

## Wilcoxon Signed Rank Test

---

This is another test that is a non-parametric equivalent of a 1-Sample  $t$ -test. The Wilcoxon Signed Rank procedure assumes that the sample we have is randomly taken from a population, with a symmetric frequency distribution. The symmetric assumption does not assume normality, simply that there seems to be roughly the same number of values above and below the median. The Wilcoxon procedure computes a test statistic  $W_{STAT}$  that is compared to an expected value.  $W_{STAT}$  is computed by summing the ranked differences of the deviation of each variable from a hypothesized median above the hypothesized value. The easiest way to illustrate this is through an example (data taken from Binford 2002).

The distribution of hunter-gatherer population densities ( $N = 86$ ) across all forest ecosystems worldwide is skewed to the right and is non-normal. The median is therefore the most reliable measure of central tendency. As such, the median population density (per 100 km) of forest hunter-gatherers is  $\eta_o = 7.38$ . An interesting question that we may want to ask is whether this value is an accurate estimate of the population density of forest hunter-gatherers on specific continents; the results might answer the question of whether hunter-gatherer population densities are determined primarily by large-scale ecological constraints (such as resource availability), or whether there seem to be other factors, possibly social and/or historic, determining population density. For this example we will look at the hunter-gatherer groups of the northern Australian forests ( $n = 13$ ).

Let  $\eta_o$  be the median population density of all forest hunter-gatherer groups ( $N = 86$ ), where  $\eta_o = 7.38$ , and let  $\eta$  be the median population density for Australian forest hunter-gatherer groups ( $n = 13$ ).

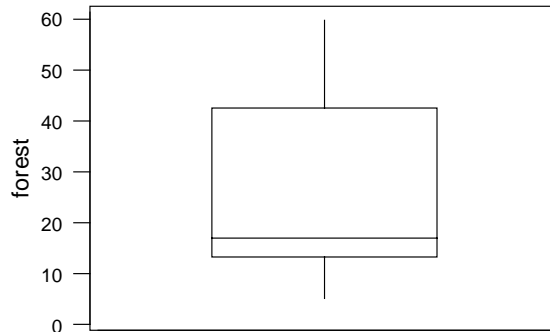
Formally, we wish to test the hypothesis at the  $\alpha = 0.05$  (95%) level:

$$H_o : \eta_o = \eta$$

$$H_A : \eta_o \neq \eta$$

name	$X_i$	$X_i - 7.38$	Sign	$ X_i - 7.38 $	Rank	Sign*Rank
jeidji	17	9.62	+	9.62	7	7
kuku	50	42.62	+	42.62	12	12
mamu	45	37.62	+	37.62	11	11
ngatjan	59.8	52.42	+	52.42	13	13
undanbi	21.74	14.36	+	14.36	8	8
jinibarra	16	8.62	+	8.62	6	6
ualaria	9	1.62	+	1.62	1	1
barkindji	15.43	8.05	+	8.05	5	5
wongaibon	5.12	-2.26	-	2.26	2	-2
jaralde	40	32.62	+	32.62	10	10
tjapwurong	35	27.62	+	27.62	9	9
tasmanians	13.35	5.97	+	5.97	3	3
badjalang	13.4	6.02	+	6.02	4	4

A boxplot of the raw data (represented on column  $X_i$ ) shows approximate symmetry:



The above table demonstrates the series of calculations for the  $W_{STAT}$ :

Column 1: the raw data ( $X_i$ )

Column 2: each variable – the hypothesized value ( $X_i - \eta_o$ )

Column 3: the sign of column 2 (+ or -)

Column 4: the absolute value of column 2 ( $\text{abs}(X_i - \eta_o)$ )

Column 5: the rank of column 4 in ascending order ( $1 \dots n$ )

Column 6: column 5 \* the sign of column 3

$W_{STAT}$  is computed by summing the non-negative values of column 6:

$$W_{STAT} = 7 + 12 + 11 + 13 + 8 + 6 + 1 + 5 + 10 + 9 + 3 + 4 = 89.$$

The  $W_{STAT}$  is then compared to an expected value. The sum of all ranks is given by  $0.5n(n + 1)$ , where  $n$  = sample size: if  $H_0$  is true you would expect half of the observations to be above  $\eta_o$  because of the assumption of symmetry. This being the case the expected value of  $W_{STAT}$  is computed as  $E(W_{STAT}) = 0.5 * 0.5n(n + 1)$ , where  $E(x)$  denotes the *expectation* of  $x$ .

In our case the sum of all ranks =  $0.5 * 13 * 14 = 91$ , and  $E(W_{STAT}) = 0.5 * 0.5 * 13 * 14 = 45.5$ .

The question now is how far from the expected value is our observed median? At this point we need to resort to MINITAB. This is because our observed median is not the median we could get from the descriptive statistics output but is estimated using a *Walsh Average* due to the discrete nature of the data (see the MINITAB help). MINITAB also calculates a  $p$  value based on a specific type of correction to the normal curve.

In MINITAB the procedure is:

```

>STAT
  >NON-PARAMETRICS
    >1 SAMPLE WILCOXON
      >Put your data into the VARIABLES box
        >Click TEST MEDIAN and input the hypothesized value
          >Leave the alternative NOT EQUAL
            >OK
  
```

Which gives the following output:

### Wilcoxon Signed Rank Test

Test of median = 7.380 versus median not = 7.380

	N	N for Test	Wilcoxon Statistic	P	Estimated Median
forest	13	13	89.0	0.003	26.00

You can also choose to calculate a confidence interval by choosing the CONFIDENCE INTERVAL option in the dialog box. This gives the output:

### Wilcoxon Signed Rank Confidence Interval

	N	Estimated Median	Achieved Confidence	Confidence Interval
forest	13	26.0	95.0	( 14.4, 37.5)

Here we see our sample size  $n = 13$ , the *N for Test* is the sample size minus values that equal the hypothesized median (in our case none), The *Wilcoxon Statistic* = 89 (the same as our hand calculation) and the *Estimated Median* is the Walsh average. So in this case MINITAB is testing whether a value of 26 is different enough from 7.38 at the  $\alpha = 0.05$  level to be statistically significant, and as the resulting  $p = 0.003$ ,  $p < \alpha$  and so we would reject the null hypothesis in favor of the alternative. For the confidence interval, MINITAB calculates this in the same way as in the *signed rank test* hence the *Achieved Confidence* as our stated  $\alpha$  level is not always possible. In this case we see the lower limit around the estimated median of 26 is 14.4, and the upper bound is 37.5. These bounds do not encompass the hypothesized value of 7.38 and so we would reject our null hypothesis. That is to say the median population density of forest-dwelling hunter-gatherer groups in northern Australia is significantly different from the global median of similar groups. As our estimated median is higher than the hypothesized value we would conclude that “on average” (because we can’t say “on median”) Australian forest groups are much denser than the global average. There are a number of reasons why this may be so and it would be interesting to follow up on this finding.