Solution: BIOM 559 Homework 05 Due date: August 17, 2020

- Exercises 4.3.4, 4.3.5 and 4.3.6 on page 108
- Exercises 4.4.2, 4.4.4 and 4.4.5 on page 113
- Review Exercise 24 on page 131
- Review Exercises 33, 34, 35 on page 132

4.3.4 The same survey database cited in exercise 4.3.1 (A-5) shows that 32 percent of U.S. adults indicated that they have been tested for HIV at some point in their life. Consider a simple random sample of 15 adults selected at that time. Find the probability that the number of adults who have been tested for HIV in the sample would be:

(a) Three. Ans: $P(X=3)=\binom{15}{3}\times .32^3\times(1-.32)^{15-3}=disp\ binomialp(15,3,.32)=.14574$

(b) Less than five

Ans: $P(X<5)=\sum_{x=0}^{4}\binom{15}{x}\times .32^x\times(1-.32)^{15-x}\ =disp\ binomial(15,4,.32)=.4477$

(c) Between five and nine, inclusive

Ans: $P(5\le X\le 9)=P(4<X\le 9)=disp\ \ binomial(15,9,.32)-binomial(15,4,.32)=.5461$

(d) More than five, but less than 10.

Ans: $P(5<X<10)=P(5<X\le 9)=disp\ \ binomial(15,9,.32)-binomial(15,5,.32)=.3331$

(e) Six or more.

Ans: $P(X\ge 6)=1-P(X<6)=1-P(X\le 5)=disp\ \ 1-binomial(15,5,.32)=.3393$

4.3.5 Refer to Exercise 4.3.4. Find the mean and variance of the number of people tested for HIV in samples of size 15.

Solution: $\mu=np=15\times .32=4.8\ and\ \sigma^2=np(1-p)=15\times .32\times .68=3.624$

4.3.6 Refer to Exercise 4.3.4. Suppose we were to take a simple random sample of 25 adults today and find that two have been tested for HIV at some point in their life. Would these results be surprising? Why or why not? Ans: $P(X=2)=disp\ \ binomial(25,2,.32)=.0043$ or $P_{value}=P(X\ge 2)=1-P(X<2)=1-P(X\le 1)=disp\ 1-\ \ binomial(25,1,.32)=.9992$. Yes, it will be a surprise for getting exactly 2 today from a sample of 25 since the probability is too low or P_value is NOT significant.

4.4.2 Tubert-Bitter et al. (A-9) found that the number of serious gastrointestinal reactions reported to the British Committee on Safety of Medicine was 538 for 9,160,000 prescriptions of the anti-inflammatory drug piroxicam. This corresponds to a rate of .058 gastrointestinal reactions per 1000 prescriptions written. Using a Poisson model for probability, with $\lambda=.06$, find the probability of

(a) Exactly one gastrointestinal reaction in 1000 prescriptions

Ans: $P(X=1)=\frac{e^{-.06}\times .06^1}{1!}=disp\ poissonp(.06,1)=.057$

(b) Exactly two gastrointestinal reactions in 1000 prescriptions

(c) Ans: $P(X=2)=\frac{e^{-.06}\times .06^2}{2!}=disp\ poissonp(.06,2)=.0017$

(d) No gastrointestinal reactions in 1000 prescriptions

Ans: $P(X=0)=\frac{e^{-.06}\times .06^0}{0!}=disp\ poissonp(.06,0)=.942$

(e) At least one gastrointestinal reaction in 1000 prescriptions

Ans: $P(X\ge 1)=1-P(X<1)=1-P(X=0)=disp\ 1-.942=.058$

4.4.4 In a study of the effectiveness of an insecticide against a certain insect, a large area of land was sprayed. Later the area was examined for live insects by randomly selecting squares and counting the number of live insects per square. Past experience has shown the average number of live insects per square after spraying to be .5. If the number of live insects per square follows a Poisson distribution, find the probability that a selected square will contain:

(a) Exactly one live insect Ans: $P(X = 1) = \frac{e^{-.5} \times .5^1}{1!} = disp\ poissonp(.5,1) = .303$

(b) No live insects Ans: $P(X = 1) = \frac{e^{-.5} \times .5^0}{1!} = disp\ poissonp(.5,0) = .607$

(c) Exactly four live insects Ans: $P(X = 4) = disp\ poissonp(.5,4) = .0016$

(d) One or more live insects Ans: $P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) = 1 - .607 = .393$

4.4.5 In a certain population an average of 13 new cases of esophageal cancer are diagnosed each year. If the annual incidence of esophageal cancer follows a Poisson distribution, find the probability that in a given year the number of newly diagnosed cases of esophageal cancer will be:

(a) Exactly 10 Ans: $P(X = 10) = \frac{e^{-13} \times 13^{10}}{10!} = disp\ poissonp(13,10) = .086$

(b) At least eight Ans: $P(X \geq 8) = 1 - P(X < 8) = 1 - P(X \leq 7) = disp\ 1 - \ poisson(13,7) = .946$

(c) No more than 12 Ans: $P(X \leq 12) = disp\ poisson(13,12) = .463$

(d) Between nine and 15, inclusive Ans: $P(9 \leq X \leq 15) = P(X \leq 15) - P(X < 9) = P(X \leq 15) - P(X \leq 8) = disp\ poisson(13,15) - poisson(13,8) = .664$

(e) Fewer than seven Ans: $P(X < 7) = P(X \leq 6) = disp\ poisson(13,6) = .026$

24. Given a binomial variable with a mean of 20 and a variance of 16, find n and p.

Solution:

$$\mu = np = 20$$
$$\sigma^2 = np(1-p) = 16$$
$$\therefore 16 = 20 \times (1-p) \to (1-p) = \frac{16}{20} \Longrightarrow p = 1 - \frac{4}{5} = \frac{1}{5}$$

Also, $$20 = np = n \times \tfrac{1}{5} \Longrightarrow n = 100$$

33. Explain why each of the following measurements is or is not the result of a Bernoulli trial:

(a) The gender of a newborn child Ans: yes, is a binomial with n = 1 if we assume that gender (boy or girl) has 50-50 chance.

(b) The classification of a hospital patient's condition as stable, critical, fair, good, or poor. Ans: Not a binomial since the outcomes are not dichotomized.

(c) The weight in grams of a newborn child. Ans: Not a binomial since the outcomes take values that are not integers.

34. Explain why each of the following measurements is or is not the result of a Bernoulli trial:

(a) The number of surgical procedures performed in a hospital in a week. Ans: Not a binomial since the outcomes are not dichotomized.

(b) A hospital patient's temperature in degrees Celsius. Ans: Not a binomial since the outcomes take values that are not integers.

(c) A hospital patient's vital signs recorded as normal or not normal. Ans: a binomial since the outcomes are dichotomized.

35. Explain why each of the following distributions is or is not a probability distribution:

(a)

x	$P(X = x)$
0	0.15
1	0.25
2	0.10
3	0.25
4	0.30

(b)

x	$P(X = x)$
0	0.15
1	0.20
2	0.30
3	0.10

(c)

x	$P(X = x)$
0	0.15
1	−0.20
2	0.30
3	0.20
4	0.15

(d)

x	$P(X = x)$
−1	0.15
0	0.30
1	0.20
2	0.15
3	0.10
4	0.10

(a). Ans: Not a probability dist. since sum of the probabilities is greater than 1.
(b). Ans: Not a probability dist. since sum of the probabilities is not 1, it is less than 1.
(c). Ans: Not a probability dist. since one of the probabilities is less than 0.
(d). Ans: It is a probability dist. since each variable has corresponding prob. And sum of the probabilities is 1.