

Homework #4 - Due Nov. 1, 2016 - Before Class

1. Starting with the Master equation for a random walker moving via nearest neighbor rates F along the sites of an infinite chain, derive the Einstein formula relating the mean square displacement (msd) of the walker initially localized at one site to the elapsed time. Do this in three ways indicated in class: by solving for the probability propagator in the Fourier domain and differentiating it with respect to q a couple of times; solving for the probability propagator in real space and using Bessel function identities, and directly from the equation WITHOUT solving for the propagator.
2. Show explicitly by evaluating the propagator in terms of special functions in real space that the motion in the horizontal and vertical directions in a 2-dimensional counterpart of the nearest-neighbor chain (without oblique i.e. diagonal hops) are independent of each other.
3. Via a program written by you for a random walk on a 2d lattice show numerically that the Einstein relation that the msd goes linearly with time is valid. Please take a large number of steps of the walker (equivalently long time) and an ensemble of a huge number of realizations. Your program should place a walker and make it hop randomly left or right or up or down. Make a video for yourself and show the grader a realization of the walk.