Syllabus for Numerical Linear Algebra
Math/CS 576
Spring, 2019, University of New Mexico
Version 1, Dated January 8, 2018

Time and Place: Tuesday/Thursday, SMLC 352, 9:30am-10:45am

Course webpage: http://math.unm.edu/~schroder/2019_Spring_576/index.html

Instructor: Prof. Jacob B. Schroder, jbschroder@unm.edu

Course Credits: 3

Office Hours: SMLC 332

1. Wednesday: 9:30am-11am
2. Thursday: 1:00pm - 2:30pm
3. By request

Prerequisites: Math 504

Texts:

1. *Iterative Methods for Sparse Linear Systems*, 2nd Edition by Y. Saad (free) https://www-users.cs.umn.edu/~saad/IterMethBook_2ndEd.pdf
2. *A Multigrid Tutorial*, 2nd Edition by W. L. Briggs, V. E. Henson, S. F. McCormick (free) https://www.researchgate.net/publication/220690328_A_Multigrid_Tutorial_2nd_Edition

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Important Dates:

- Spring Break: March 10-17
- Final Project: *See **Final Project** section for details on what's due on each day.*
  - Topic Selection: April 5, Bring your topic to class!
  - Final Outline: April 15
  - **Present Outline to Class: April 16**
  - Final Progress on Numerics: April 26
  - Final Rough Draft: May 2
  - **Final Version Due: May 9**

Course Description: This is an introductory course to iterative methods for solving systems of (non)linear equations, focusing on Krylov and multigrid methods. The target systems of equations are large (in the millions or billions of degrees-of-freedom), sparse, and usually derived from discretized partial differential equations (PDEs). Traditional direct solution methods are impractical, so instead iterative solution methods must be used. Some possible application areas for Krylov and multigrid methods include (but are not limited to) elliptic PDEs, elasticity, fluid dynamics, and machine learning.

This is an advanced course for graduate students seeking to know how to efficiently solve large sparse linear systems, potentially for their own research.

Schedule of Topics:

- 1/6 Introduction
  - Sparse matrix storage formats: CSR, CSC, and LiL
  - Iterative methods background
  - Numerical PDEs (review)
- 2/6 Krylov Methods

    - Basic iterative methods (Jacobi, Gauss-Seidel, SOR)
    - One-step projection methods (steepest descent, minimum residual)
    - Arnoldi's method and GMRES
    - The symmetric Lanczos method and CG
    - Convergence properties of GMRES and CG
    - Other Krylov methods, e.g., BICGSTAB
- 1/6 Preconditioning
    - What is preconditioning?
    - Preconditioned CG and GMRES
    - Preconditioning with Jacobi, ILU, and SPAI
- 2/6 Multigrid methods
    - Geometric multigrid
    - Algebraic multigrid
    - Multigrid-in-time methods (Parareal, and multigrid-reduction-in-time)
- Additional topics may be covered at the instructor's discretion depending on time and student interest

Goals:

- Introduce students to the field of sparse iterative methods.
- Leave students with the necessary skills to choose and use appropriate iterative methods for various sparse matrix classes (i.e., problem types).
    - Such sparse matrix equations frequently appear in the sciences and engineering (especially when discretizing partial differential equations). Thus, this is a very useful skill.
- Conduct a course project on multigrid methods, or on an iterative solver topic from your own research.

Learning Outcomes:

- Give you a working knowledge of sparse matrices and how to efficiently store and manipulate such matrices on a computer
- Understand Krylov methods, both from the projection approach perspective, and an efficiency perspective.
- Understand the importance of preconditioning when efficiently using Krylov methods, and know how to choose preconditioners for common problem types
- Construct and test a working collection of iterative solvers (Krylov, multigrid, ...)
- Develop an understanding of spatial multigrid methods at a research level, by
    - Devoting at least 2/6 of the course lectures to multigrid methods,
    - Carrying out a course project on a multigrid method,
    - And as a part of that project, reading multigrid journal article(s)
- Give you a practical understanding of iterative and multigrid methods, with an emphasis on choosing the right solver for common problem types and application areas

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Grading: The course grade will be determined by

- Homework: 45%
- Participation: 10%
- Final Project: 45%

The final grade for the class will be based on the summed weighted percentages above. Letter grades will be assigned as follows:

- A, 90% or above
- B, 80% or above
- C, 70% or above
- D, 60% or above
- F, below 60%

The instructor reserves the right to curve grades to offset unforeseen circumstances. Such a curve will never decrease a student's letter grade below that from the above scheme.

Absences Policy: It is expected that each student regularly attend class, and participate by asking questions and commenting on the material.

- Course follows UNM handbook https://handbook.unm.edu/policies/section-d/d170.html
- Note, the handbook says "A student with excessive absences may be dropped from a course by the instructor with a grade of W/P or W/F."
- If you need to miss more than two classes during the semester, please contact the instructor. Conference attendance, illness, etc... are all excusable.
- For information on how absences affect your participation grade, see below.

Homework: Homework will be posted every 1 to 2 weeks on the course webpage, during the first part of course. Once the final project due dates start, other homework assignments will cease. Each homework will consist of computer and theoretical problems. You need to hand in a written report on the due date in class.

You are strongly encouraged to work in pairs (a group of two students) for the homework and hand in a single report, but it is expected that both of you can explain the theory and computer codes. Groups of more than two students are not allowed.

Late homeworks are not accepted; however, if an extenuating circumstance (hardship) arises, please let the instructor know before the homework due date (if at all possible). In such cases, the instructor will be flexible.

Homework grading disputes must be submitted in writing within one week after the work is returned.

Software: Use of numerical Python with the open-source PyAMG package will be required to complete the course homework assignments. For tips on installing PYAMG, please see https://github.com/pyamg/pyamg/wiki/Installing

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Participation: 10% of the course grade is from participation, which is calculated based a student's attendance and in-class participation (which includes in-class handouts). If you (1) attend each class, (2) pay attention, and (3) participate with any in-class hand-outs, then you will receive all 10%. Unexplained absences (beyond your two "freebies") will lower this grade.

Final Project: The final project will occur in stages over the latter part of the course, with various part-way due dates to keep everyone on-track. Please see the "**Important Dates**" section for the due dates on these project milestones.

- Topic Selection

    - Simply select a topic from your own research, from the class material, or in consultation with the instructor.
    - Due: A short abstract and title
- Final Outline
    - Due: A detailed outline of 1-2 pages, paying attention to the final rubrics (to be provided later).
- **Present Outline to Class**
    - Due: Give a 20-25 minute presentation to the class of your outline. You've likely read 1 or 2 papers on this topic by now, so use these paper(s) to give an overview of what the problem is, what previous approaches have tried, and what you plan to do.
    - The presentation can be a mixture of projecting your outline and/or slides, and work on the chalkboard.
- Final Progress on Numerics
    - Due: Your outline, but with a numerical results section with data and some exposition. Generally, you should stop doing numerical tests after this, and focus on writing.
- Final Rough Draft
    - Due: A rough draft in Latex using the provided SIAM style files (linked on the course webpage). It is anticipated that your writeup, with figures and references, will be 7-11 pages. If you will deviate from this length, check with the instructor.
    - This document will be like a conference paper. The final rubric will outline the required sections.
- **Final Version Due**
    - Due: A polished final draft of the above document.

You are strongly encouraged to work in pairs (a group of two students) for the project and hand in a single report, but it is expected that both of you can explain the theory and computer codes. Groups of more than two students are not allowed.

Academic Integrity: Each student is expected to maintain the highest standards of honesty and integrity in academic and professional matters. The University reserves the right to take disciplinary action, including dismissal, against any student who is found responsible for academic dishonesty or otherwise fails to meet the standards. Any student judged to have engaged in academic dishonesty in course work may receive a reduced or failing grade for the work in question and/or for the course.

Academic dishonesty includes, but is not limited to, dishonesty on quizzes, tests or assignments; claiming credit for work not done or done by others; and hindering the academic work of other students; misrepresenting academic or professional qualifications within or without the University; and nondisclosure or misrepresentation in filling out applications or other University records.

Accommodation Statement and Americans with Disabilities Act: In accordance with University Policy 2310 and the Americans with Disabilities Act (ADA), academic accommodations may be made for any student who notifies the instructor of the need for an accommodation. It is imperative that you take the initiative to bring such needs to the instructor's attention, as I am not legally permitted to inquire. Students who may require assistance in emergency evacuations should contact the instructor as to the most appropriate procedures to follow. Contact Accessibility Resource Center at 277-3506 for additional information.

Title IX: In an effort to meet obligations under Title IX, UNM faculty, Teaching Assistants (TAs), and Graduate Assistants (GAs) are considered "responsible employees" by the Department of Education (see page 15, http://www2.ed.gov/about/offices/list/ocr/docs/qa-201404-title-ix.pdf). This designation requires that any report of gender discrimination which includes sexual harassment, sexual misconduct and sexual violence made to a faculty member, TA, or GA must be reported to the Title IX Coordinator at the Office of Equal Opportunity (http://oeo.unm.edu). For more information on the campus policy regarding sexual misconduct, see: https://policy.unm.edu/university-policies/2000/2740.html

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Citizenship and/or Immigration Status: All students are welcome in this class regardless of citizenship, residency, or immigration status. Your professor will respect your privacy if you choose to disclose your status. As for all students in the class, family emergency-related absences are normally excused with reasonable notice to the professor, as noted in the attendance guidelines above. UNM as an institution has made a core commitment to the success of all our students, including members of our undocumented community. The Administration's welcome is found on our website: http://undocumented.unm.edu/.

Disclaimer: I reserve the right to make reasonable and necessary changes to the policies outlined in this syllabus. Whenever possible, the class will be notified in advance of such changes. An up-to-date copy of the syllabus can always be found on the course website. It is your responsibility to know and understand the policies discussed therein and to be up-to-date. If in doubt, please ask questions.