Ph.D. Qualifying Examination

Dynamics

Spring 2008

Notes:

- Time allowed: 2 hours
- Exam is closed-book and closed-notes
- Please state your assumptions, methods, and procedures. Show your work on these exam sheets (add additional sheets if needed)
- You may use a calculator
- Problems count as marked
- Laptops and cell phones are not allowed

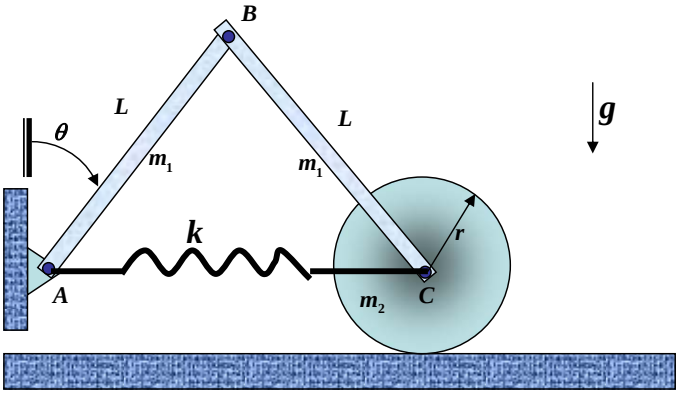
Problem 1. (40 points) The assembly shown consists of two slender rods of mass m_1, and a homogeneous thin disk of radius r and mass m_2. If the spring k is unstretched when $\theta = \theta_o$ and the assembly is released from rest at this position, determine:

(a) The angular velocity of the rod AB at the instant $\theta = \theta_f$

(b) The velocity of the center of mass of the disk at the instant $\theta = \theta_f$

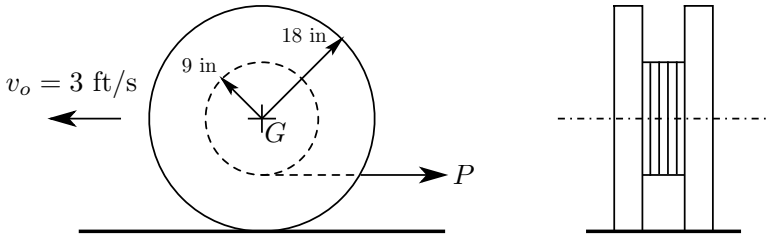
How must you modify your analysis if the disk were replaced with a block of the same mass that slides on the surface with a coefficient of friction μ. Assume the system overcomes the static coefficient of friction upon release from rest. What difficulties will you encounter?

NOTE: Mass moments of inertia about center of mass: $I_{rod} = \frac{1}{12}mL^2$, $I_{disk} = \frac{1}{2}mr^2$.



Problem 2. (30 points) The force P, which is applied to the cable wrapped around the central hub of the symmetrical wheel, is increased slowly according to $P = 1.5t$, where P is in pounds and t is the time in seconds after P is first applied. Determine the angular velocity ω of the wheel 10 s after P is applied if the wheel is rolling to the left with a velocity of its center of 3 ft/s at time $t = 0$. The wheel weighs 120 lb with a radius of gyration around its center of 10 in, and rolls without slipping.

NOTE: Please solve this problem using *impulse-momentum* methods.



Problem 3. (30 points) In the pendulum shown in the figure below, two particles with masses $m_1 = 1$ kg and $m_2 = 1$ kg are suspended through massless links of length $L_1 = 1$ m and $L_2 = 1$ m, respectively. The forces exerted on the particles are due to gravity and friction, and the latter is modeled as a single viscous force of the form: $\mathbf{F} = -b\mathbf{v}$ acting on m_1. In the expression for F, $b = 0.1$ N·s/m is a constant, and $\mathbf{v}$ is the velocity vector of the particle.

(a) Express the horizontal and vertical components of the linear velocities and accelerations of the two particles in terms of θ_1 and θ_2 and their derivatives.

(b) Derive the equations of motion in terms of the linear velocity and acceleration components (without substituting from (a)).

Take $g = 9.81$ m/s^2.