

Mechanical Engineering Department
University of New Mexico

Ph.D. Qualifying Examination

Mathematics

Spring 2010

INSTRUCTIONS:

- Closed Book / Closed Notes.
- Time Allowed: 2 Hours.
- Problems count 25 points each (total = 100 points).
- Calculators allowed.
- You **MUST** show work to get credit. Show your work on these exam sheets. (Add additional sheets, if needed.)
- Laptops and cell phones not allowed.

Problem 1

Determine the solution to the following ordinary differential equation only in terms of cos, tan, exponentials and constants:

$$\ddot{y} + 4\dot{y} + 5y = 0$$

for the initial conditions:

$$y(0) = 4$$

$$\dot{y}(0) = 3$$

Problem 2

Integrate the following:

$$\int x^2 e^x dx$$

Problem 3

(a) Is $\sum_{n=1}^{\infty} \frac{n^2}{5n^2 + 4}$ convergent or divergent? Justify your answer.

(b) If $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent, then is $\sum_{n=1}^{\infty} \frac{2}{\sqrt{11}n^\pi}$ convergent or divergent? Justify your answer.

Problem 4

- (a) A plane in space passes through the points P(1, 0, 2), Q(3, -1, 6) and R(5, 2, 4). Find a unit vector normal to this plane? Also, find an in-plane unit vector that is perpendicular to the line joining points P and Q.

(b) Find the following: $\lim_{\xi \rightarrow \infty} \frac{\sin(\cos(\xi))}{\xi^e}$ and $\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{\tan \theta}{e^\theta}$