

Ph.D. Qualifying Examination

Engineering Mathematics

Spring 2012

Logistics Notes:

- Time allowed: 2 hours.
- Problems count 25 points each (total=100 points).
- Exam is closed-book and closed-notes (except for one sheet of “notes”: 8.5×11 in., both sides)
- State your assumptions, methods, and procedures. Show your work on these exam sheets. (Add additional sheets, if needed.)
- Calculators are allowed.
- Laptops and cell phones are not allowed.

1) Solve the following differential equations

a) $x^2 y^2 y' + 1 = y$

b) $(x^2 + y^2)y' = 2xy$

2) Find the following indefinite integrals

a) $\int \frac{x+1}{\sqrt{x}} dx$

b) $\int x^2 e^{-2x} dx$

3) For the matrix $\mathbf{A} = \begin{bmatrix} a & 1 & 0 \\ 1 & a & 1 \\ 0 & 1 & a \end{bmatrix}$

a) Find the eigenvalues of $\mathbf{A}$

b) Find the eigenvectors of $\mathbf{A}$

c) Show that the eigenvectors are orthogonal

4) If $w = f(x, y, z)$ and $z = g(x, y)$

Find $\frac{\partial w}{\partial x}$, $\frac{\partial w}{\partial y}$ and $\frac{\partial w}{\partial z}$