Homework III (Due Wednesday, Aug 3) -Solutions

1. Exercises 3.5.1 and 3.5.2 on page 83 (10th Ed)

2. Review exercises 9 (page 88) and 22 (page 90)

3.5.1 A medical research team wishes to assess the usefulness of a certain symptom (call it S) in the diagnosis of a particular disease. In a random sample of 775 patients with the disease, 744 reported having the symptom. In an independent random sample of 1380 subjects without the disease, 21 reported that they had the symptom.
(a) In the context of this exercise, what is a false positive?
(b) What is a false negative?
(c) Compute the sensitivity of the symptom.
(d) Compute the specificity of the symptom.
(e) Suppose it is known that the rate of the disease in the general population is. 001. What is the predictive value positive of the symptom?
(f) What is the predictive value negative of the symptom?
(g) Find the predictive value positive and the predictive value negative for the symptom for the following hypothetical disease rates: .0001, .01, and .10.
(h) What do you conclude about the predictive value of the symptom on the basis of the results obtained in part g?

Solution:

	Disease (D)	No Disease($\bar{D}$)	Total
Symptom (S)	744	21	765
No Symptom ($\bar{s}$)	31	1359	1390
Total	775	1380	2155

.(a) *Had symptom but not having the disease*

.(b) *Not having symptom but had the disease.*

$.(c)\ \text{sensitivity of the symptom} = \mathrm{P(S|D)} = \frac{744}{775} = .96$

$.(d)\ \text{specificity of the symptom} = \mathrm{P(\bar{S}|\bar{D})} = \frac{1359}{1380}$

$.(e)\ P(D) = .001, P(S|\bar{D}) = \frac{21}{1380}, P(\bar{D}) = 1 - P(D) = 1 - .001 = .999\ \ then$

$$P(D|S) = \frac{P(S|D)P(D)}{P(S)} = \frac{P(S|D)P(D)}{P(S|D)P(D) + P(S|\bar{D})P(\bar{D})} = \frac{.96 \times .001}{.96 \times .001 + .015 \times .999} = .0595$$

$.(f)\ P(D) = .001, P(\bar{S}|D) = \frac{31}{775}, P(\bar{D}) = 1 - P(D) = 1 - .001 = .999\ \ then$

$$P(\bar{D}|\bar{S}) = \frac{P(\bar{S}|\bar{D})P(\bar{D})}{P(\bar{S})} = \frac{P(\bar{S}|\bar{D})P(\bar{D})}{P(\bar{S}|\bar{D})P(\bar{D}) + P(\bar{S}|D)P(D)} = \frac{.9848 \times .999}{.9848 \times .999 + .04 \times .001} = .99996$$

$.(g)\ for\ P(D) = .0001$

$$P(D|S) = \frac{P(S|D)P(D)}{P(S|D)P(D) + P(S|\bar{D})P(\bar{D})} = \frac{.96 \times .0001}{.96 \times .0001 + .015 \times .9999} = .0063$$

$$P(\bar{D}|\bar{S}) = \frac{P(\bar{S}|\bar{D})P(\bar{D})}{P(\bar{S}|\bar{D})P(\bar{D}) + P(\bar{S}|D)P(D)} = \frac{.9848 \times .9999}{.9848 \times .9999 + .04 \times .0001} = .999996$$

$.\ for\ P(D) = .01$

$$P(D|S) = \frac{P(S|D)P(D)}{P(S|D)P(D) + P(S|\bar{D})P(\bar{D})} = \frac{.96 \times .01}{.96 \times .01 + .015 \times .99} = .3895$$

$$P(\bar{D}|\bar{S}) = \frac{P(\bar{S}|\bar{D})P(\bar{D})}{P(\bar{S}|\bar{D})P(\bar{D}) + P(\bar{S}|D)P(D)} = \frac{.9848 \times .99}{.9848 \times .99 + .04 \times .01} = .9996$$

$. for\ P(D) = .10$

$$P(D|S) = \frac{P(S|D)P(D)}{P(S|D)P(D) + P(S|\bar{D})P(\bar{D})} = \frac{.96 \times .10}{.96 \times .10 + .015 \times .90} = .875$$

$$P(\bar{D}|\bar{S}) = \frac{P(\bar{S}|\bar{D})P(\bar{D})}{P(\bar{S}|\bar{D})P(\bar{D}) + P(\bar{S}|D)P(D)} = \frac{.9848 \times .90}{.9848 \times .90 + .04 \times .10} = .996$$

$.(h)$ Conclusion: the values increase as the disease rate increase.

3.5.2 In an article entitled "Bucket-Handle Meniscal Tears of the Knee: Sensitivity and Specificity of MRI signs," Dorsay and Helms (A-6) performed a retrospective study of 71 knees scanned by MRI. One of the indicators they examined was the absence of the "bow-tie sign" in the MRI as evidence of a bucket-handle or "bucket-handle type" tear of the meniscus. In the study, surgery confirmed that 43 of the 71 cases were bucket-handle tears. The cases may be cross-classified by "bow-tie sign" status and surgical results as follows:

	Tear Surgically Confirmed (D)	Tear Surgically Confirmed As Not Present ($\bar{D}$)	Total
Positive Test (absent bow-tie sign) (T)	38	10	48
Negative Test (bow-tie sign present) ($\bar{T}$)	5	18	23
Total	43	28	71

Source: Theodore A. Dorsay and Clyde A. Helms, "Bucket-handle Meniscal Tears of the Knee: Sensitivity and Specificity of MRI Signs," *Skeletal Radiology*, 32 (2003), 266–272.

(a) What is the sensitivity of testing to see if the absent bow tie sign indicates a meniscal tear?
(b) What is the specificity of testing to see if the absent bow tie sign indicates a meniscal tear?
(c) What additional information would you need to determine the predictive value of the test?

Solution:

$.(a)\ sensitivity = P(T|D) = \frac{38}{43}$

$.(b)\ specificity = P(\bar{T}|\bar{D}) = \frac{18}{28}$

$.(c)\ P(Bucket - Handle\ Tear)$

9. For a variety of reasons, self-reported disease outcomes are frequently used without verification in epidemiologic research. In a study by Parikh-Patel et al. (A-12), researchers looked at the relationship between self-reported cancer cases and actual cases. They used the self-reported cancer data from a California Teachers Study and validated the cancer cases by using the California Cancer Registry data. The following table reports their findings for breast cancer:

Cancer Reported (A)	Cancer in Registry (B)	Cancer Not in Registry	Total
Yes	2991	2244	5235
No	112	115849	115961
Total	3103	118093	121196

(a) Let A be the event of reporting breast cancer in the California Teachers Study. Find the probability of A in this study. $Ans: P(A) = \frac{5235}{121196}$

(b) Let B be the event of having breast cancer confirmed in the California Cancer Registry. Find the probability of B in this study. $Ans: P(B) = \frac{3103}{121196}$

(c) Find $P(A \cap B)$ $Ans: P(A \cap B) = \frac{2991}{121196}$

(d) Find $P(A|B)$ $Ans: P(A|B) = \frac{2991}{3103}$

(e) Find $P(B|A)$ $Ans: P(B|A) = \frac{2991}{5235}$

(f) Find the sensitivity of using self-reported breast cancer as a predictor of actual breast cancer in the California registry. $Ans: Sensitivity = P(A|B) = \frac{2991}{3103} = .964$

(g) Find the specificity of using self-reported breast cancer as a predictor of actual breast cancer in the California registry. $Ans: Specificity = P(\bar{A}|\bar{B}) = \frac{115849}{118093} = 0.981$

22. Verma et al. (A-14) examined the use of heparin-PF4 ELISA screening for heparin-induced thrombocytopenia (HIT) in critically ill patients. Using C-serotonin release assay (SRA) as the way of validating HIT, the authors found that in 31 patients tested negative by SRA, 22 also tested negative by heparin-PF4 ELISA.

(a) Calculate the specificity of the heparin-PF4 ELISA testing for HIT.

(b) Using a "literature derived sensitivity" of 95 percent and a prior probability of HIT occurrence as 3.1 percent find the positive predictive value.

(c) Using the same information as part (b), find the negative predictive value.

Solution:

Based on the study SRA validated that 33 does not have the disease, which means or assumes that 31 patients do not have the disease, that is $\bar{D} = 31$. Further, PF4 ELISA test showed that 22 of those 31 do not have the disease. Then

$.(a)\ \ P(\bar{T}\ |\bar{D}) = \frac{22}{31} = .71$

$.(b)\ P(T|D) = .95\ and\ P(D) = .031\ then\ P(D|T) = \frac{P(T|D)P(D)}{P(T|D)P(D)+P(T|\bar{D})P(\bar{D})}$

$$= \frac{.95 \times .031}{.95 \times .031 + (1-.71)(1-.031)} = .095$$

$.(c)\ P(\bar{D}|\bar{T}) = \frac{P(\bar{T}|\bar{D})P(\bar{D})}{P(\bar{T}|\bar{D})P(\bar{D})+P(\bar{T}|D)P(D)} \quad = \frac{.71\times(1-.031)}{.71\times(1-.031)+(1-.95)\times.031} = .998$