Mechanical Engineering Department
University of New Mexico

Ph.D. Qualifying Examination

Math Section

Spring 2008

INSTRUCTIONS:

- Closed Book
- 2 Hours
- Calculator allowed
- **You should start each problem on a <u>new</u> sheet of paper**
- You ***<u>MUST</u>*** show work to get credit.

Problem 1. (25 points)

a. (20 points) Find the eigenvalues and eigenvectors of the following matrix:

$$\begin{bmatrix} 13 & 5 \\ 2 & 4 \end{bmatrix}$$

b. (5 points) Show that the eigenvectors are orthogonal.

Problem 2 (25 points)

Determine the complete solution to the following ordinary differential equation:

$$\ddot{x} + 2\dot{x} + 16x = 10\,sin(3t)$$

for the initial conditions:

$$x(0) = 1 \qquad \dot{x}(0) = -4$$

Assume consistent units.

What is the steady state value of x?

Problem 3 (25 points)

Evaluate the following integrals:

(a) $I_1 = \int \frac{x^2}{x^3 + 1} dx$

(b) $I_2 = \int x\sqrt{1 + \pi x^2}\, dx$

(c) $I_3 = \int \frac{x^2 - 1}{x + 1} dx$

Problem 4 (25 points)

(a) What is the gradient of the following scalar function: $\phi = \phi(x, y, z) = x^2 + y^2 + z^2$, i.e. what is $grad\phi$?

(b) What is the dot product between $grad\phi$ and $\vec{n}$, where $\vec{n} = \frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{3}}\vec{k}$?

(c) Evaluate the cross product between $grad\phi$ and $\vec{n}$, i.e. $grad\phi \times \vec{n}$, at the point (1,1,1).

(d) Is the result from part (c) surprising considering what 3-D object or geometry is represented with $\phi =$ constant?