Real-World Ratios

Modeling and Scaling Using a Spreadsheet

By William J. Sumrall

Subject: Modeling, ratios and proportions, problem solving

Audience: Teachers, teacher educators

Grade Level: K–12 (Ages 5–18)

Technology: Internet/Web, spreadsheets

Standards: *NETS•S* 3; *NETS•T* II, III (http://www.iste.org/standards)

Supplement: http://www.iste.org/LL/

Model building and scaling activities that emulate the activities that take place in the workforce present teachers with multiple instructional opportunities to address key components of the National Council of Teachers of Mathematics (NCTM) 2000 standards. For instance, spreadsheet/graphing skills are recommended as early as Grades 3–5 under the Data Analysis and Probability Standard. A strong focus on ratios and proportions is recommended as early as Grades 6–8 under the Number and Operations Standard. And, making connections such that mathematics is applied in contexts outside of mathematics is recommended for Grades PK–12 under the Connections Standard. Hence, making the connections between differing math concepts (e.g., geometric shapes, ratios and proportions, and algebraic manipulations) is an inherent part of both activities that use model building as a focus and those that adhere to the NCTM standards.

Scaling and model building are an integral part of many jobs. Aeronautical engineers build models of an airplane to test the stress, lift, and stability of the model prior to the actual building of a plane. Architects use models to improve designs, discuss changes, and provide an audience a 3-D view of a proposed project.

Making models, either smaller or larger than the actual object, requires a keen understanding of ratios and proportions. Also, a lot of models involve the understanding of geometric figures and shapes (e.g., a model of the Washington monument would require the builder to learn the formula and, more importantly, understand the properties of an obelisk). Furthermore, if multiple related models are being created (e.g., building furniture and people for a doll house or building a miniature city for a train station), then algebraic manipulations and problem solving are required.

Seeing the Value of a Spreadsheet

Because the size of a model can be dependent on the amount of materials available or the size of the space in which the model is being built (e.g., room, garage, or backyard) and can require extensive proportional calculations, a builder would be wise to use tools to accurately and quickly make various calculations. A computer spreadsheet offers tools with which a model builder can determine appropriate dimensions based on the materials and the size of a room or yard at hand. Furthermore, the computer spreadsheet enables the builder to quickly change dimensions if a series of models are being built as a unit. For example, by entering the dimensions of and the distances between every planet and the sun in the solar system, a student can use a computer spreadsheet to quickly change models of a solar system while remaining true to scale.

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Getting Started Using Road Sign Models

A good beginning for students is to build models based on spreadsheet calculations involving road and advertising sign measurements. Signs offer opportunities that involve measuring, geometric understanding, proportional reasoning, and algebraic manipulations. In addition, these uniquely shaped signs can be easily found in a community. I have approached this assignment using various strategies. Problem-based approaches where students create large road signs that fit a classroom wall or miniature signs that fit proportionally with model cars are two examples I've used where spreadsheets can come into use. Students should have skills in measuring length prior to beginning these activities and possibly the use of spreadsheets, but these activities could be used as a means to learn about spreadsheets.

Begin the lesson with a discussion of models. Display model planes, cars, dollhouses, trains, dolls, and so on. It is particularly effective to display large and small versions of the same kind (e.g., various dolls) to discuss accuracy and proportional measurements. Tell students you want them to locate a yield sign and measure its post height, sign height, bottom edge of sign width, and post width. Allow them to make any other measurements they deem appropriate as well (such as letter sizes). Set a standard unit (e.g., centimeters) so that data is measured in that unit. All other data received through different sources should be converted to your standard.

Have students working in pairs measure the dimensions of various other road signs or advertising signs as homework assignments. Encourage them to use various resources such as the Internet to find the formulas for an equilateral triangle (yield sign), a rectangle (miles per hour sign), and for the advanced students, a regular octagon (stop sign). Then have them develop a table that lists class average measurements for their different signs. Discuss setting a standard from those measurements and have them begin developing ratios that can be used to scale down or scale up their sign for model building.

Explain that understanding ratios and proportions is the foundation by which model building occurs. Simply stated, if one dimension on an object is changed (for example, the length of one edge of a yield sign), then other dimensions will change accordingly if the model is to look like a reduced or enlarged version of the original. Explain that model building allows one to scale an object up or down in size. The example in Table 1 displays data averaged for a class of students with ratios that can be used for modifying the dimensions of a yield sign. In the case of the yield sign, students developed various ratios. Prior to developing a spreadsheet, have students work in small groups to develop ratios based on class averages for a standard yield sign. If necessary, you may

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want to show them one ratio to get them started. One ratio they should develop involves the relationship between the original heights of the sign (Hso) and the post (Hpo). Another involves the relationship between the original height of the sign (Hso) and the original length of the bottom edge (Lbo). Similarly, the original width of the post (Wpo) that holds the sign is related to the original height of the post (Hpo). With all these dimensions identified through actual measurements, have students create a spreadsheet program to change dimensions based on changing the post height. Let them know that the more ratios they develop, the more accurate their model will be when constructed. Remind students that the ratios designed for the yield sign need to be connected such that by changing one variable (e.g., post height), all other variables change proportionally based on the ratios they have developed.

Using a Spreadsheet to Model a Yield Sign

The data inTable 2 was calculated in Microsoft Excel, but many other spreadsheet programs are available. Although formatting may differ, the formulas to derive answers will be the same for individual cells. Using the ratios in Table 1 as a source, students worked in groups to find the dimensions of the yield sign if a model had a new post height (Hpn) of 25 cm. Thus the formulas to determine various dimensions were:

New post width

```
Hpo/Wpo = Hpn/Wpn
25.3 = 25 cm/Wpn
Wpn = .99 cm or 9.9 mm
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New sign height

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Hpo/Hso = Hpn/Hsn
3.2 = 25cm/Hsn
Hsn = 7.81 cm or 78.1 mm
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New sign base length

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Hso/Lbo = Hsn/Lbn
.89 = 7.81 cm/Lbn
Lbn = 8.78 cm or 87.8 mm
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After saving the formulas into a spreadsheet, a student can quickly determine the proportional sizes if the post height is changed.

The beauty of using a spreadsheet and developing models is that detail, accuracy, and the speed at which different models are completed can be continuously maintained once the spreadsheet is built. Some of my students sought more detail and accuracy than was shown in the earlier example. Specifically, they included letter sizes in the word "yield" and holes in the post in some of the spreadsheet calculations and scaling processes.

Encouraging Problem Solving

To reinforce learning, students may develop additional spreadsheets based on other signs they measured. Tell the class you would like them to determine various ratios that can be used to build models around a toy car or doll. Thus, you would like for them to determine appropriately scaled dimensions for a chair, a yield sign, and one other item of their choice that would be proportional to their toy and to one another. As

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Table 1. Actual Yield Sign Measurements and Scaling Ratios (Numbers Based on Class Averages)

Actual Post Width (Wpo)	*Actual Post Height (Hpo)*	*Actual Length of Bottom Edge of Sign (Lbo)*	*Actual Height of Sign (Hso)*	*Ratio Hpo/Hso*	*Ratio Hso/Lbo*	*Ratio Hpo/Wpo*
7.9 cm	199 cm	70 cm	62 cm	3.2	.89	25.3

p = post s = sign o = original b = base W = width H = height L = length

Table 2. Yield Sign Dimensions Based on 25- and 50-cm Post Heights Using Numerical Data Calculated in a Spreadsheet

Model Post Height (Hpn)	*Model Post Width (Wpn)*	*Model Sign Height (Hsn)*	*Model Sign Base Length (Lbn)*
25.00 cm	0.99 cm	7.81 cm	8.78 cm
250.00 mm	9.88 mm	78.13 mm	87.78 mm
Model Post Height (Hpn)	*Model Post Width (Wpn)*	*Model Sign Height (Hsn)*	*Model Sign Base Length (Lbn)*
50.00 cm	1.98 cm	15.63 cm	17.56 cm
500.00 mm	19.76 mm	156.25 mm	175.56 mm

n = new

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a closure, have them create other proportional signs using paper, modeling clay, and so on.

Another approach for gaining an understanding of ratios and to encourage problem solving is to set parameters where students are required to fit their models into a box or onto a plane surface. For example, you could ask them to develop model dimensions if they were to reduce their classroom to Hobbit size by making it have only five-foot ceilings. Remind them that they need to be consistent with units (e.g., centimeters) and may need to make conversions. On completion, have them create spreadsheets that would allow them to change the ceiling height and other dimensions rapidly.

Extending the Activity

Although the development of scaling ratios is an important part of the model-building process, allow time for students to construct different spreadsheet-determined models. A good starting point is for students working in groups to construct a chair or a road sign using materials of their choice. The accuracy for detail as well as the number of models that can be developed is never ending once students have mastered the concept of ratios and spreadsheet use. Then, students can build scale models of famous structures (e.g., Taj Mahal, Buckingham Palace, the White House, Washington Monument, and Stonehenge). See the supplement at http://www.iste.org/LL/ for useful Web sites and connections to other subject areas as well as more information on projects such as scaling famous buildings and extending these lessons into cross-curricular projects that can include math, technology, geography, and social studies. An assignment and grading rubric is provided in the Web supplement describing requirements for an oral presentation.

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