Ph.D. Qualifying Examination

Dynamics

Fall 2011

Notes:

- Duration: 2.5 hours.
- Closed book, closed notes (one sheet of formulæ is allowed).
- State your assumptions, methods, and procedures. Show your work on these exam sheets. Add additional sheets, if needed.
- Calculator allowed.
- Laptops and cell phones are not allowed.

Problem 1. (35) A four-bar "slider-crank" linkage is shown in Figure 1. The angular velocity of link 2 is $\omega_2 = 100$ rad/s (ccw), and is constant. Link 2 is "counterbalanced," so its center of mass is at point A. The center of mass of link 3 is at point G. The dimensions are:

- AB=6 cm
- BC=10 cm
- $BG = 5$ cm
- $\angle BAC = 90°$

Slider (link 4) translates horizontally without friction. There is no friction at pin joints A, B, or C. You may neglect gravity. Please use the xyz coordinate system shown (and unit vectors $\mathbf{i}\,\mathbf{j}\,\mathbf{k}$) for indicating directions.

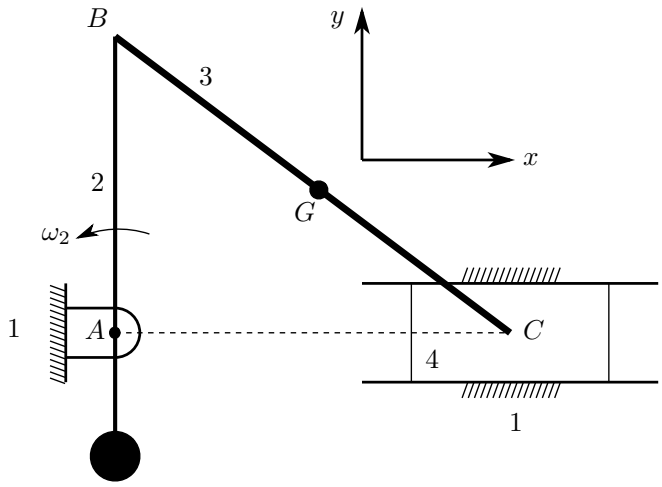

Figure 1: Slider-crank mechanism.

(a) Find the angular velocity $\boldsymbol{\omega}_3$ (rad/s), and the velocity $\mathbf{v}_C$ of point C (m/s) at this instant.

(b) Find the angular acceleration $\boldsymbol{\alpha}_3$ (rad/s^2), and the accelerations $\mathbf{a}_C$ and $\mathbf{a}_G$ of points C and G (m/s^2) at this instant.

(c) What must be true about the resultant force on link 2? Draw a free-body diagram of link 2, showing the direction of all forces (remember to neglect gravity).

(d) What must be true about the resultant force on link 4? Draw a free-body diagram of link 4, showing the direction of all forces.

Problem 2. (35) In the mechanism shown in Figure 2, each of the two wheels has a mass of 30 kg and a centroidal radius of gyration of 100 mm. Each link OB has a mass of 10 kg and may be treated as a slender bar ($I_{cm} = ml^2/12$). The 7 kg collar B slides on the fixed vertical shaft with negligible friction. The spring has a stiffness $k = 30$ kN/m and is contacted by the bottom of the collar when the links reach the horizontal position. If the collar is released from rest at the position $\theta = 45°$ and if the friction is sufficient to prevent the wheels from slipping, determine:

(a) the velocity of the collar as it first strikes the spring

(b) the maximum deformation x of the spring

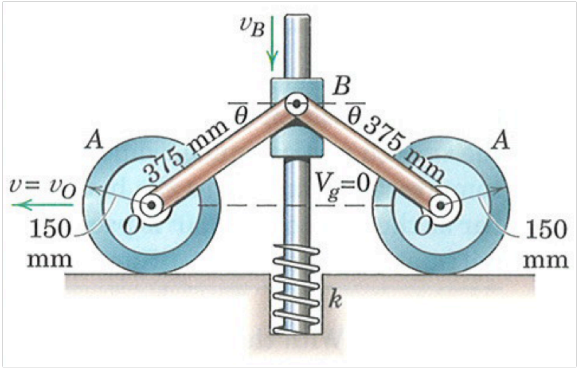

Figure 2: Wheel, collar, and spring mechanism.

Problem 3. (30) In Figure 3, a massless rope carries two masses of 5 and 7 kg hanging on a pulley of mass 5 kg, radius 600 mm, and radius of gyration 450 mm. How long will it take to change the speed of the masses from 3 to 6 m/s? Solve this problem using the equations of impulse and momentum.

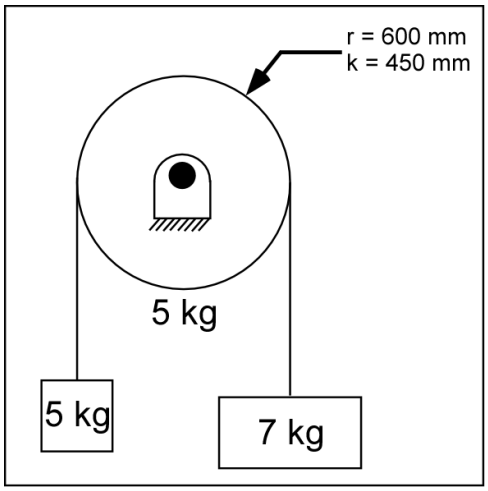

Figure 3: Mass and pulley system.