505 STAT MECH TAKE-HOME (MIDTERM 2)

All questions (there are four of them) are equally weighted. Attempt them all. You may consult any notes or other source of information but not another person. You are starting a bit earlier than 2:30 pm on Thu Nov 17. You must finish in 48 hours. The deadline for submission of the answers (a pdf to kenkre@unm.edu with copy to your TA) is 2:30 pm on Sat Nov 19. Additionally put a hard copy in my mailbox on Monday Nov21. Feel free to email me for any clarification.- V. M. Kenkre

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1. Write down a Master equation to describe the nearest-neighbor motion of a random walker (on an infinite linear chain in 1-dimension) that is attracted towards a specific single site, the attraction being more intense the farther the walker is from the site, using rates that are linearly dependent on the number of sites the walker is away from the attractive center. Do a discrete Fourier transform of the Master equation to obtain a first order linear partial differential equation. Solve the equation for an initial condition that the walker is at a site two to the right of the attractor site. The aim is to obtain all probabilities in real space and the evolution and show a PLOT of the mean square displacement versus time. Note that the motion is NOT in the continuum. Writing continuum equations will give you no credit

2. Write down an equation for the time rate of change of the velocity of a 1-dimensional Brownian particle of mass m given that the random force is R(t) and the systematic (damping) force is opposed to the velocity and proportional to the CUBE of the velocity. Given an initial velocity v(0), calculate the time dependence of the energy of the particle at zero temperature (think what the temperature might have to do here!). Now try to solve the problem at finite temperature and explain what problem you run into which you would not have run into if the systematic force were to be linear in velocity. (B). Finally assume that the force is indeed linear (as in class) and solve for the average of the energy for arbitrary time using random properties of R(t) which you should state clearly.

3. A particle initially occupies the opposite corners of a square ring with all four sites equivalent in every way.Write down a Master equation of motion for the evolution of the probabilities of occupation in terms of a rate F that connects nearest-neighbor only sites. (A) Solve the equation for all probabilities for all times. Now assume that, rather than a Master equation, a Schroedinger equation governs the evolution of the amplitudes in the square, the rates being replaced by nearest-neighbor inter site matrix elements V, the four sites being otherwise (in the absence of V) degenerate (i.e. equi-energetic). (B) Solve the equation for the amplitudes for an initial condition identical to that in A and get the probabilities of occupation. Calculate and plot the H-function on the same plot for the two cases. Comment on what you see and its significance.

4. A walker moves on an infinite linear chain via nearest neighbor rates F except for the sites 0 and 1 between which the motion occurs with rate f (f is not equal to F). Formulate the description in terms of a Master equation and solve it in the Laplace domain explicitly for the initial condition that the walker is at site 0. Exhibit the Laplace transform of the difference in the probabilities at 0 and 1.