ME PhD Math Qualifying Examination, Fall 2007

Linear Algebra

(a) Find the eigenvalues of the matrix $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$.

(b) Find the eigenvectors which are associated with the eigenvalues you found in (a).

(c) Compute the determinant of A.

(d) What is the rank of A? (Extra Credit)

Differential and Integral Calculus

(a) Find the derivative h'(z) of the function

$$h(z) = \left(\frac{z-1}{z+1}\right)^5$$

(b) Find $\int x^2\sqrt{x^3+9}dx$

(c) Find $\int x^2 e^{-x} dx$

Differential Equations

(a) Solve the following differential equation:

$$y'' + 5y' + 6y = 0 \text{ with } y(0) = 2 \text{ and } y'(0) = 3$$