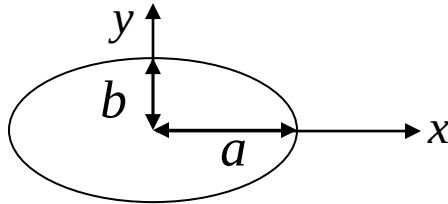


Spring 2005
PhD Qualifying Examination in
Mathematics

Department of Mechanical Engineering

Problem 1:

- a) Using integration, find the volume of a sphere of radius R . Note that the equation for the surface of a sphere is given by $x^2 + y^2 + z^2 = R^2$
- b) Using integration, find the area of the following ellipse:



Note: the equation of the ellipse is $x^2/a^2 + y^2/b^2 = 1$

Hint: $\int \sqrt{\alpha^2 - z^2} = \frac{1}{2} \left[z\sqrt{\alpha^2 - z^2} + \alpha^2 \sin^{-1} \frac{z}{\alpha} \right]$

Problem 2:

Let A be an $n \times n$ matrix with real elements, and B its transpose, $B = A^T$ (such that $b_{ij} = a_{ji}$). Show that if the inverse of A , A^{-1} , exists, then the inverse of B will exist too and will be equal to $(A^{-1})^T$. (Hint: Recall that $(C D)^T = D^T C^T$ and work with the definition of an inverse matrix)

Problem 3:

Solve the following differential equation:

$$\ddot{y}(t) + 6\dot{y}(t) + 9y(t) = 0; y(0) = 0, \dot{y}(0) = 2$$