

Mechanical Engineering Department University of New Mexico

Ph.D. Qualifying Examination

Math Section

Spring 2009

INSTRUCTIONS:

- Closed Book
- 2 Hours
- Calculator allowed
- **You should start each problem on a new sheet of paper**
- You **MUST** show work to get credit.

Problem 1 (25 points)

Determine the complete solution to the following ordinary differential equation:

$$\ddot{y} - 4\dot{y} - 12y = 3e^{5t}$$

for the initial conditions:

$$y(0) = \frac{18}{7}$$

$$\dot{y}(0) = -\frac{1}{7}$$

Problem 2 (25 points):

Find the limit for the following:

(a) $\frac{\cos x}{\sqrt{x}}$ as $x \rightarrow \infty$,

(b) $\frac{x^2 - 1}{x + 1}$ as $x \rightarrow -1$,

(c) $\frac{e^{-x}}{e^x}$ as $x \rightarrow \infty$

Problem 3 (25 points):

Integrate the following:

(a) $\int \frac{x+5}{x^2+x-2} dx$

(b) $\int \frac{\cos x}{\sin x} dx$

(c) $\int x^2 \sin 2x dx$

Problem 4 (25 points):

Given the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, find its eigenvalues and eigenvectors. Express each eigenvector so its norm is unity.