

Ph.D. Qualifying Examination

Mathematics

Spring 2013

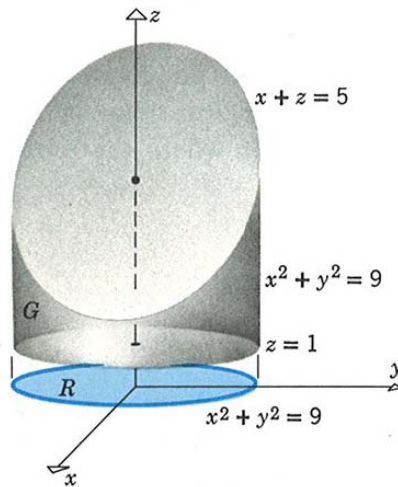
Logistics Notes:

- Time allowed: 2 hours.
- Closed book / Closed Notes (one 8.5 x 11.00 in. sheet of formulas is allowed)
- Calculators are allowed.
- Laptops, cell phones, and similar electronic devices are not allowed.

**Mathematics Qualifying Exam
Spring 2013**

1. Find the volume integral $I = \iiint_V dV$ to get the volume of the solid enclosed between the cylinder $x^2 + y^2 = 9$ and the planes $z = 1$ and $z = 5 - x$ as shown in the figure.

(Hint: $\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \sin^{-1}(x/a) \right] + C$)



2. Find the solution to the second order differential equation

$$\frac{\partial^2 u(x)}{\partial x^2} + u(x) = \delta\left(x - \frac{1}{2}\right)$$

with boundary conditions $u(0) = u(\pi/4) = 0$.

3. Find and classify the stationary points of the following function of two variables

$$f(x, y) = x(x + y)e^{y-x}$$

4. Solve the following initial value problem by using the Laplace transform

$$\ddot{y}(t) + 4\dot{y}(t) + 5y(t) = 8e^{-2t}$$

with $y(0) = -1$ and $\dot{y}(0) = 0$.