In quantum mechanics, when we are concerned with the transition between states, one approach which is commonly applied is the time-dependent theory. The Fermi golden rule is derived from the series expansion of the propagator $e^{\frac{-iHt}{\hbar}}$ to the first order. The higher order terms are neglected via the assumption that the interaction resulting in the transition is weak. To obtain the neat expression eventually, we also take advantage of several approximations which involve with time scales. In other words, the Fermi golden rule is restricted for the use in the proper time range to which you should keep in mind.

- Small interaction or short time approximation
  We already know the small interaction V makes the first order perturbation theory works. What this really means is that the time scale we are discussing must satisfies the condition $t \ll \hbar/V$. It stems from the fact that envision we have a two degenerate level system in which the interaction between states induces the periodic motion of the occupation probability given by the initial condition we place a particle at a certain one level. The occupation changes in time with the period $\pi\hbar/V$. However, this periodic motion is a reversible process rather than the irreversible transition in the sense The transition has to occur before the recurrence shows up. that is why we would like to make a short time approximation.

- Long time approximation
  This condition basically means the energy-time uncertainty principle. If time is long enough and difference between the initial and final state energy is extreme small, the *sinc* function could be treated as a delta function evaluated at $\omega_n - \omega_m = 0$.

- short time approximation
  The probability increases linearly in time so that we have to consider the short time scale to prevent the probability from blowing up.