

$$\rho = \frac{8\pi kT}{\lambda^4}$$

ρ: energy density
k: Boltzmann constant (1.38×10^{-23} J/K)

Assumption: blackbody consists of a collection of oscillators with arbitrary energies.

Fits the experimental curve well at long wavelengths,
But when $\lambda \to 0$, $\rho \to \infty$ (ultraviolet catastrophe).

Planck's distribution (1900, phenomenological):

$$\rho = \frac{8\pi hc}{\lambda^5}\left(\frac{1}{e^{hc/\lambda kT} - 1}\right)$$

Assumption: energies of oscillators are quantized!

$$E = nh\nu = nhc/\lambda$$

h: Planck's constant (6.626×10^{-34} Js)

When $\lambda \to 0$, $\rho \to 0$.

When $hc/\lambda kT << 1$, $\rho \approx \dfrac{8\pi kT}{\lambda^4}$. $\qquad (e^x \approx 1 + x)$

The photoelectric effect

(1887 Hertz)

Observations:

- No e^- if $\nu < \nu_t$ even at high light intensity.
- Above ν_t, # of e^- increases with light intensity.
- Emit e^- if $\nu > \nu_t$ even at low light intensity.
- Kinetic energy of e^- linearly proportional to ν.

Classically, e^- emission proportional to light intensity, not ν.

Einstein's theory (1905, quantization of radiation field)

$$\frac{1}{2}m_e\upsilon^2 = h\nu - \Phi$$

Work function (E_{min} for e^- escape) $\Phi = h\nu_t$.

Light is made of particles called photons, each with $E = h\nu$!

Electron diffraction (1927, Davisson and Germer)

Diffraction of e^- beam
on crystal surfaces

Classically, e^- cannot interfere because it is a particle.

Conclusions:

i. electron is wave-like (interference).
ii. its λ is comparable to lattice size.

de Broglie wave (1923):

For photon:

$$E = mc^2 = h\nu$$

$$mc^2 = \frac{hc}{\lambda} \qquad (\lambda\nu = c)$$

so

$$\lambda = \frac{h}{p} \qquad (p = mc)$$

<u>Matter-wave dualism</u>: photon and other quantum objects have both wave and particle nature.

	Classical object	Quantum object
Light :	electromagnetic wave	photon and electromagnetic wave
Electron:	particle	electron and matter wave

Example: Calculate de Broglie wavelength for an e^- accelerated from rest thru a potential difference of 1.5 kV

Kinetic energy

$$m_e \upsilon^2/2 \ = \ e\Delta\phi \qquad \text{or} \qquad \upsilon = (2e\Delta\phi/m_e)^{1/2}$$

and momentum

$$p \ = \ m_e \upsilon = (2m_e e\Delta\phi)^{1/2}$$

de Broglie wavelength

$$\begin{aligned} \lambda \ &= \ h/p \\ &= \ h/(2m_e e\Delta\phi)^{1/2} \\ &= \frac{6.626\times10^{-34}\,Js}{[(2\times9.109\times10^{-31}kg)(1.602\times10^{-19}C)(1500.00V)]^{1/2}} \\ &= 3.17\times10^{-11}m = 3.17\times10^{-2}\,nm \end{aligned}$$

Crystal lattice const. is approx 0.1 nm, so e^- can be diffracted.

Atomic spectrum

In 1909, Rutherford established the planetary model for atoms. However, this model predicts an unstable atom since accelerate charges (electrons) emit radiation according to classical theory, thus they lose energy and will eventually collapse. The classical model also predicts continuous spectra.

Observations:

- Stable atoms
- Line spectrum

Bohr's atom model (1911)

Assume electron moves in an orbital associated with quantized energy. Transitions between orbitals correspond to absorption or emission

$$h\nu = |E_n - E_m|$$

but it did not explain why.

Summary:

- classical theory is inadequate in describing microscopic world
- quantum objects have both wave and particle nature
- quantum objects often have discrete energies