Further Notes for CM course Fall 2007

V. M. Kenkre

October 30, 2007

1 Anatomy of Fermi Golden Rule Derivation

One finds in typical texts the following derivation of the Fermi Golden Rule.

The Schroedinger equation allows one to write, if the Hamiltonian is $H = H_0 + V$ with H_0 having eigenstates labelled say by m, with energy $E_m =< m|H_0|m >$, the matrix elements $< m|V|n >$ being V_{mn},

$$\frac{dc_m(t)}{dt} = -iE_m c_m(t) - i\sum_n V_{mn}c_n \tag{1}$$

The transformation $C_m(t) = e^{iE_m t}c_m(t)$ removes the diagonal term and allows one to write the solution, for the initial condition that state I is intially occupied, as

$$C_m(t) = -i\int_0^t e^{-i\omega_{mn}s}\sum_n V_{mn}C_n(s)ds \tag{2}$$

Here m represents any initially *un*occupied state and ω_{mn} is the energy difference $E_m - E_n$.

So far the only assumptions made have been about the initial condition. We now assume that *the time and the interaction are small enough* so that the amplitude C_I of the initially occupied state remains close to 1 and all others remain close to 0. Then the summation collapses to a single term allowing one to write for the probability of occupation of the mth state $P_m(t) = C_m^*(t)C_m(t)$

$$P_m(t) = |-i\int_0^t e^{-i\omega_{mI}s}V_{mI}ds|^2 = |V_{mI}|^2|\frac{\sin^2(\omega_{mn}t/2)}{(\omega_{mn}/2)^2} \tag{3}$$

Notice that the probability of occupation of state m is oscillatory. If, however, one looks *for large enough times, the square of the sync function looks like a delta-function IN THE SPACE OF* ω_{mn}*!!*

This leads one to say that if the space of the energies E_m is continuous, one can talk about the probability of occupation of the mth state to be *linearly increasing* in time so that

$$\frac{dP_m(t)}{dt} = \left[const.|V_{mI}|^2|\delta(E_m - E_I)\right]$$

So what are the assumptions we have made?

One of short times. Another of long times. another of weak interactions V. Yet another of short times which you might have missed. Notice that if you take the last equation too seriously, the constant rate will make $P_m(t)$ increase linearly beyond 1 and mess you up. The earlier short time assumption was so that you could collapse the sum over n. The new short time assumption is so that the probability should not exceed 1. Both are mixed with a weak V assumption but not in a clear way. The intermediate long time assumption was so that you could make the sync function behave the way you wanted.

This is a mess but a nice mess looking closely at which might teach us a lot. This we will now do, first by invoking one of the most clear thinkers of physics, Pauli, then taking him down a notch for muddled thinking, learning passingly about the van Hove λ^2 limit, and then more seriously about the projection techniques of Zwanzig and how we should modify them to understand the Fermi Golden Rule and generally transport theory.