

Mechanical Engineering Department University of New Mexico

Ph.D. Qualifying Examination

Math Section

Fall 2010

INSTRUCTIONS:

- Closed Book
- 2 Hours
- Calculator allowed
- **You should start each problem on a new sheet of paper**
- You **MUST** show work to get credit.

Problem 1 (30 points)

Determine the complete solution to the following ordinary differential equation:

$$\ddot{y} - \dot{y} - 2y = \sin(2t)$$

The initial conditions are:

$$y(0) = 0 \quad \text{and} \quad \dot{y}(0) = \frac{1}{20}$$

Problem 2 (30 points)

a) Integrate the following expression:

$$\int \ln(x+1) dx$$

b) Integrate the following expression:

$$\int \tan^{-1} x \, dx$$

Problem 3 (40 points)

A plane (plane 1) is defined by the equation: $x - y - z = 2$

A second plane (plane 2) is defined by: $-2x + y - 2z = 4$

- (a) Find a parametric equation for the line L , which is the intersection between the two planes.
- (b) Find a vector $\mathbf{c}$ parallel to L .
- (c) Find the dot product of $\mathbf{c}$ and the vector $\mathbf{v} = \mathbf{i} - \mathbf{j} - \mathbf{k}$, where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors along the positive x , y and z axes.