

Ph.D. Qualifying Examination

Mathematics

Spring 2011

Logistics Notes:

1. Time allowed: 2 hours.
2. Problems count 25 points each (total=100 points).
4. State your assumptions, methods, and procedures. Show your work on the blank exam sheets provided by the proctor.
5. Calculators are allowed.
6. **Laptops and cell phones are not allowed.**

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1. Given the matrix $\mathbf{A} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$:

- Show that $\mathbf{A}^{-1} = \mathbf{A}^T$
- Find the eigenvalues and eigenvectors of \mathbf{A} .

2. If $x^2 + y^2 + z^2 = 0$, $x = u^2 - v^{-1}$ and $y = \frac{2v}{u}$.

Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ in terms of u and v .

3. Solve the following ordinary differential equation for $y(t)$, given that $y(1)=-8$.

$$2t \frac{dy}{dt} + 4y = 3$$

4. Find the angle between the planes $x + y + z = 1$ and $x + 2y + 3z = 6$.