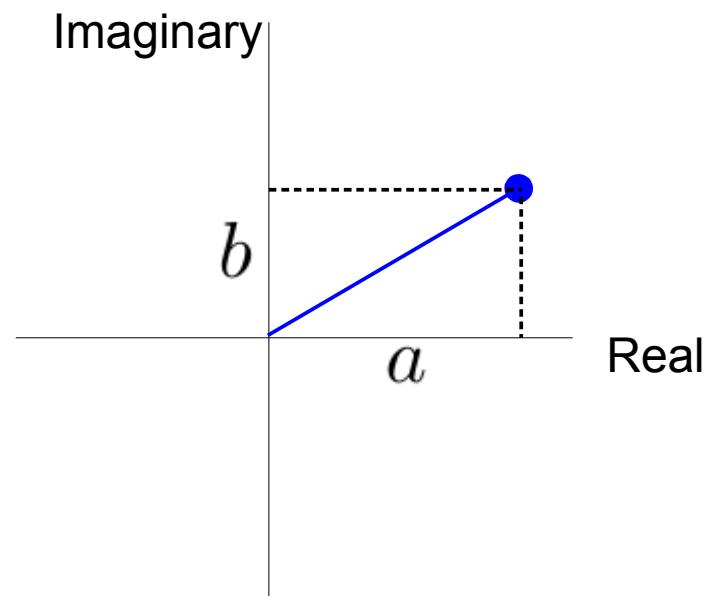


# **Lab 3: Capacitors and Inductors in AC circuits**

# Review of Complex Numbers

$$j = \sqrt{-1}$$

$$z = a + jb$$



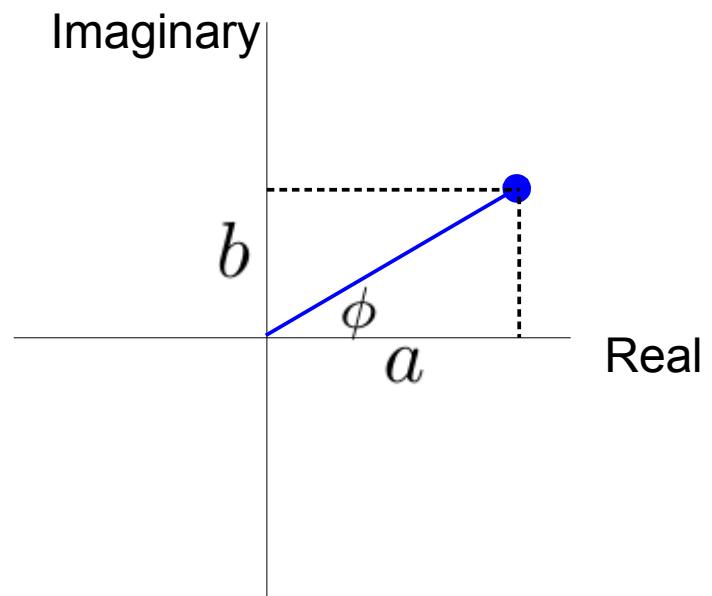
# Review of Complex Numbers

$$j = \sqrt{-1}$$

$$z = a + jb = Ae^{j\phi}$$

$$A = \sqrt{a^2 + b^2}$$

$$\phi = \tan^{-1} \left( \frac{b}{a} \right)$$



# Review of Complex Numbers

$$j = \sqrt{-1}$$

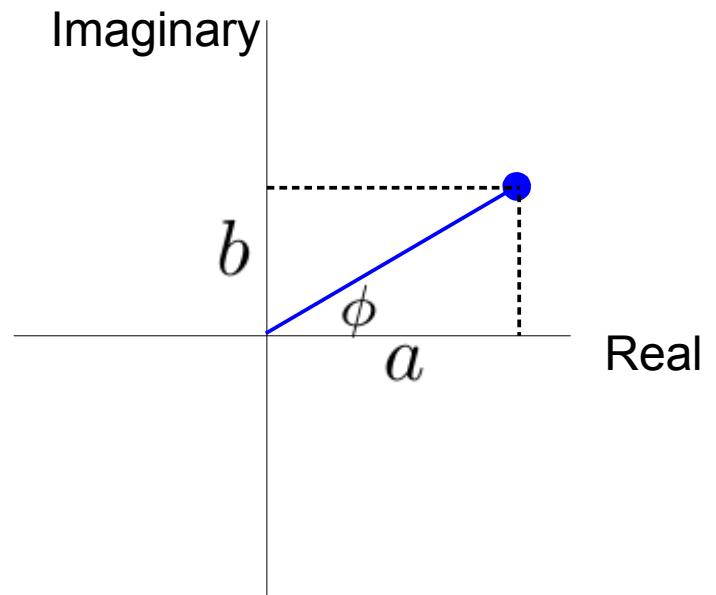
$$z = a + jb = Ae^{j\phi}$$

$$A = \sqrt{a^2 + b^2}$$

$$\phi = \tan^{-1} \left( \frac{b}{a} \right)$$

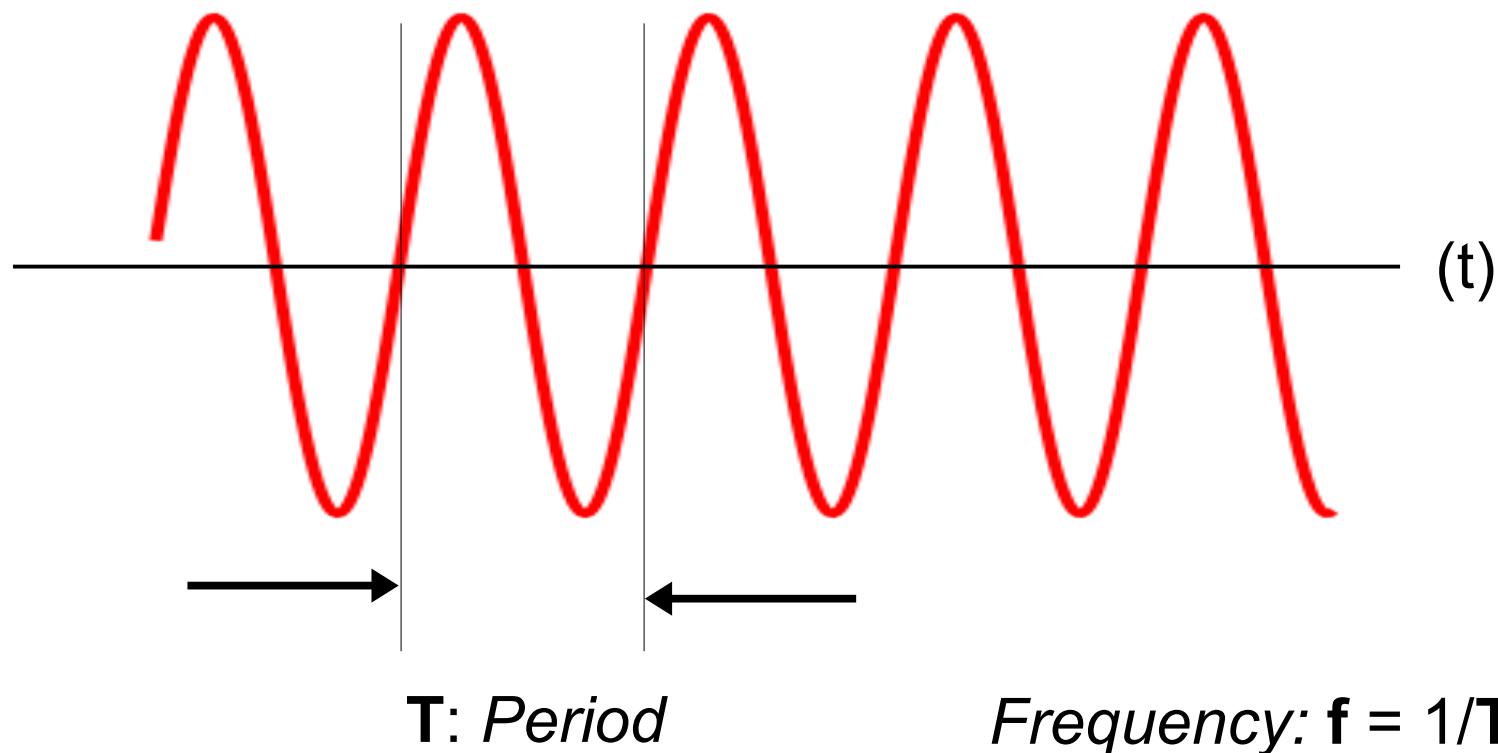
$$e^{j\phi} = \cos \phi + j \sin \phi$$

$$e^{j\frac{\pi}{2}} = \cos(\frac{\pi}{2}) + j \sin(\frac{\pi}{2}) = j$$



OUR ESSENTIAL WORKING ASSUMPTION:

**AC  $\equiv$  SINUSOIDAL FUNCTION**



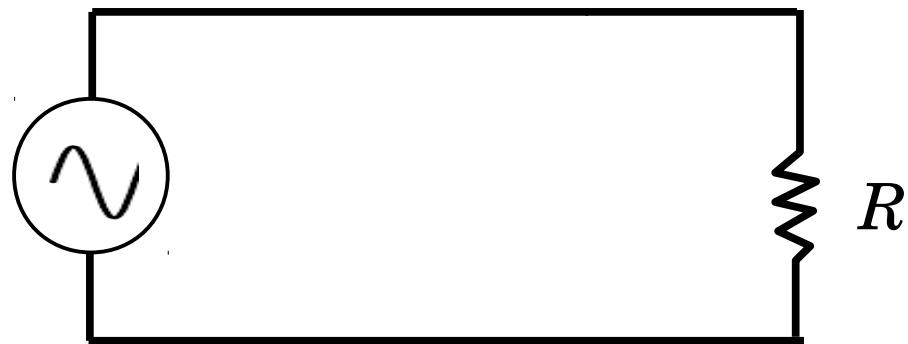
# Resistor in AC circuit

$$V_{ac} = V_o \sin \omega t$$

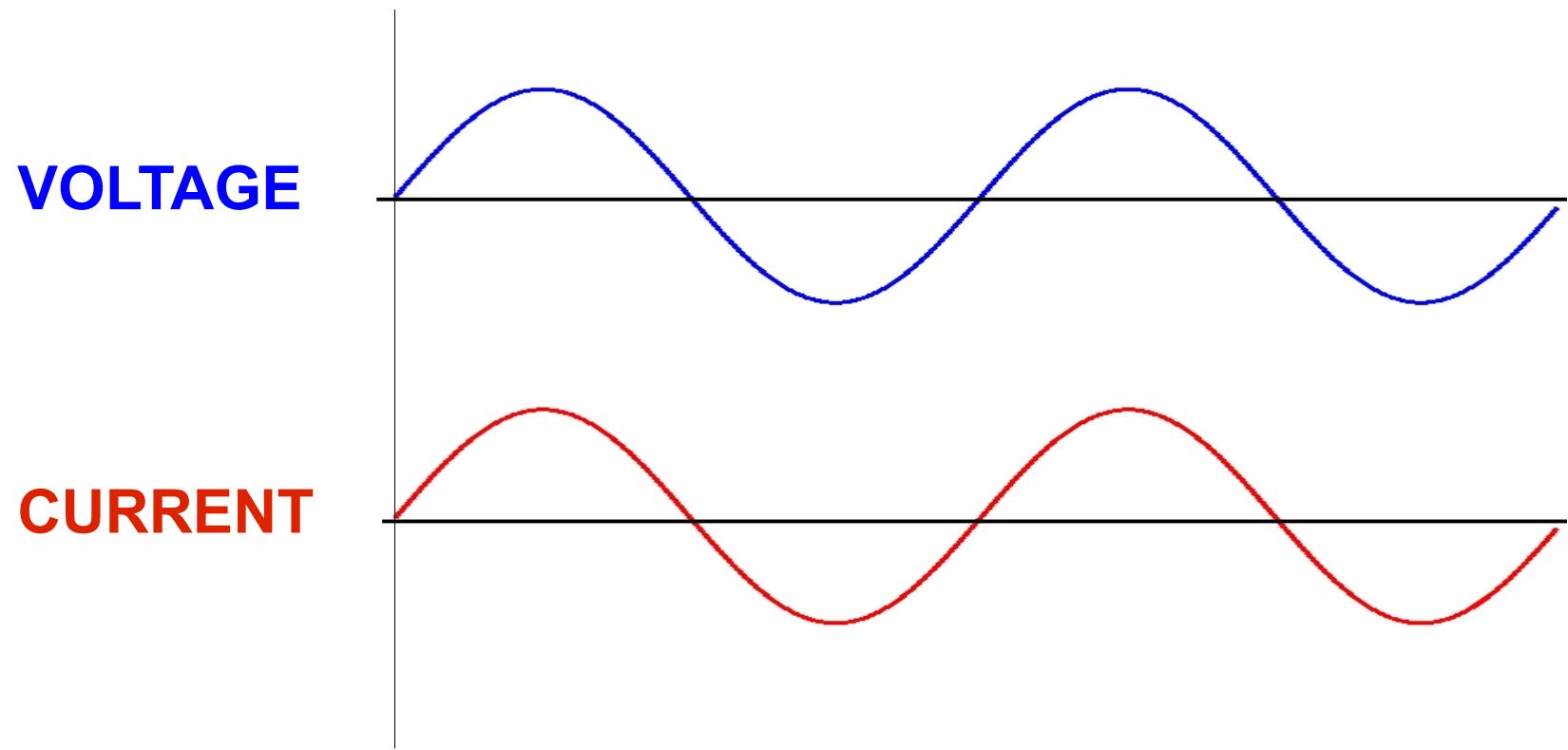
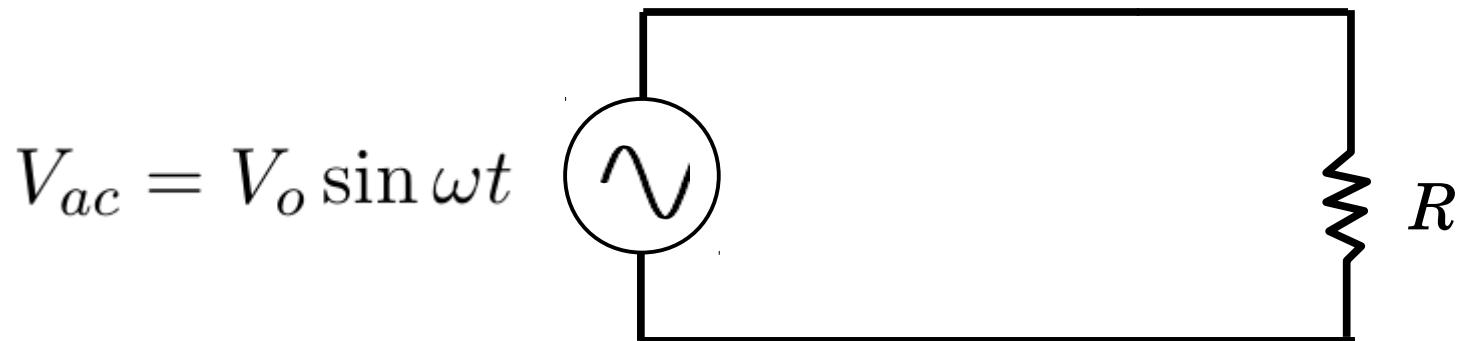
$$\omega = 2\pi f$$

$\omega$ : radians/sec

$f$ : Herz (1/sec)



# Resistor in AC circuit



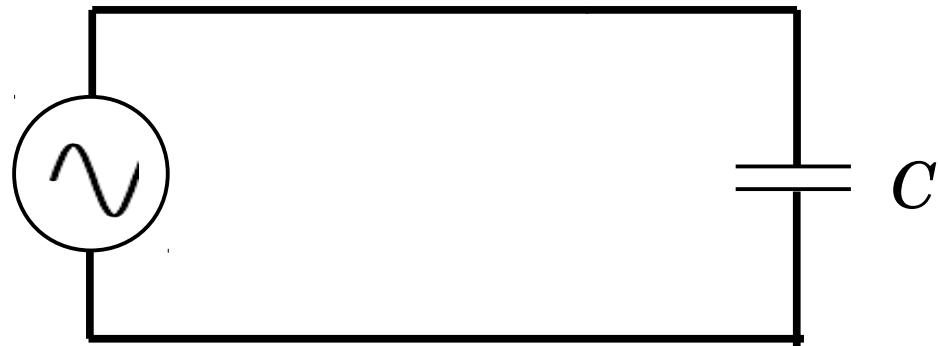
# Capacitor in AC circuit

$$V_{ac} = V_o \sin \omega t$$

$$\omega = 2\pi f$$

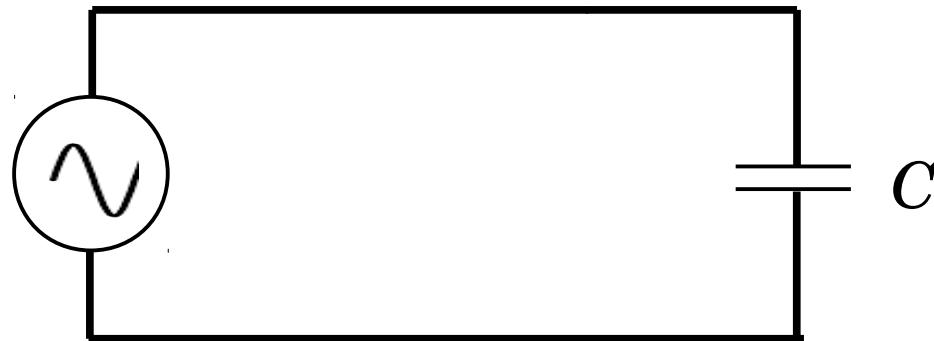
$\omega$ : radians/sec

$f$ : Herz (1/sec)



# Capacitor in AC circuit

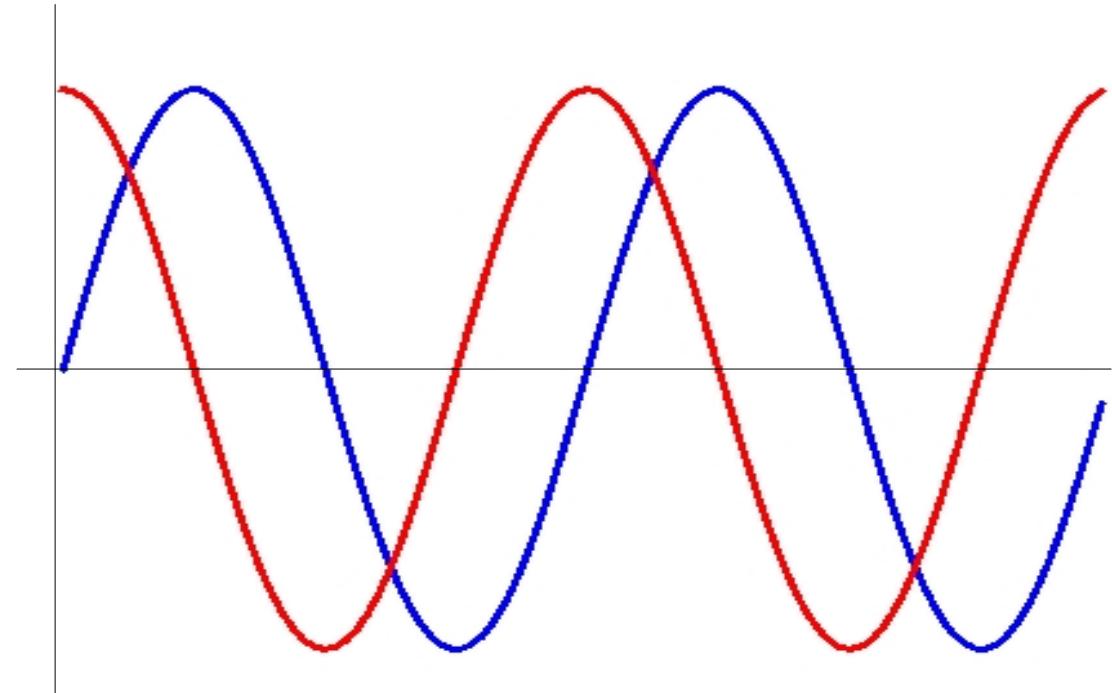
$$V_{ac} = V_o \sin \omega t$$



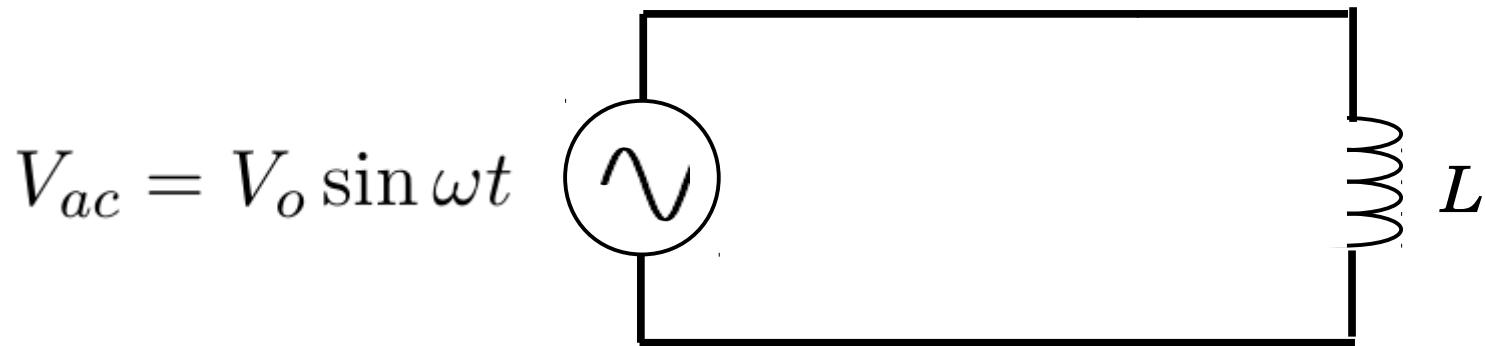
$$\omega = 2\pi f$$

$\omega$ : radians/sec  
 $f$ : Herz (1/sec)

**VOLTAGE**  
**CURRENT**

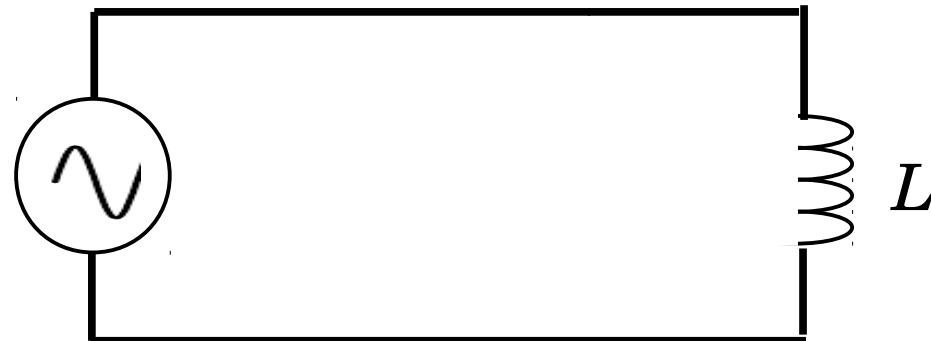


# Inductor in AC circuit

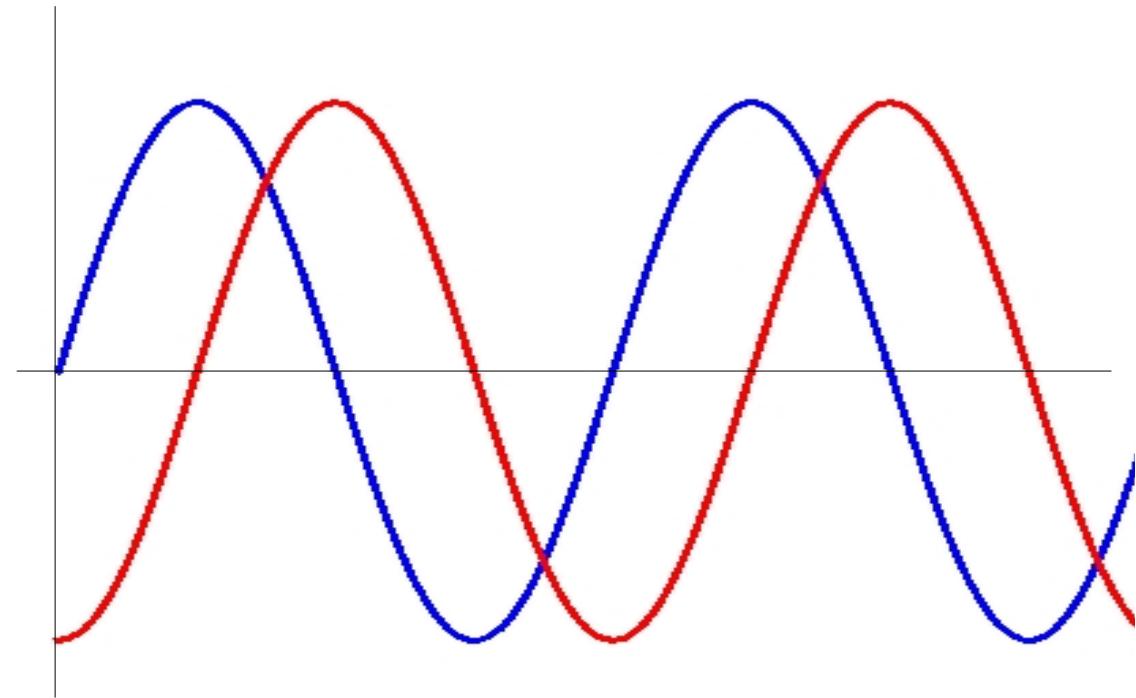


# Inductor in AC circuit

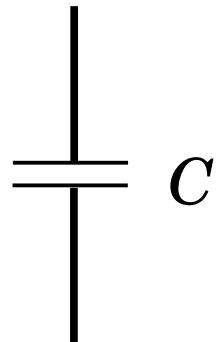
$$V_{ac} = V_o \sin \omega t$$



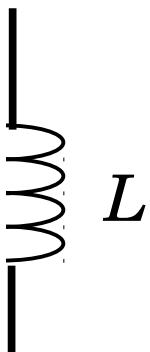
**VOLTAGE**  
**CURRENT**



# Ohm's Law for L and C: Impedance (Z)

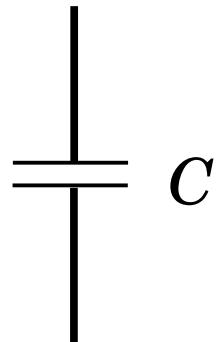


$$C \quad Z_C = \frac{V_C}{I_C} = \frac{V_o \sin \omega t}{\omega C V_o \cos \omega t} = \frac{V_o \sin \omega t}{\omega C V_o \sin \left( \omega t + \frac{\pi}{2} \right)}$$



*L*

# Ohm's Law for L and C: Impedance (Z)

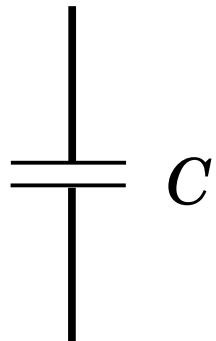


$$C \quad Z_C = \frac{V_C}{I_C} = \frac{V_o \sin \omega t}{\omega C V_o \cos \omega t} = \frac{V_o \sin \omega t}{\omega C V_o \sin \left( \omega t + \frac{\pi}{2} \right)}$$



$$L \quad Z_L = \frac{V_L}{I_L} = \frac{V_o \sin \omega t}{-(V_o / \omega L) \cos \omega t} = \frac{V_o \sin \omega t}{-(V_o / \omega L) \sin \left( \omega t + \frac{\pi}{2} \right)}$$

# Ohm's Law for L and C: Impedance (Z)



$$C \quad Z_C = \frac{V_C}{I_C} = \frac{V_o \sin \omega t}{\omega C V_o \cos \omega t} = \frac{V_o \sin \omega t}{\omega C V_o \sin \left( \omega t + \frac{\pi}{2} \right)}$$

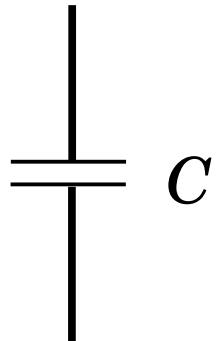
90° phase shift



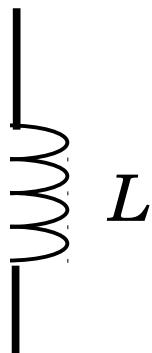
$$L \quad Z_L = \frac{V_L}{I_L} = \frac{V_o \sin \omega t}{-(V_o / \omega L) \cos \omega t} = \frac{V_o \sin \omega t}{-(V_o / \omega L) \sin \left( \omega t + \frac{\pi}{2} \right)}$$

90° phase shift

# Ohm's Law for L and C: Impedance (Z)



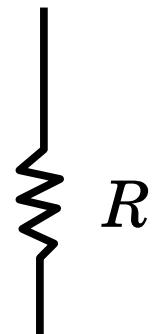
$$Z_C = \frac{V_C}{I_C} = \frac{1}{j\omega C}$$



$$Z_L = \frac{V_L}{I_L} = \frac{\omega L}{-j} = j\omega L$$

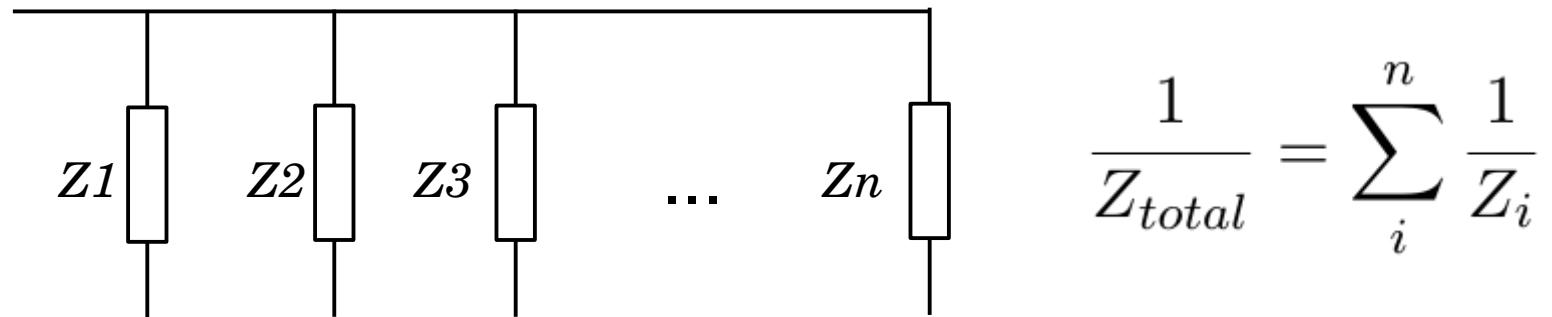
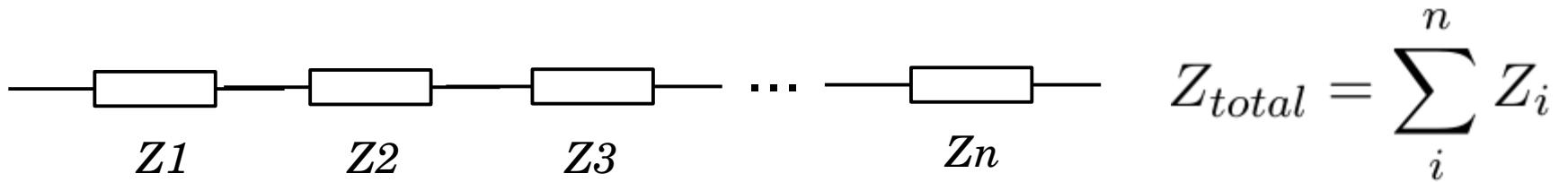
90° phase-shift in polar form:  $e^{j\frac{\pi}{2}} = \cos(\frac{\pi}{2}) + j \sin(\frac{\pi}{2}) = j$

# Impedance of a Resistor:

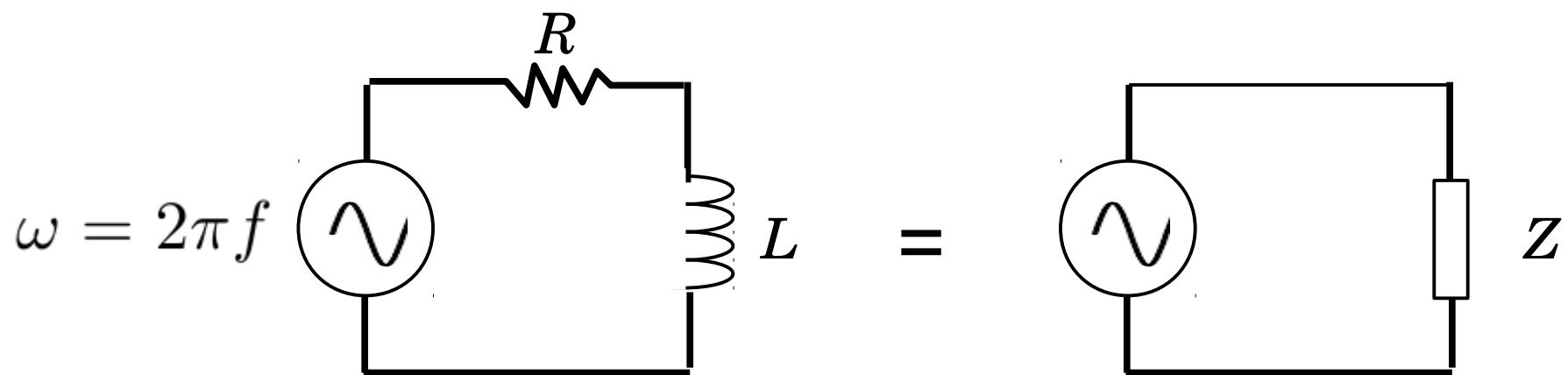


$$Z_R = \frac{V_R}{I_R} = R$$

# Impedance arithmetic: same as a resistor



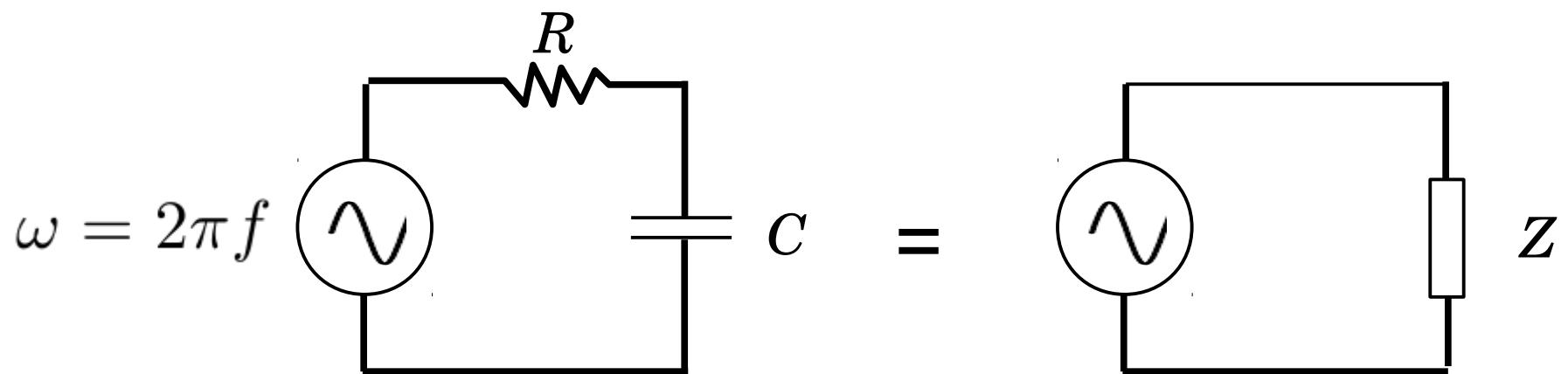
# EXAMPLE 1



$$Z = Z_R + Z_L = \text{Resistance} + j\omega L$$

Reactance

## EXAMPLE 2

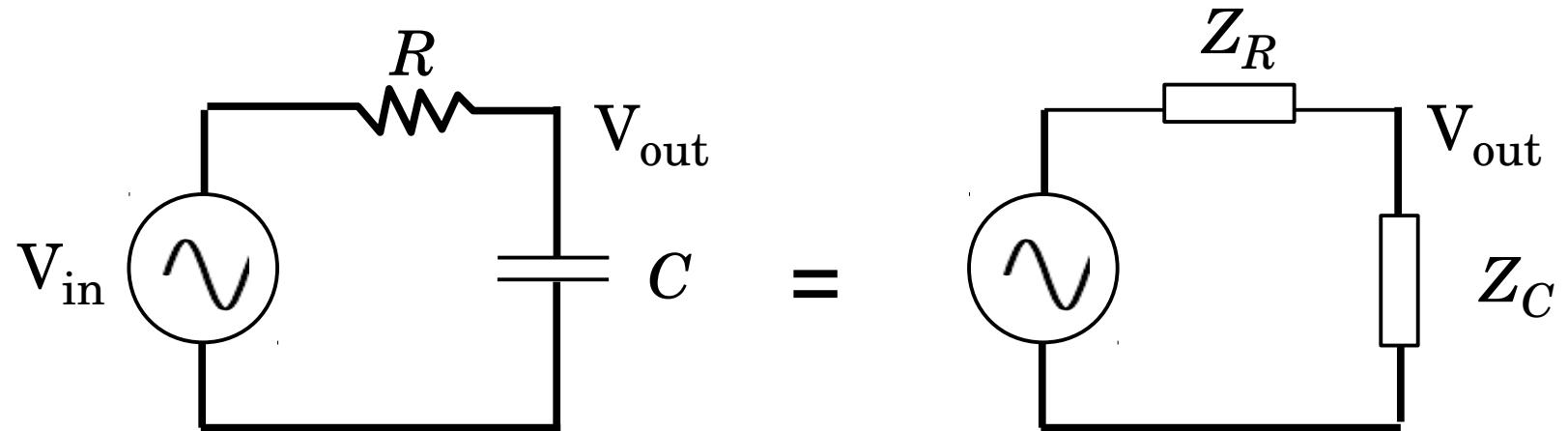


$$Z = Z_R + Z_C = R + \frac{j}{\omega C}$$

Resistance      Reactance

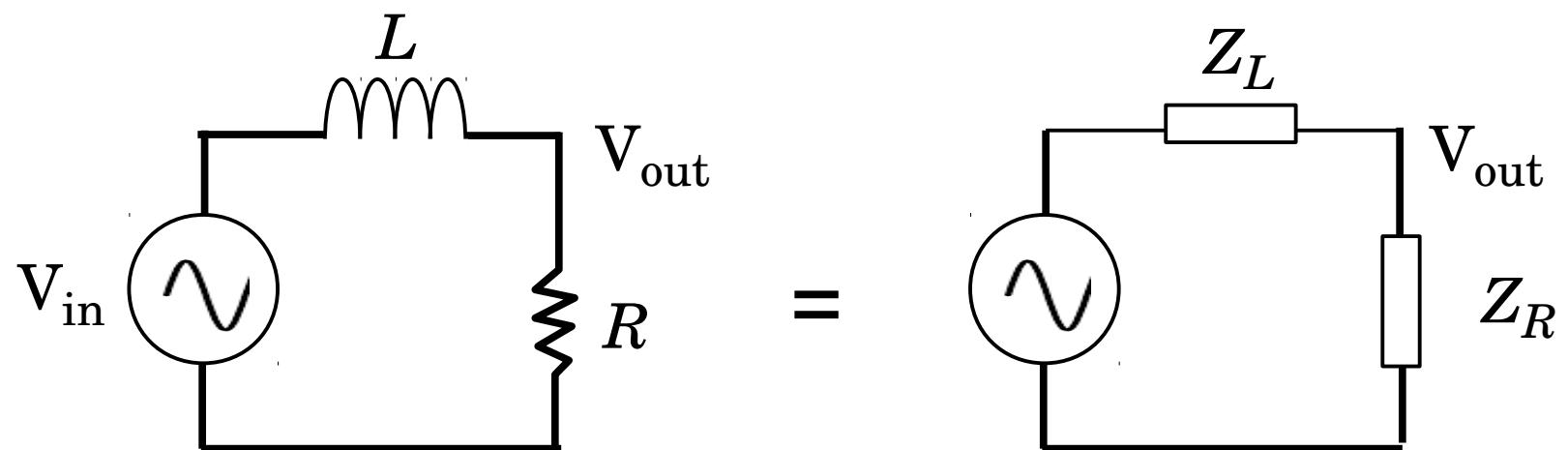
The diagram shows the parallel R-C circuit from the previous section. The resistor R is highlighted with a red circle and a magnifying glass icon. The capacitor  $\frac{j}{\omega C}$  is highlighted with a blue circle and a magnifying glass icon. Below the circuit, the words "Resistance" and "Reactance" are written in red and blue respectively, corresponding to the highlighted components.

# EXAMPLE 3: Voltage Divider

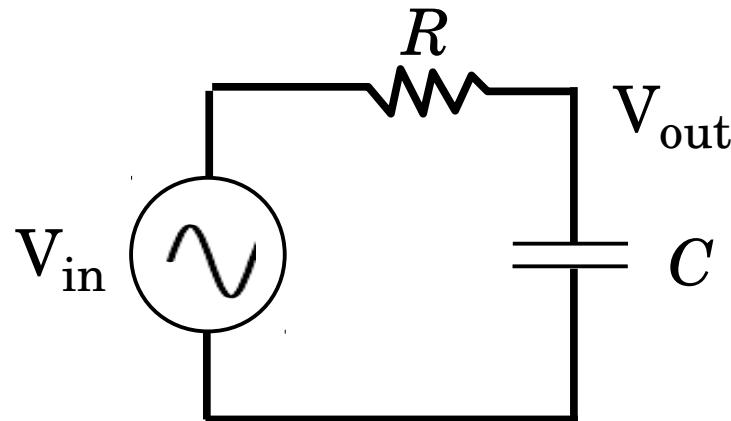


$$\frac{V_{out}}{V_{in}} = \frac{Z_C}{Z_R + Z_C} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$$

## EXAMPLE 4: Voltage Divider



$$\frac{V_{out}}{V_{in}} = \frac{Z_R}{Z_R + Z_L} = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega L/R}$$



$$R = 217.8 \Omega$$

$$C = 145.6 \text{ nF}$$

