Ph.D. Qualifying Examination

Mathematics

Fall 2011

Notes:

- Time allowed: 2 hours.
- Exam is closed book and closed-notes (except for one sheet of "notes": 8.5x 11 in., both sides).
- State your assumptions, methods, and procedures. Show your work on these exam sheets. (Add additional sheets, if needed.)
- You may use a calculator – however note that all problems require a hand solution. It is unacceptable to write down a result determined by a preprogrammed calculator routine.
- Problems value is 25 percentage points per problem (total 100%).
- Laptops and cell phones are not allowed.

1. A plane passes through three points A, B,and C, whose Cartesian coordinates are A = (1,3,1), B = (3,0,-5), and C = (-2,-2,4). Find the equation of the plane and the angle that the normal to the plane would form with a line $z = 4x+0.6y+4$.

2. Find the invariants, eigenvalues, and eigenvectors of matrix
$$A=\begin{bmatrix} 5 & -3 \\ 2 & 8 \end{bmatrix}$$
Is this matrix orthogonal?

3. Solve the following differential equation
$$y''+y=4e^{x},\quad y'(0)=-3$$

4. Find the following indefinite integrals
$$\int \frac{dx}{e^{x}+e^{(-x)}},\quad \int (1+\sin x+\cos x)\,dx$$