Homework 2, due 2/6

1. Express the Hamiltonian of the He atom in the coordinate representation.

2. Prove that the eigenvalues of a Hermitian operator are real and the corresponding non-degenerate eigenfunctions are orthogonal.

3. The three components of the total angular momentum operator are

$$\hat{l}_x = \hat{y}\hat{p}_z - \hat{z}\hat{p}_y, \quad \hat{l}_y = \hat{z}\hat{p}_x - \hat{x}\hat{p}_z, \qquad \hat{l}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$$

Prove the following commutation relation:

$$[\hat{l}_x, \hat{l}_y] = i\hbar\hat{l}_z$$

using the definitions of the position and momentum operators in the coordinate representation.

4. Identify the eigenfunction(s) of the momentum operator ($\hat{p}_q$) from the following functions:

a. aq^2, b. aq, c. $A\sin kq$, d. De^{-aq}, e. $Be^{\beta q^2}$.

For the eigenfunction(s), give the eigenvalue(s).