Solution: Homework 07&08

Due August 24, 2020

- Exercises 4.7.3 and 4.7.5 (Page 128)

- Review exercise 27 (page 132)

- Supplement exercise. Based on a survey conducted in 1988 – 1994, the distribution of height of males in the 20 – 29 age bracket (U.S. Census Bureau 1999) has the mean 69.3 (inches) and standard deviation 2.92 and follows a normal distribution reasonably well.

- If a male is picked at random from that population, what is the probability that (1) he is higher than 71 inches, (2) his height is somewhere between 67 and 69 inches?

- If 10 males were picked at random, (3) what is the probability that at least 5 of them are higher than the average height and (4) what is the probability that at least 5 of them are higher than 71 inches? (hint: you need to use both normal and binomial distributions)

- Exercises 5.3.8 (Page 145, 10th Ed)

4.7.3 One of the variables collected in the North Carolina Birth Registry data (A-3) is pounds gained during pregnancy. According to data from the entire registry for 2001, the number of pounds gained during pregnancy was approximately normally distributed with a mean of 30.23 pounds and a standard deviation of 13.84 pounds. Calculate the probability that a randomly selected mother in North Carolina in 2001 gained:
(a) Less than 15 pounds during pregnancy (b) More than 40 pounds
(c) Between 14 and 40 pounds (d) Less than 10 pounds
(e) Between 10 and 20 pounds

Solutions: $\mu = 30.23 \text{ and } \sigma = 13.84$

(a) $P(X < 15) = P\left(\frac{X-30.23}{13.84} < \frac{15-30.23}{13.84}\right) = P\left(Z < \frac{15-30.23}{13.84}\right) = disp\ normal\left(\frac{15-30.23}{13.84}\right) = .1356$

(b). $P(X > 40) = P\left(\frac{X-30.23}{13.84} > \frac{40-30.23}{13.84}\right) = 1 - P\left(Z \le \frac{40-30.23}{13.84}\right) = disp\ 1 - normal\left(\frac{40-30.23}{13.84}\right) = .241$

(c). $P(14 < X < 40) = P\left(\frac{14-30.23}{13.84} < \frac{X-30.23}{13.84} < \frac{40-30.23}{13.84}\right) = P\left(\frac{14-30.23}{13.84} < Z < \frac{40-30.23}{13.84}\right)$

$$= P\left(Z < \frac{40-30.23}{13.84}\right) - P\left(Z < \frac{14-30.23}{13.84}\right) = disp\ normal\left(\frac{40-30.23}{13.84}\right) - normal\left(\frac{14-30.23}{13.84}\right) = .639$$

(d). $P(X < 10) = P\left(\frac{X-30.23}{13.84} < \frac{10-30.23}{13.84}\right) = P\left(Z < \frac{10-30.23}{13.84}\right) = disp\ normal\left(\frac{10-30.23}{13.84}\right) = .072$

(e). $P(10 < X < 20) = disp\ normal\left(\frac{20-30.23}{13.84}\right) - normal\left(\frac{10-30.23}{13.84}\right) = .158$

4.7.5 If the total cholesterol values for a certain population are approximately normally distributed with a mean of 200 mg/100 ml and a standard deviation of 20 mg/100 ml, find the probability that an individual picked at random from this population will have a cholesterol value:
(a) Between 180 and 200 mg/100 ml (b) Greater than 225 mg/100 ml
(c) Less than 150 mg/100 ml (d) Between 190 and 210 mg/100 ml

Solutions: $\mu = 200mg/100ml \text{ and } \sigma = 20mg/100ml$

(a) $P(180 < X < 200) = disp\ normal\left(\frac{200-200}{20}\right) - normal\left(\frac{180-200}{20}\right) = .341$

(b). $P(X > 225) = disp\ 1 - \ normal\left(\frac{225-200}{20}\right) = .106$

(c). $P(X < 150) = disp\ \ normal\left(\frac{150-200}{20}\right) = .0062$

(d). $P(190 < X < 210) = disp\ normal\left(\frac{210-200}{20}\right) - normal\left(\frac{190-200}{20}\right) = .383$

27. Given the normally distributed random variable X with mean 100 and standard deviation 15, find the numerical value of k such that:

(a) $P(X \leq k) = .0094$
(b) $P(X \geq k) = .1093$
(c) $P(100 \leq X \leq k) = .4778$
(d) $P(k' \leq X \leq k) = .9660$, where k' and k are equidistant from μ

Solutions: $\mu = 100 \text{ and } \sigma = 15$

(a). $P(X \leq k) = .0094 = disp\ \ normal\left(\frac{k-100}{15}\right), then\ \frac{k-100}{15} = disp\ invnormal(.0094) = -2.35$

$$\therefore k = -2.35 \times 15 + 100 = 64.75$$

(b). $P(X \geq k) = .1093 = 1 - P(X < k) \rightarrow P(X < k) = 1 - .1093 = .8907$

$$= disp\ \ normal\left(\frac{k-100}{15}\right), then\ \frac{k-100}{15} = disp\ invnormal(.8907) = 1.23$$

$$\therefore k = 1.23 \times 15 + 100 = 118.45$$

(c). $P(100 \leq X \leq k) = .4778 = disp\ \ normal\left(\frac{k-100}{15}\right) - normal\left(\frac{100-100}{15}\right) \rightarrow$
$disp\ \ normal\left(\frac{k-100}{15}\right) = disp\ .4778 + normal\left(\frac{100-100}{15}\right) = .9778,$

$$then\ \frac{k-100}{15} = disp\ invnormal(.9778) = 2.01$$

$$\therefore k = 2.01 \times 15 + 100 = 130.15$$

(d). (c). $P(k' \leq X \leq k) = .9660 = disp\ \ normal\left(\frac{k-100}{15}\right) - normal\left(\frac{k'-100}{15}\right)$

But k and k' are equidistant from center. Then $\Phi(z') = 1 - \Phi(z), where\ z = \frac{k-100}{15}$. So,

$$.9660 = P(X \leq k) - P(X \leq k') = P(X \leq k) - \big(1 - P(X \leq k)\big) = 2P(X \leq k) - 1 \rightarrow 2P(X \leq k)$$
$$= 1 + .9660 = 1.9660 \rightarrow P(X \leq k) = \frac{1.9660}{2} = .983$$

$\rightarrow disp\ \ normal\left(\frac{k-100}{15}\right) = .983, then\ \frac{k-100}{15} = disp\ invnormal(.983) = 2.12$

$$\therefore k = 2.12 \times 15 + 100 = 131.8$$

Supplement exercise. Based on a survey conducted in 1988 – 1994, the distribution of height of males in the 20 – 29 age bracket (U.S. Census Bureau 1999) has the mean 69.3 (inches) and standard deviation 2.92 and follows a normal distribution reasonably well.

• If a male is picked at random from that population, what is the probability that (1) he is higher than 71 inches, (2) his height is somewhere between 67 and 69 inches?

• If 10 males were picked at random, (3) what is the probability that at least 5 of them are higher than the average height and (4) what is the probability that at least 5 of them are higher than 71 inches? (hint: you need to use both normal and binomial distributions)

Solutions: $\mu = 69.3\ and\ \sigma = 2.92$

(1) $P(X > 71) = disp\ 1 - normal\left(\frac{71-69.3}{2.92}\right) = .280$

(2). $P(67 < X < 69) = disp\ normal\left(\frac{69-69.3}{2.92}\right) - normal\left(\frac{67-69.3}{2.92}\right) = .244$

(c). $Sample\ size\ (n) = 10, X > average\ and\ M \geq 5.$

$.First, use\ normal\ to\ find\ the\ probability\ of\ success.$

$$So, \qquad P(X > average) = P(X > 69.3) = disp\ 1 - normal(0) = .5$$

$.Then\ use\ binomial\ dist.\ to\ find\ the\ prob.\ that\ M \geq 5.$

$$So, \qquad P(M \geq 5) = 1 - P(M < 5) = 1 - P(M \leq 4) = disp\ 1 - binomial(10,4,.5) = .623$$

(d). $Sample\ size\ (n) = 10, X > 71\ and\ M \geq 5.$

$.First, use\ normal\ to\ find\ the\ probability\ of\ success.$

$$So, \qquad P(X > 71) = disp\ 1 - normal\left(\frac{71 - 69.3}{2.92}\right) = .280$$

$.Then\ use\ binomial\ dist.\ to\ find\ the\ prob.\ that\ M \geq 5.$

$$So, \qquad P(M \geq 5) = 1 - P(M < 5) = 1 - P(M \leq 4) = disp\ 1 - binomial(10,4,.280) = .118$$

5.3.8 Suppose a population consists of the following values: 1, 3, 5, 7, 9. Construct the sampling distribution of $\bar{X}$ based on samples of size 2 selected without replacement. Find the mean and variance of the sampling distribution.

Solution: Sampling condition is without replacement and one at a time.

Using table, we displayed all possible samples of size 2 and values of $\bar{X}$.

		2^{nd} pick (say, X_2)				
		1	3	5	7	9
	1		$1{,}3 \to \bar{x} = 2$	$1{,}5 \to \bar{x} = 3$	$1{,}7 \to \bar{x} = 4$	$1{,}9 \to \bar{x} = 5$
	3			$3{,}5 \to \bar{x} = 4$	$3{,}7 \to \bar{x} = 5$	$3{,}9 \to \bar{x} = 6$
1^{st} pick (say, X_1)	5				$5{,}7 \to \bar{x} = 6$	$5{,}9 \to \bar{x} = 7$
	7					$7{,}9 \to \bar{x} = 8$
	9					

Number of possible outcomes is ${}^{5}C_2 = 10$, then

$$Mean = E(\bar{X}) = \sum \bar{x} P(\bar{x}) = (2 + 3 + 7 + 8) \times \frac{1}{10} + (4 + 5 + 6) \times \frac{2}{10} = 5$$

$$Variance = E\left(\bar{X} - E(\bar{X})\right)^2 = \sum\left(\bar{x} - E(\bar{X})\right)^2 P(\bar{x})$$

$$= ((2-5)^2 + (3-5)^2 + (7-5)^2 + (8-5)^2) \times \frac{1}{10}$$

$$+ ((4-5)^2 + (5-5)^2 + (6-5)^2) \times \frac{2}{10} = 3$$