

Lab 5: AC Circuits (III)

Poles and Zeroes

$$\frac{V_{OUT}(\omega)}{V_{IN}(\omega)} = G(\omega) = \frac{N(\omega)}{D(\omega)}$$

Numerator: $N(\omega) = (\omega_{N1} + j\omega)(\omega_{N2} + j\omega) \times \dots (\omega_{Nm} + j\omega)$

Denominator: $D(\omega) = (\omega_{D1} + j\omega)(\omega_{D2} + j\omega) \times \dots (\omega_{Dm'} + j\omega)$

$$m \leq m'$$

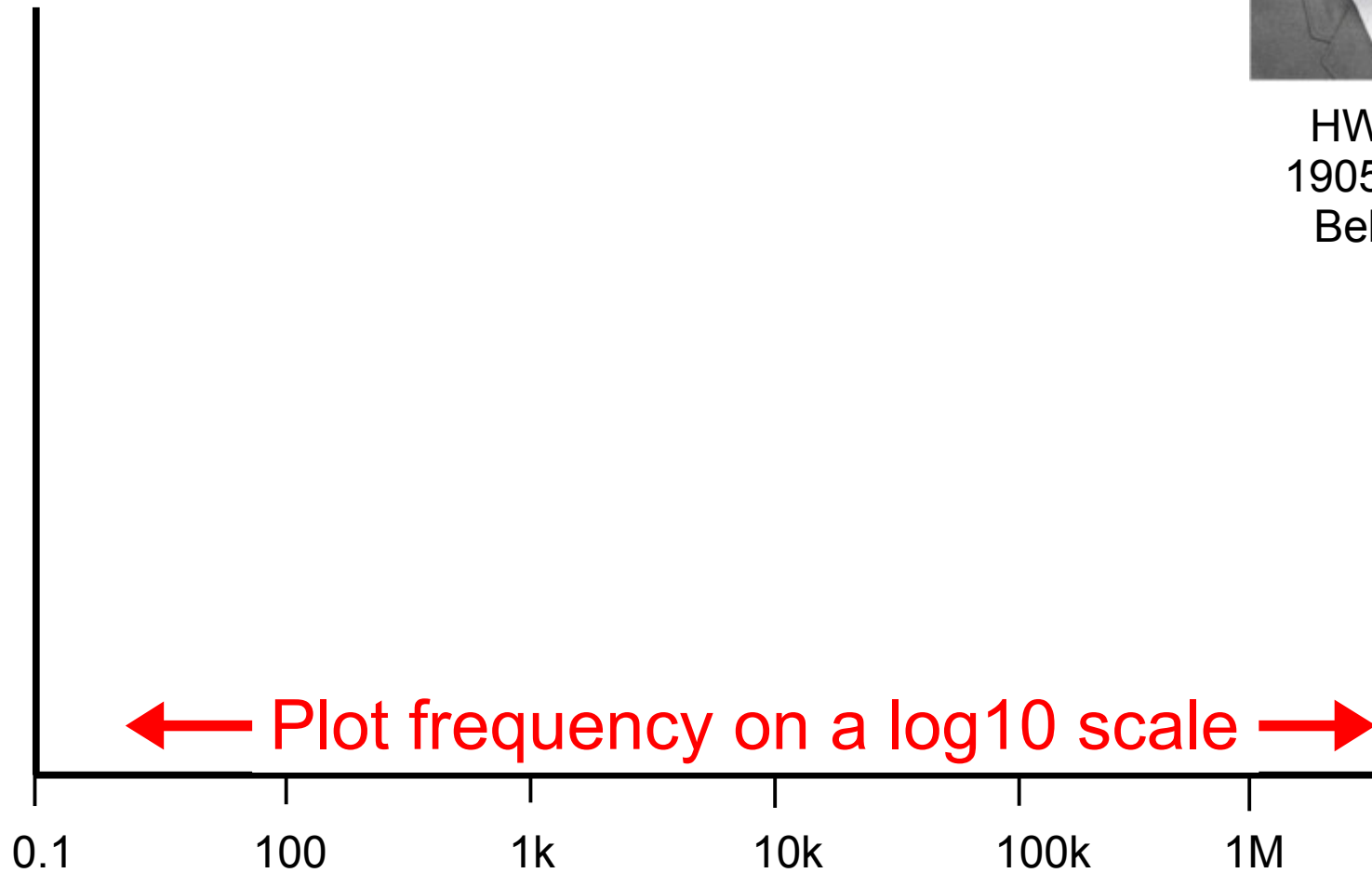
Zeroes: Frequencies (ω_N) that make the numerator zero

Poles: Frequencies (ω_D) that make the denominator zero

The Bode Plot for $|G(\omega)|$



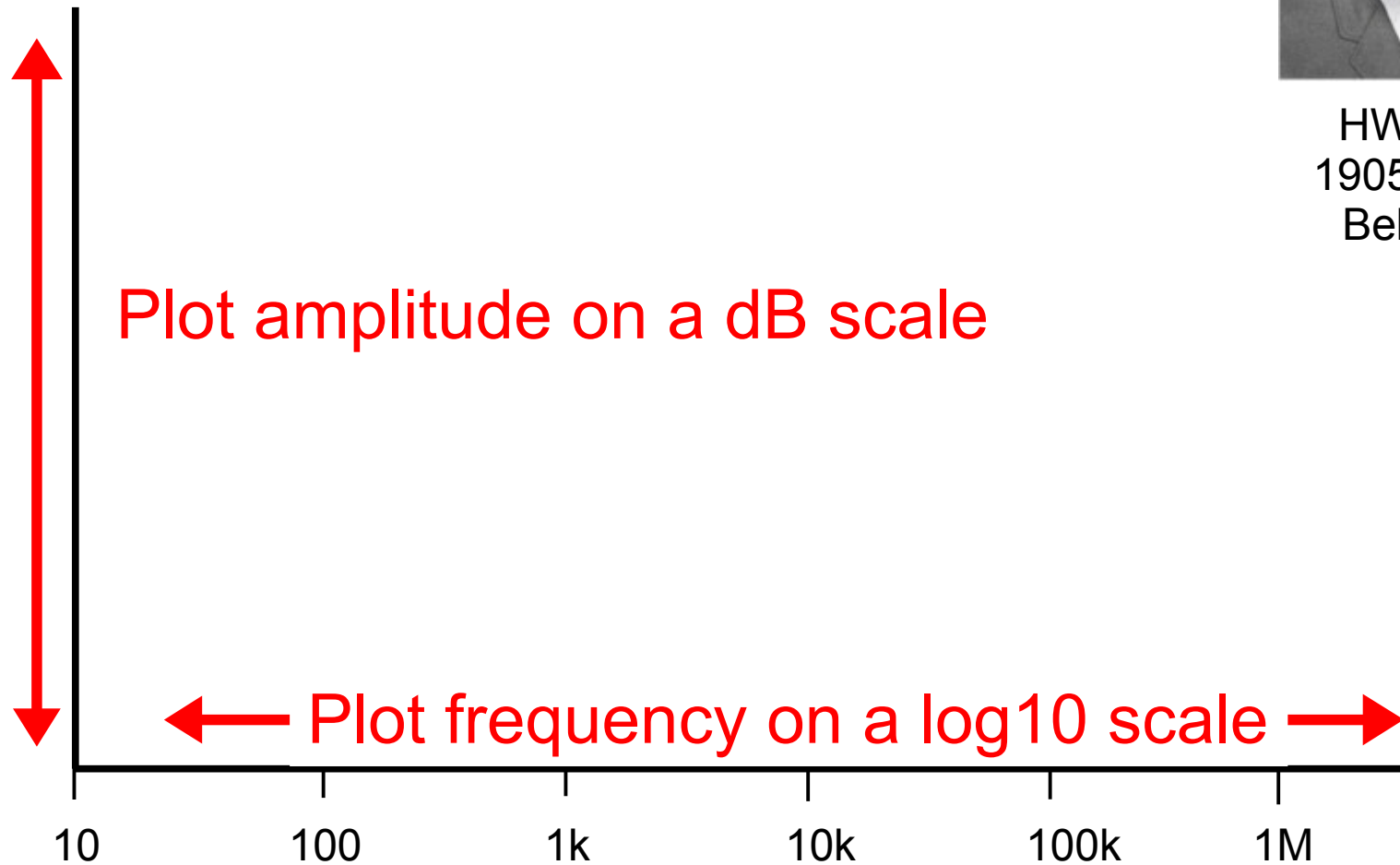
HW Bode
1905—1982
Bell Labs



The Bode Plot for $|G(\omega)|$



HW Bode
1905—1982
Bell Labs



POLES and ***ZEROES*** change the slopes on the Bode Plot

Amplitude response

ZERO: + 20 dB/decade

POLE: – 20 dB/decade

Phase response

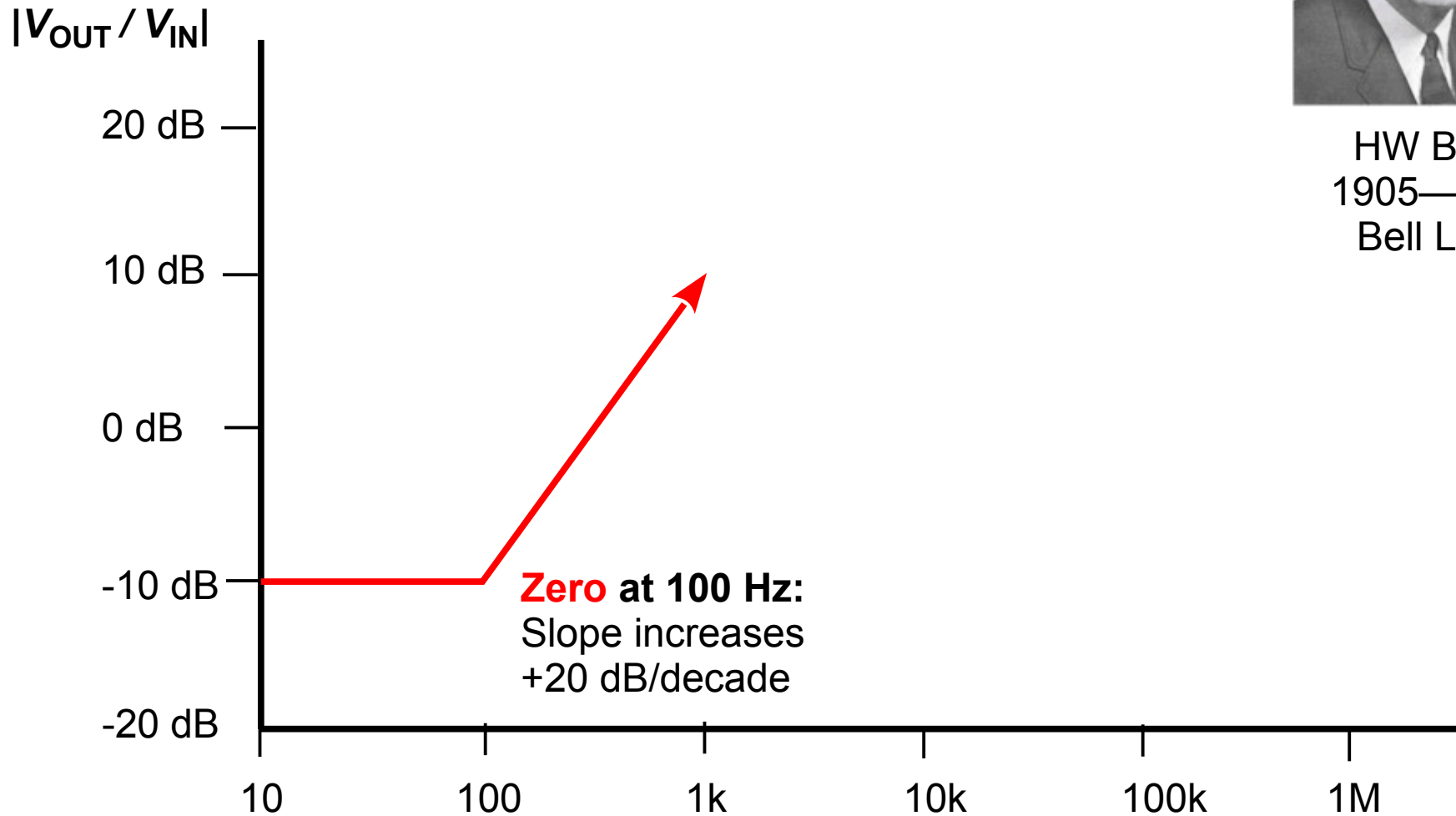
ZERO: + 45 deg/decade

POLE: – 45 deg/decade

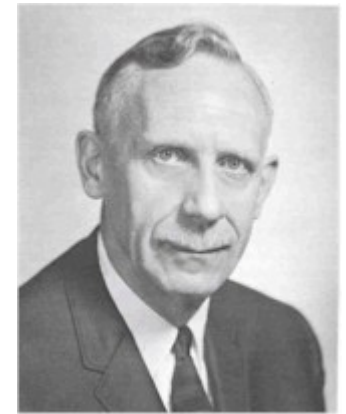
POLES and ZEROES dramatically affect the shape of a Bode plot



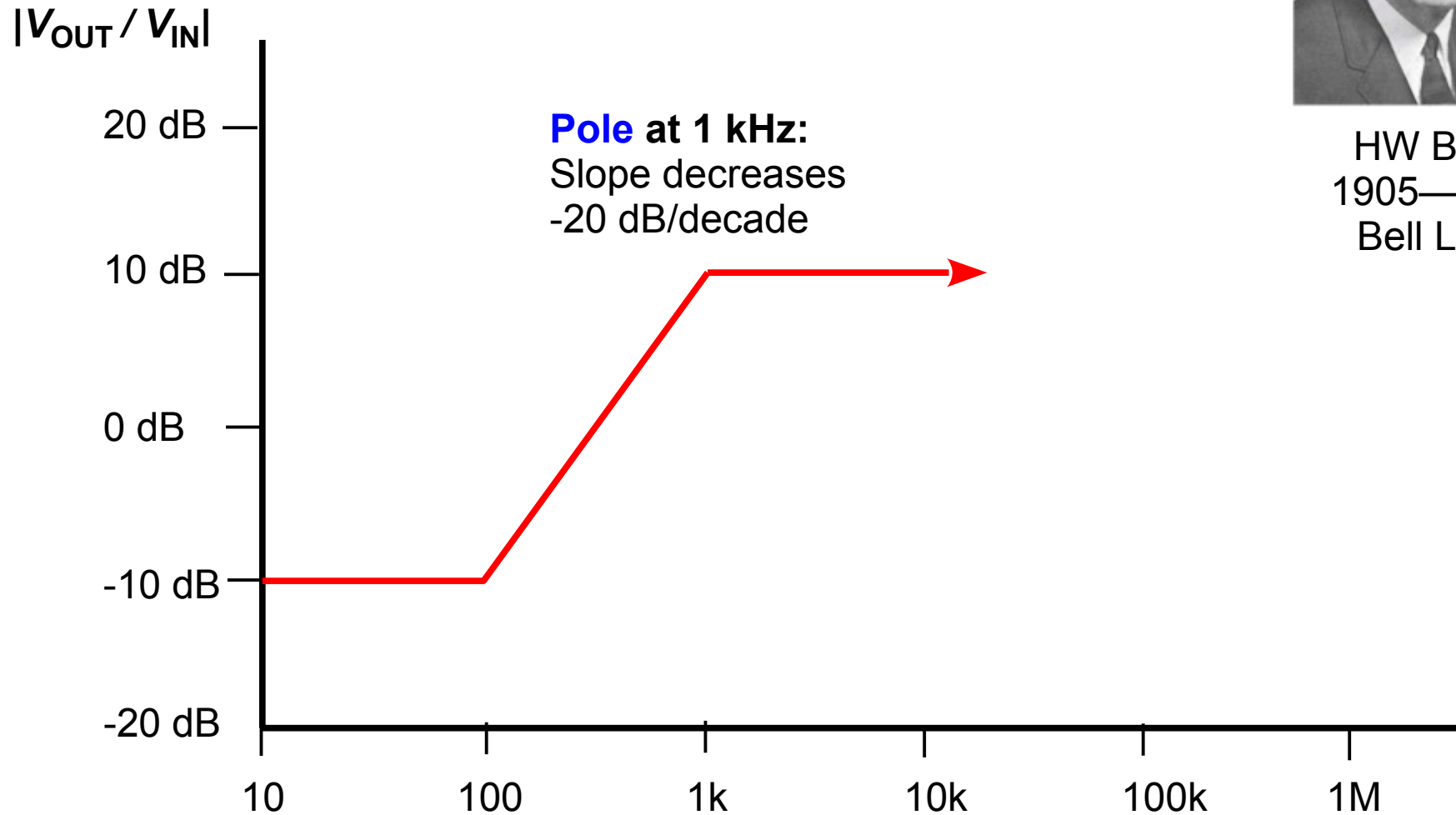
HW Bode
1905—1982
Bell Labs



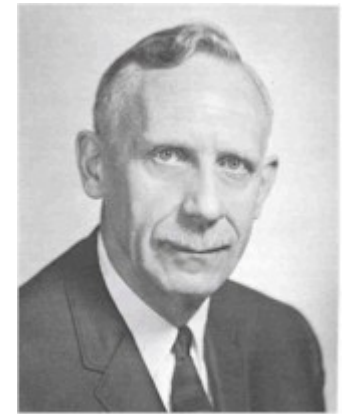
POLES and ZEROES dramatically affect the shape of a Bode plot



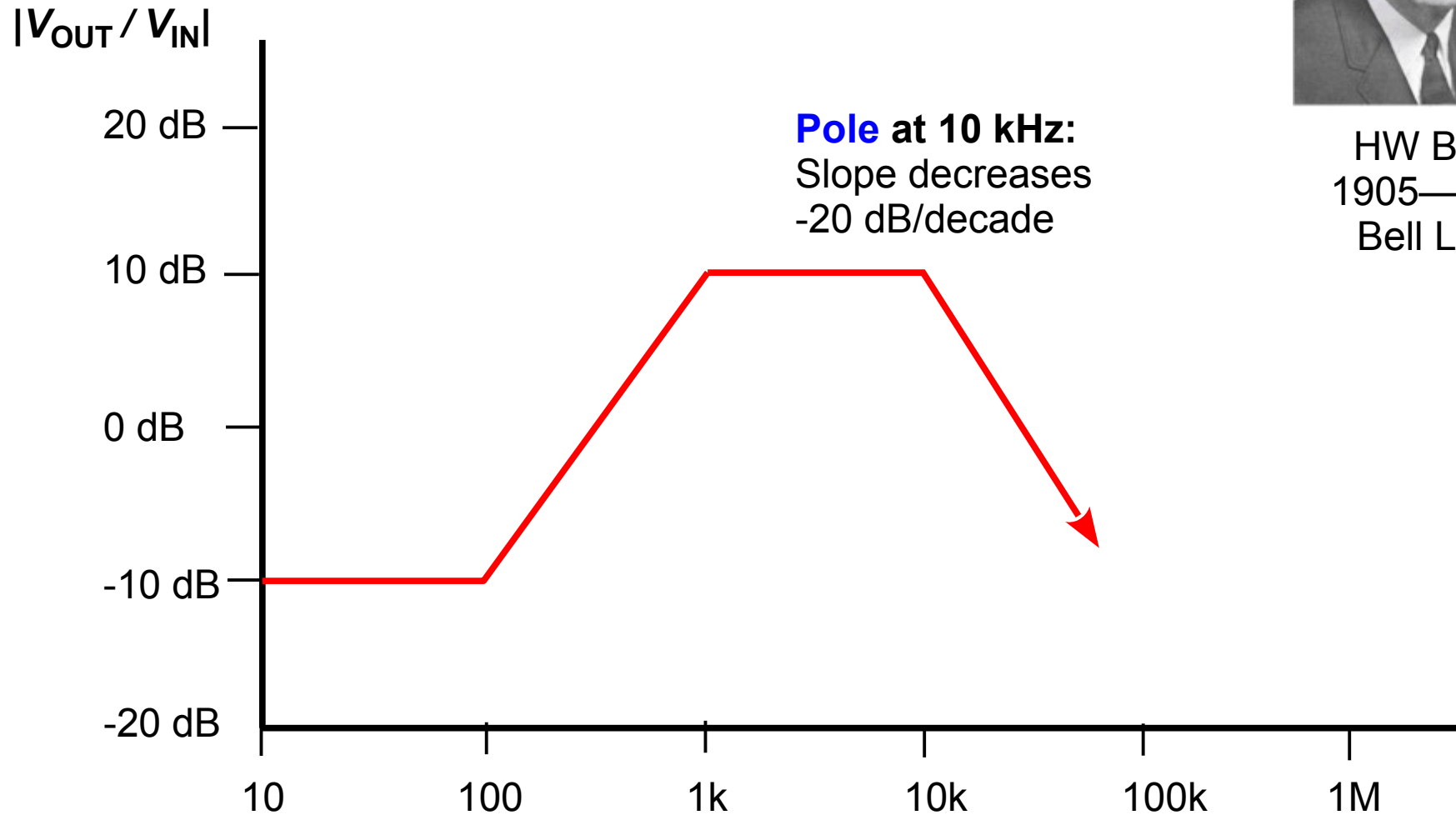
HW Bode
1905—1982
Bell Labs



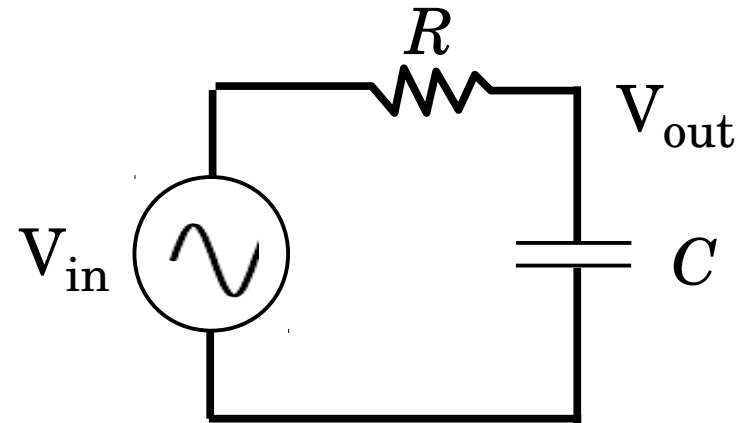
POLES and ZEROES dramatically affect the shape of a Bode plot



HW Bode
1905—1982
Bell Labs

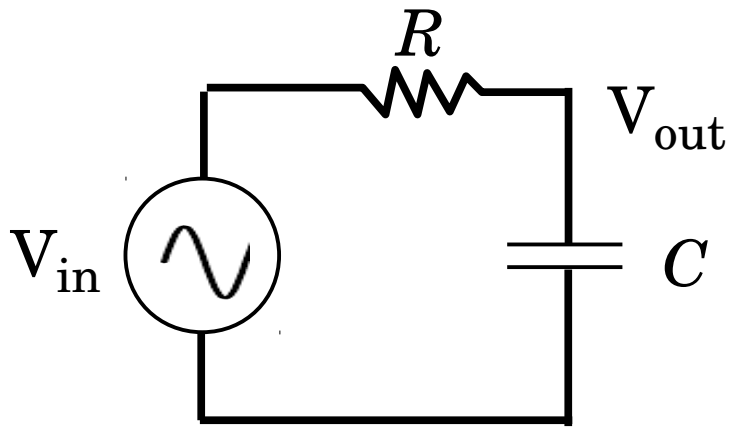


Example 1: Low-pass filter



$$\frac{V_{out}}{V_{in}} = \frac{Z_C}{Z_R + Z_C} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$$

Pole in denominator

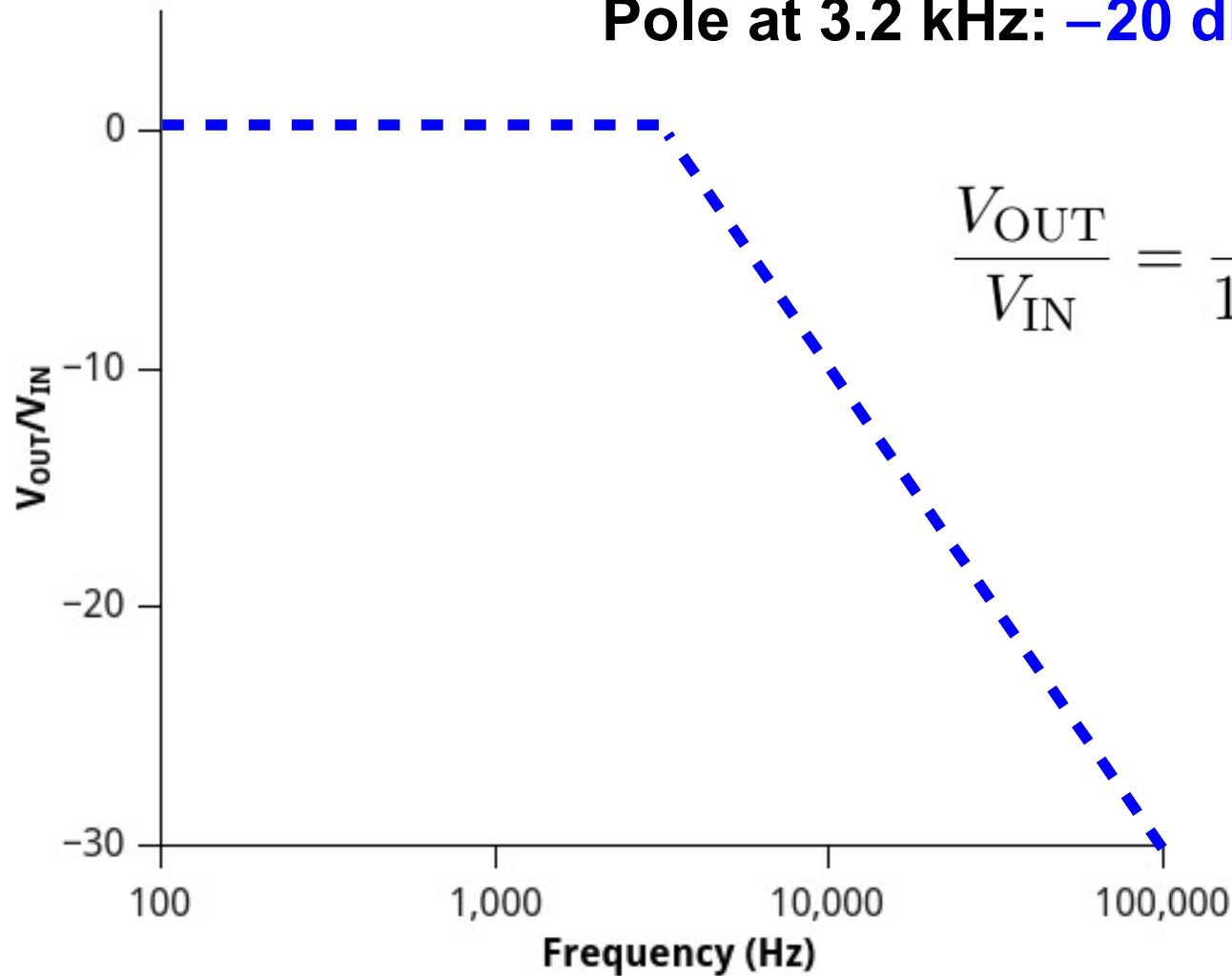


$$R = 5000 \Omega$$

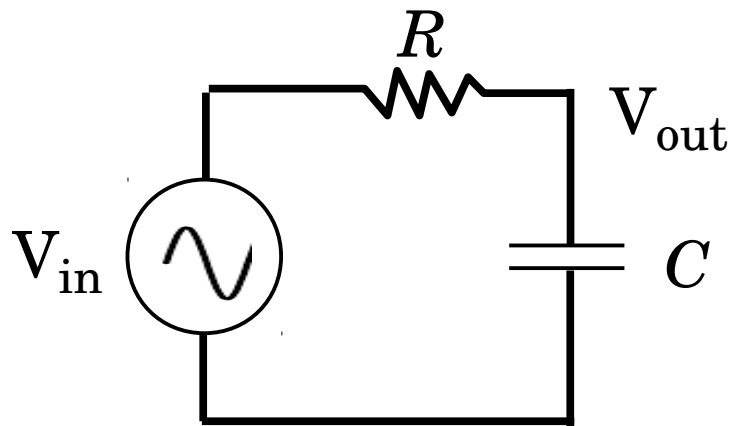
$$C = 10 \text{ nF}$$

$$f = 3.2 \text{ kHz}$$

Pole at 3.2 kHz: -20 dB/decade



$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + j\omega RC}$$

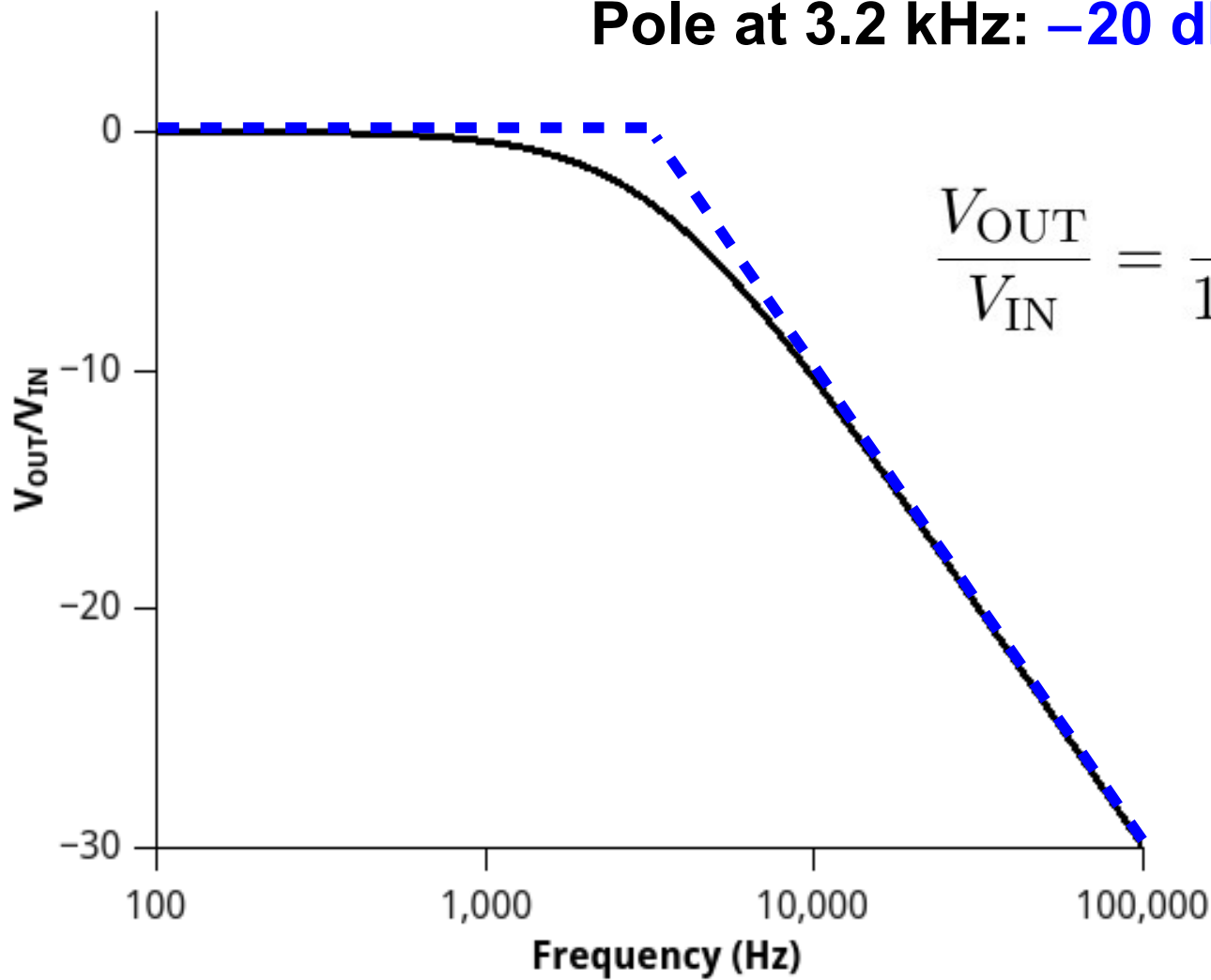


$$R = 5000 \Omega$$

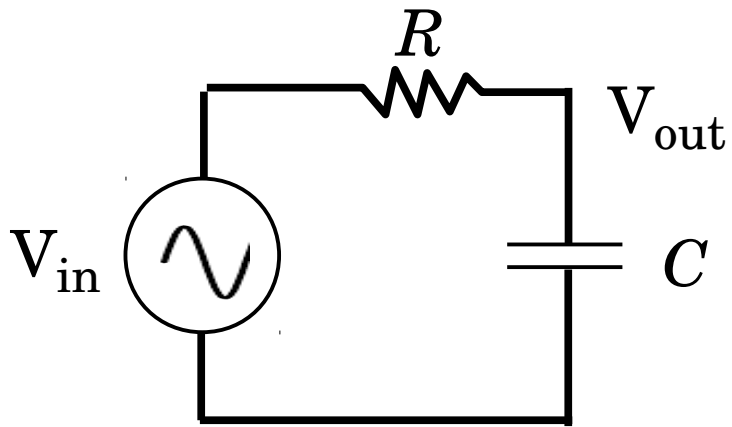
$$C = 10 \text{ nF}$$

$$f = 3.2 \text{ kHz}$$

Pole at 3.2 kHz: -20 dB/decade



$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + j\omega RC}$$

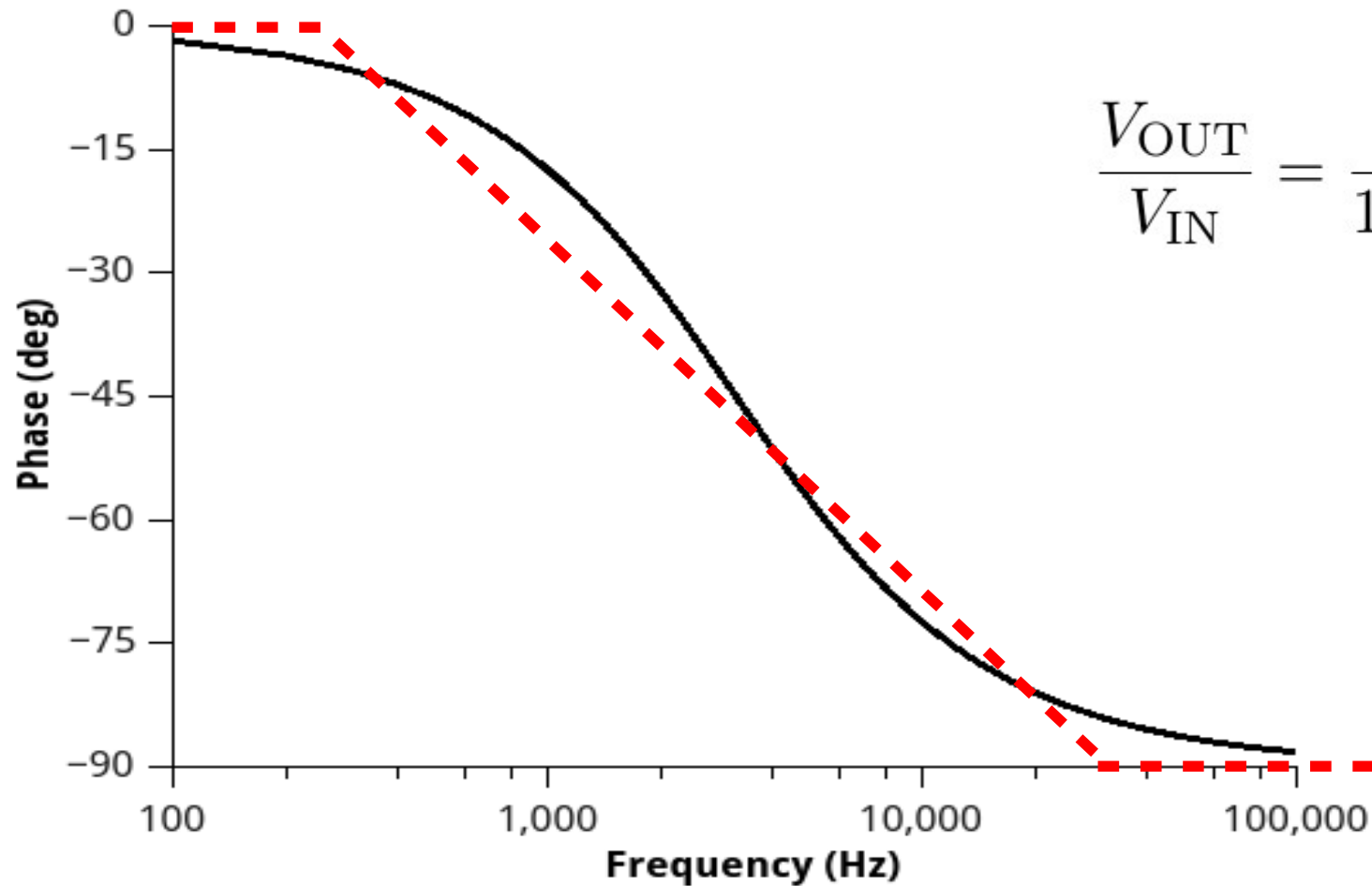


$$R = 5000 \Omega$$

$$C = 10 \text{ nF}$$

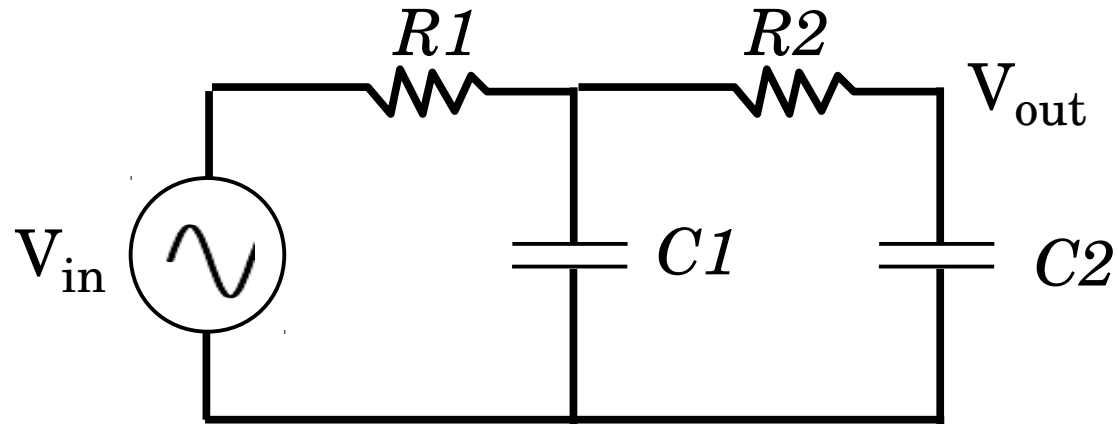
$$f = 3.2 \text{ kHz}$$

Pole at 3.2 kHz: -45 deg/decade



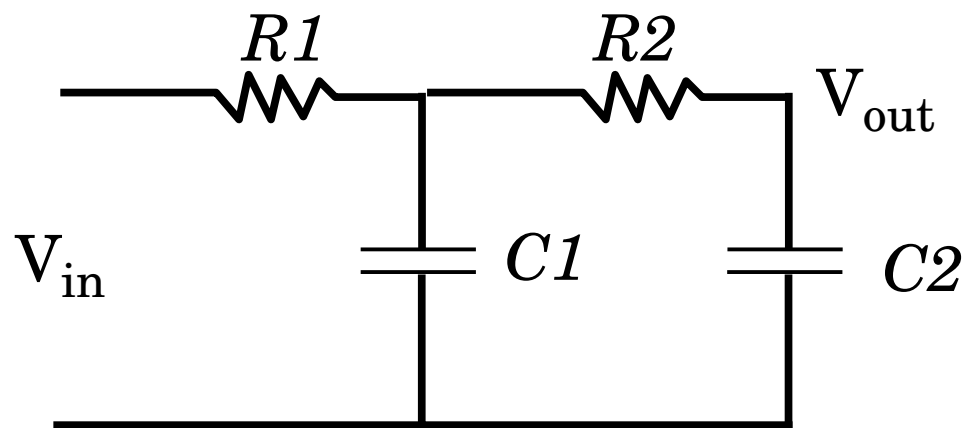
$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + j\omega RC}$$

Example 2: 2-pole low-pass filter



Assume "no loading"

$$\frac{V_{OUT}}{V_{IN}} \approx \underbrace{\left[\frac{1}{1 + j\omega R_1 C_1} \right]}_{\text{1st pole}} \underbrace{\left[\frac{1}{1 + j\omega R_2 C_2} \right]}_{\text{2nd pole}}$$



$$R1 = 5000 \, \Omega$$

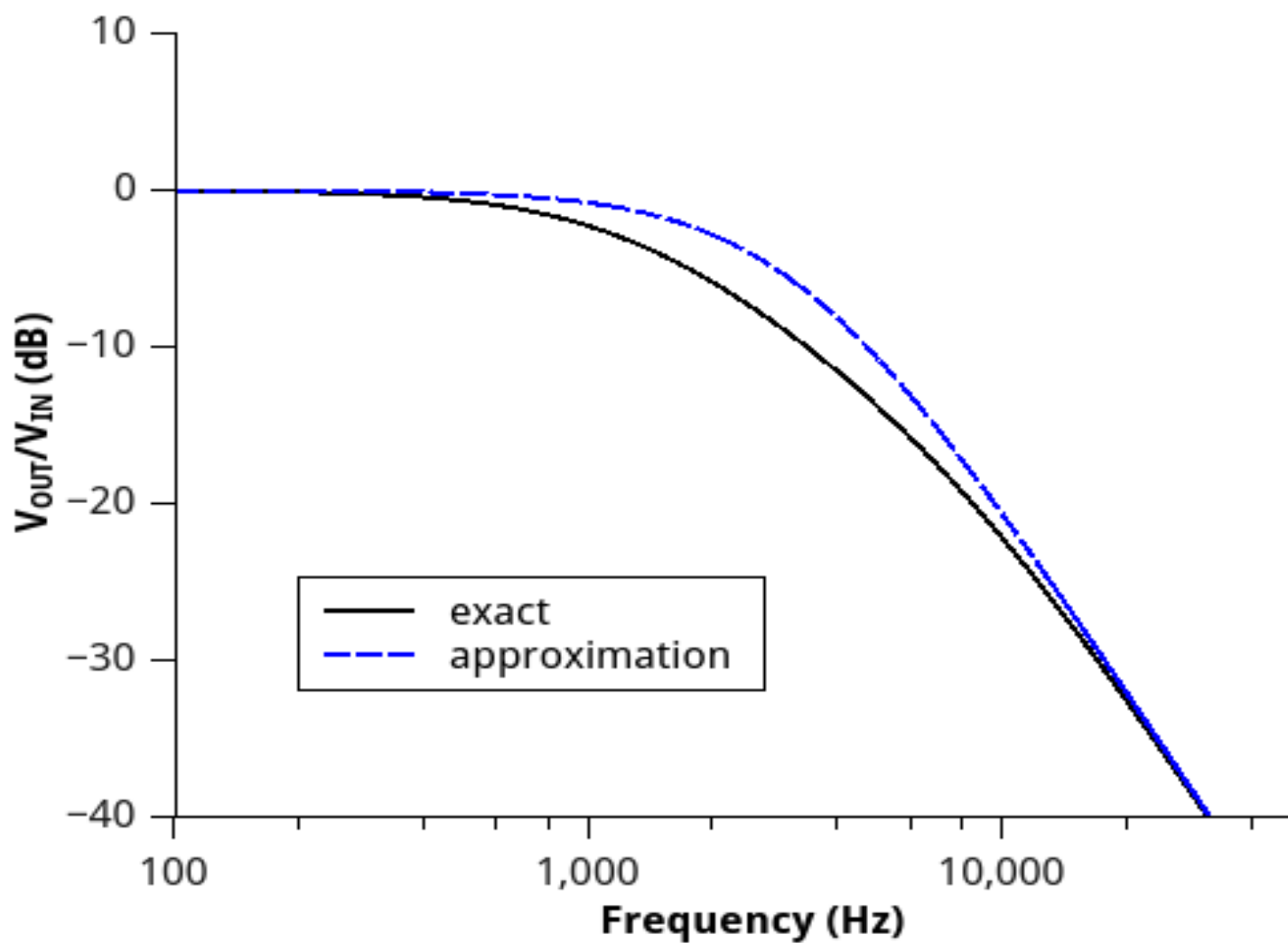
$$R2 = 500 \, \Omega$$

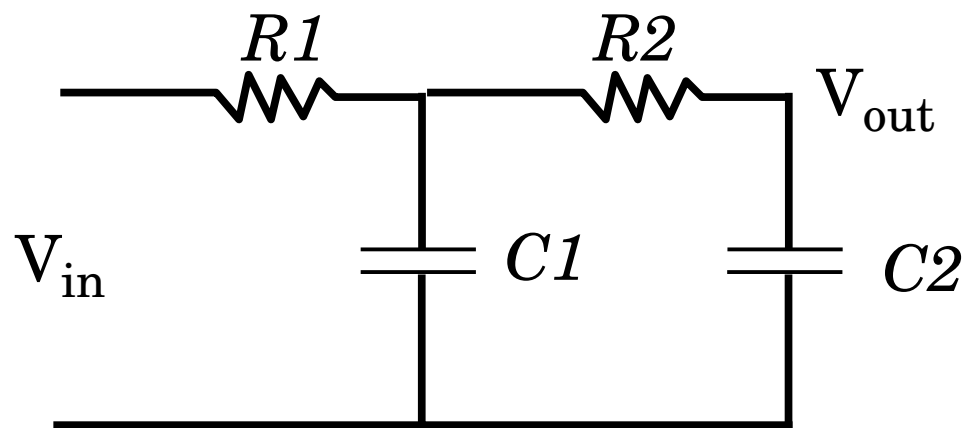
$$C1 = 10 \, \text{nF}$$

$$C2 = 3 \, \text{nF}$$

$$f = 3.2 \, \text{kHz}$$

$$f = 10.6 \, \text{kHz}$$





$$R1 = 5000 \, \Omega$$

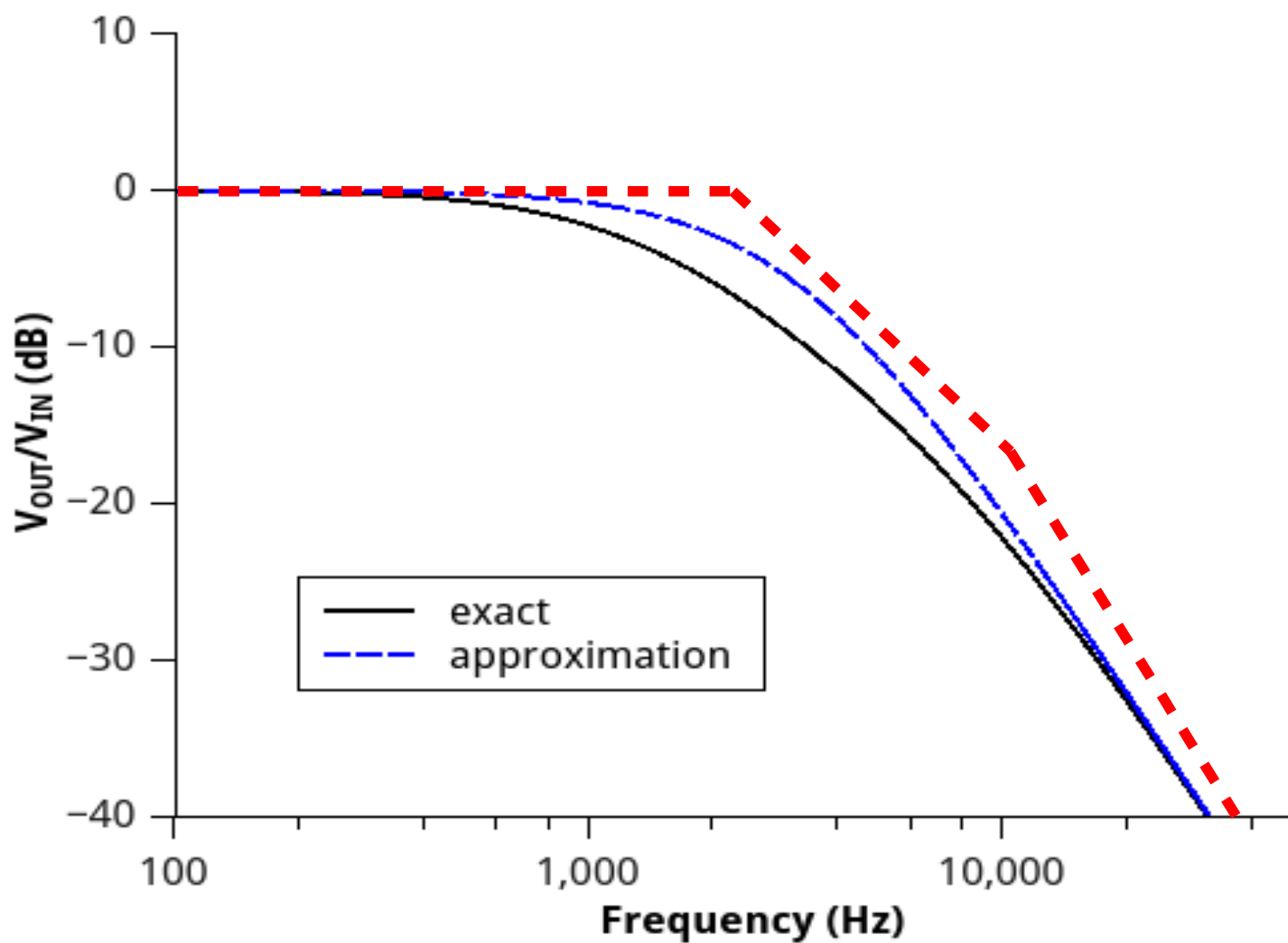
$$R2 = 500 \, \Omega$$

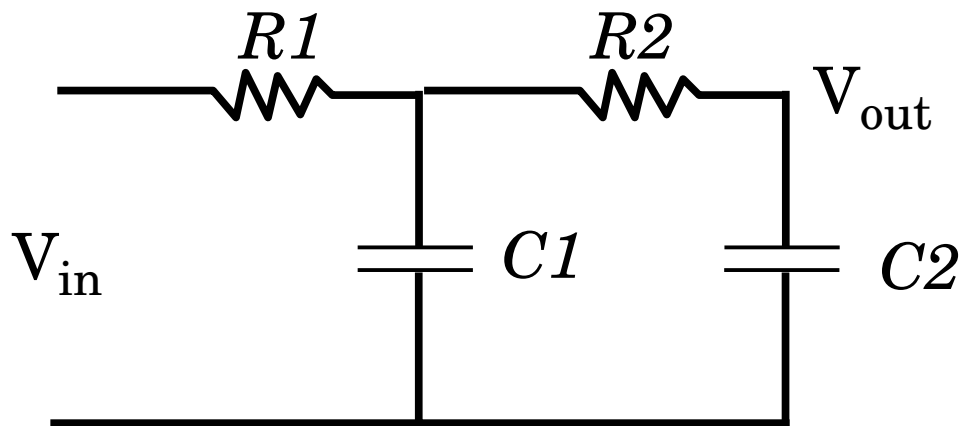
$$C1 = 10 \, \text{nF}$$

$$C2 = 3 \, \text{nF}$$

$$f = 3.2 \, \text{kHz}$$

$$f = 10.6 \, \text{kHz}$$





$$R1 = 5000 \, \Omega$$

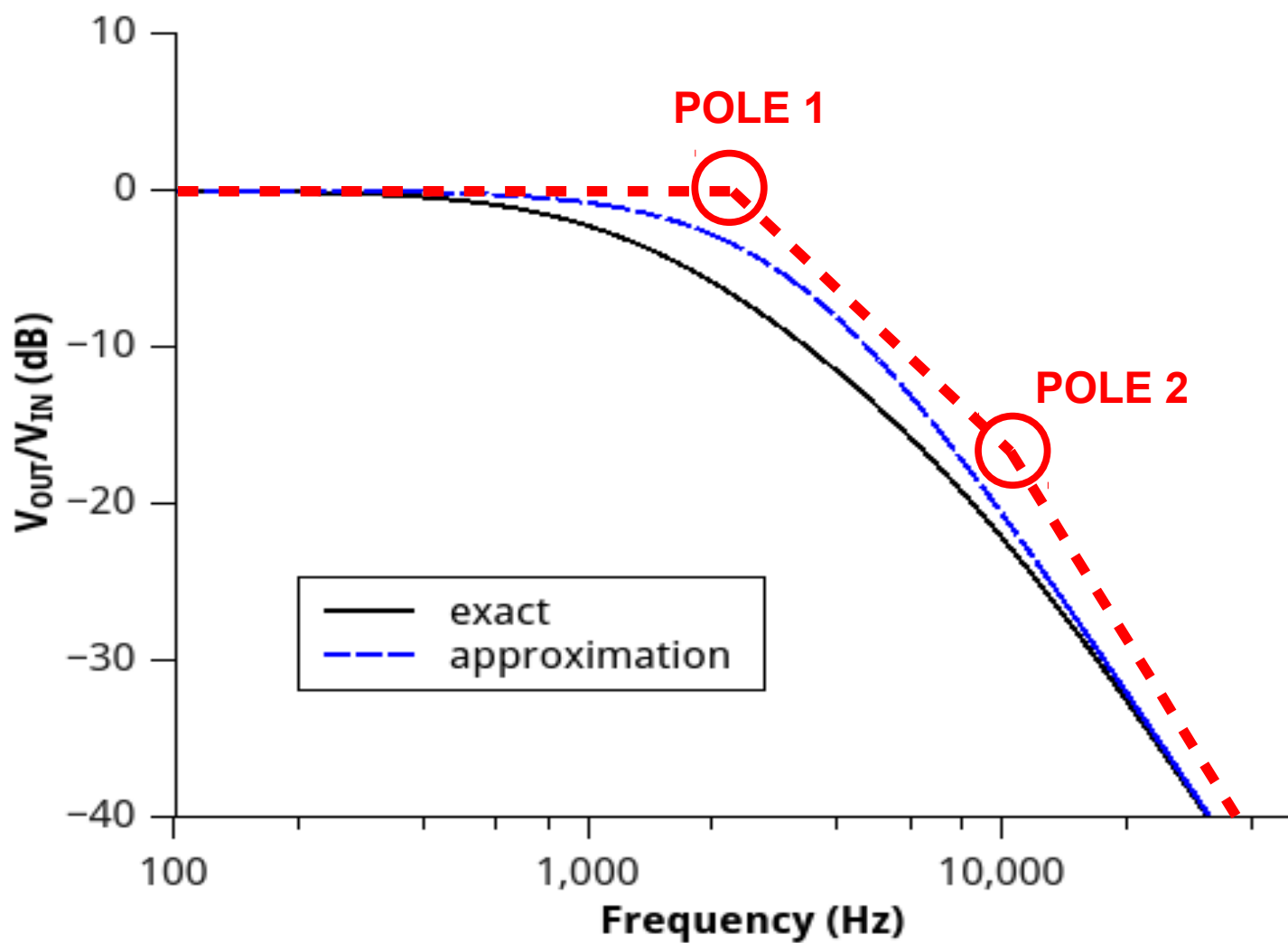
$$R2 = 500 \, \Omega$$

$$C1 = 10 \, \text{nF}$$

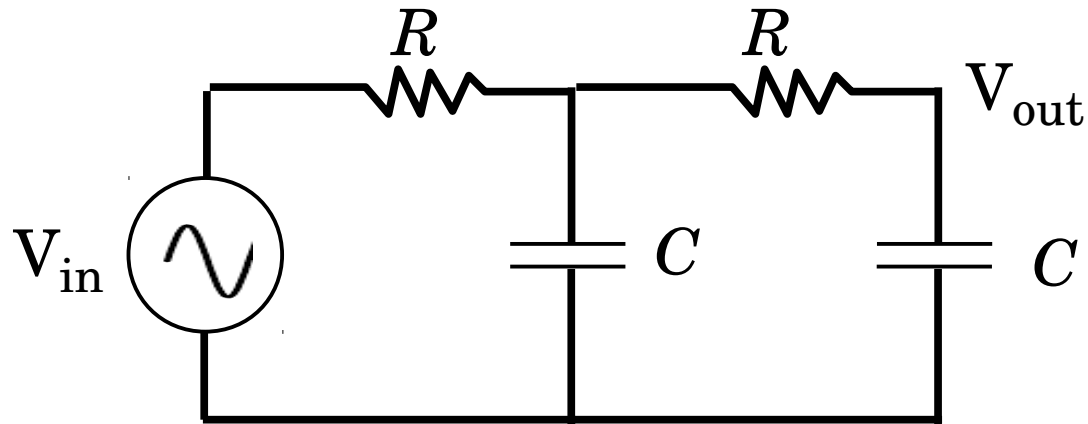
$$C2 = 3 \, \text{nF}$$

$$f = 3.2 \, \text{kHz}$$

$$f = 10.6 \, \text{kHz}$$



Example 3: 2-pole low-pass filter



$$R = 5000 \Omega$$

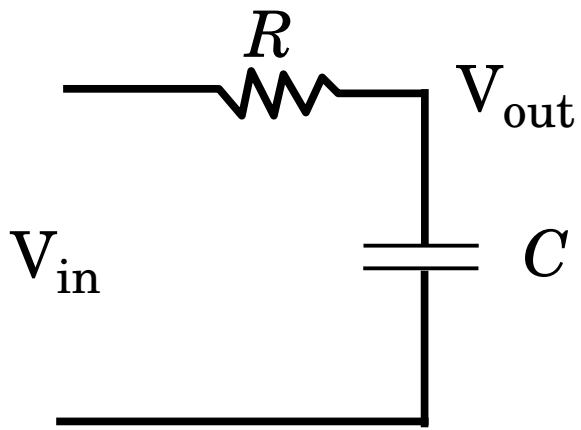
$$C = 10 \text{ nF}$$

$$f = 3.2 \text{ kHz}$$

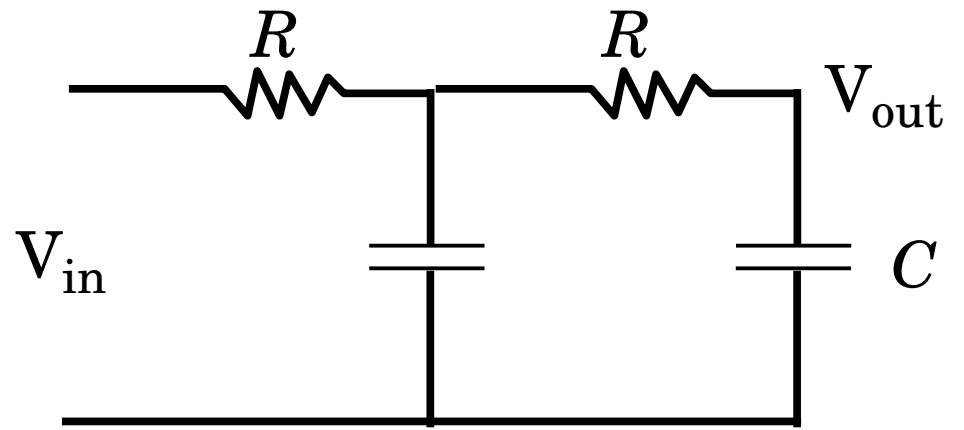
Assume “no loading”

$$\frac{V_{OUT}}{V_{IN}} \approx \left[\frac{1}{1 + j\omega RC} \right]^2$$

2 identical poles

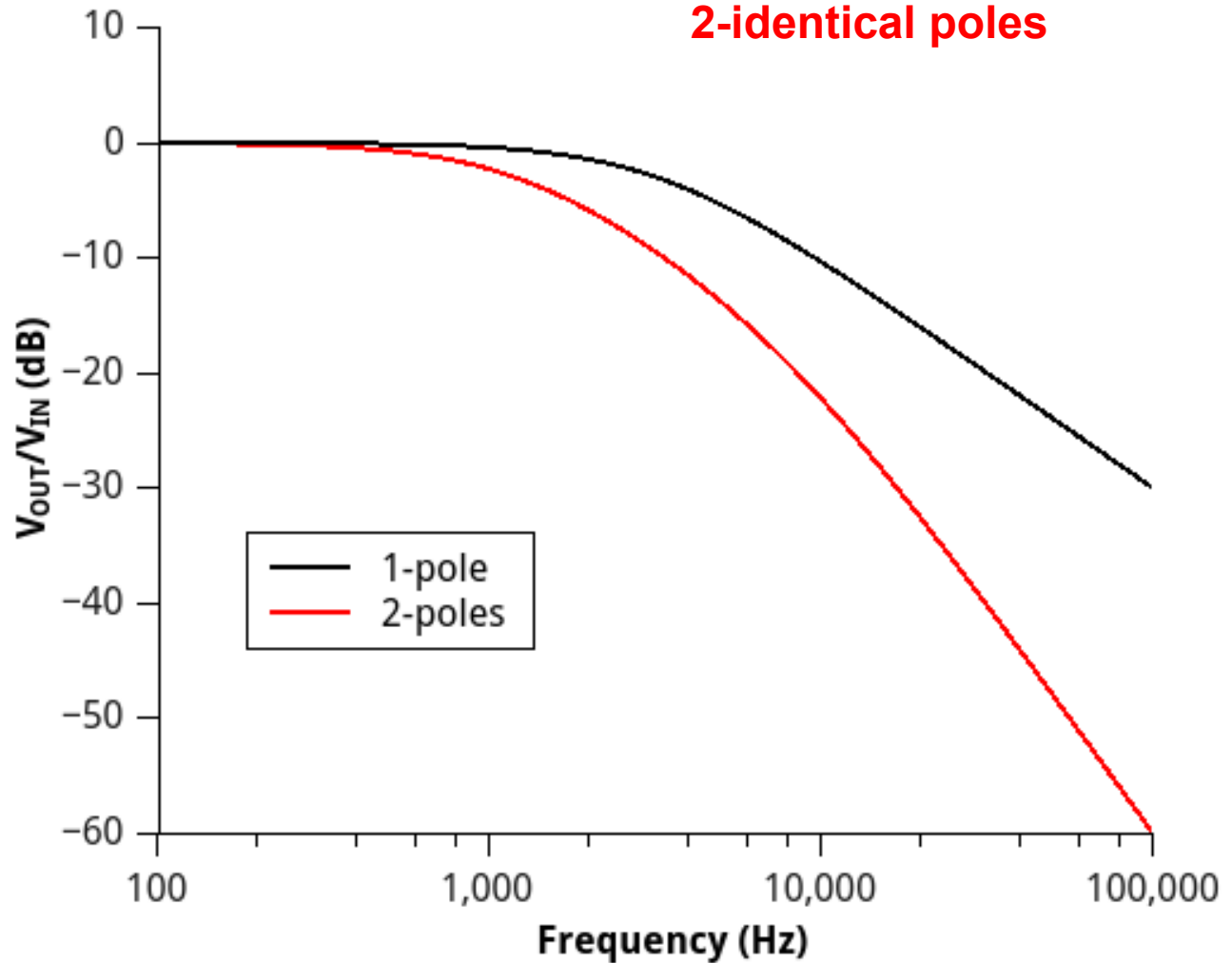


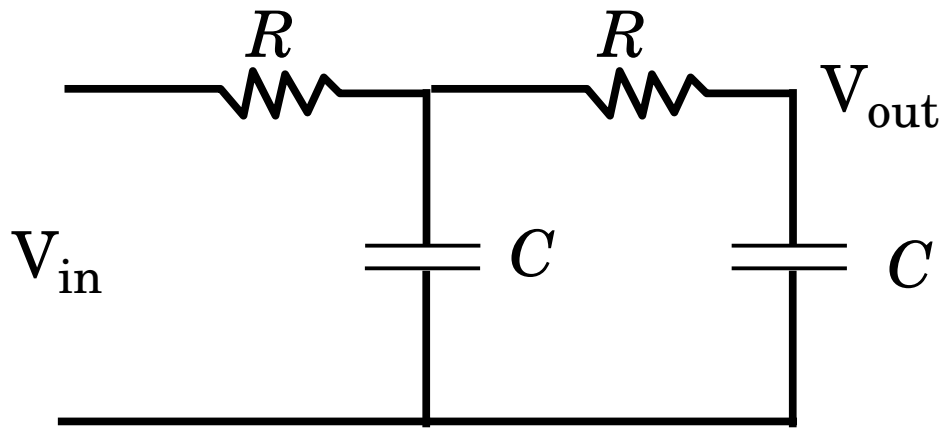
1-pole



2-identical poles

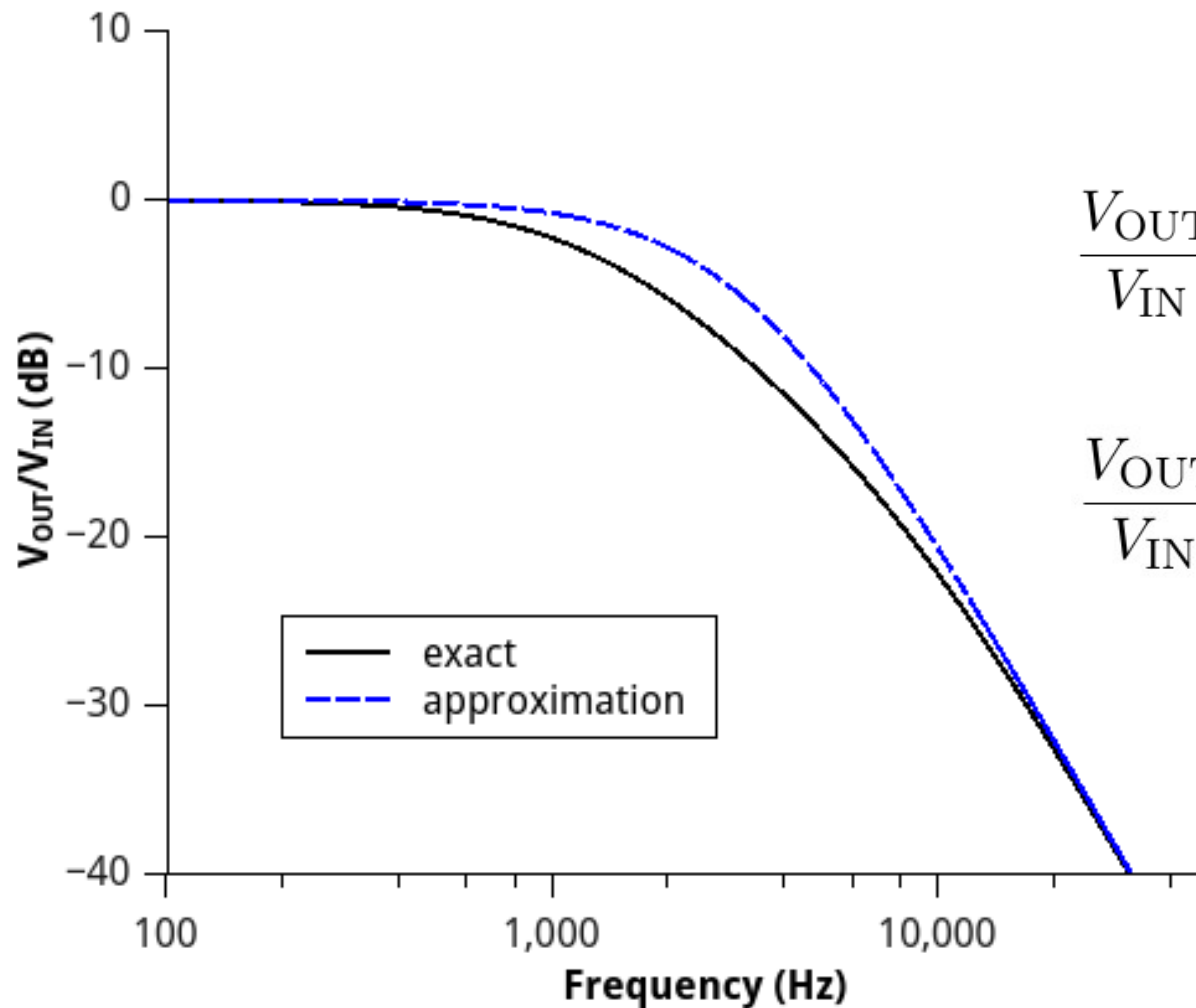
$R = 5000 \Omega$
 $C = 10 \text{ nF}$
 $f = 3.2 \text{ kHz}$





2 identical poles

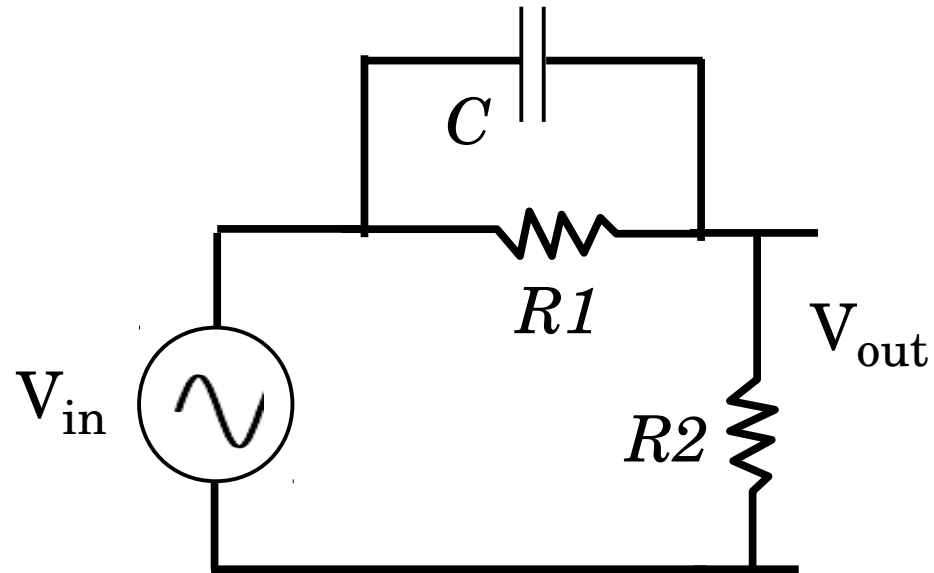
- Effect of Loading
- Analyze as uncoupled stages



$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{1 - (\omega RC)^2 + j\omega 3RC}$$

$$\frac{V_{OUT}}{V_{IN}} \approx \left[\frac{1}{1 + j\omega RC} \right]^2$$

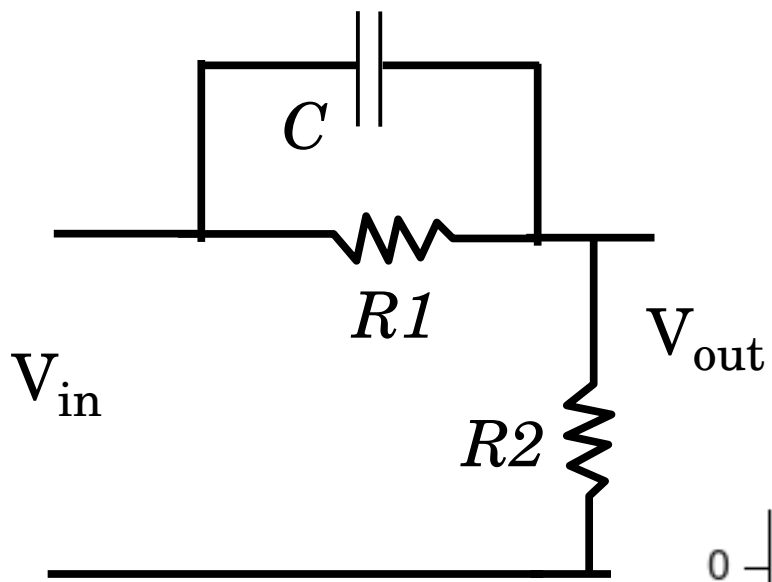
Example 4: 1-pole + 1-zero



$$\frac{V_{OUT}}{V_{IN}} = \left[\frac{R2}{R1 + R2} \right] \left[\frac{1 + j\omega R1C}{1 + j\omega C \frac{R1R2}{R1+R2}} \right]$$

ZERO in numerator

POLE in denominator

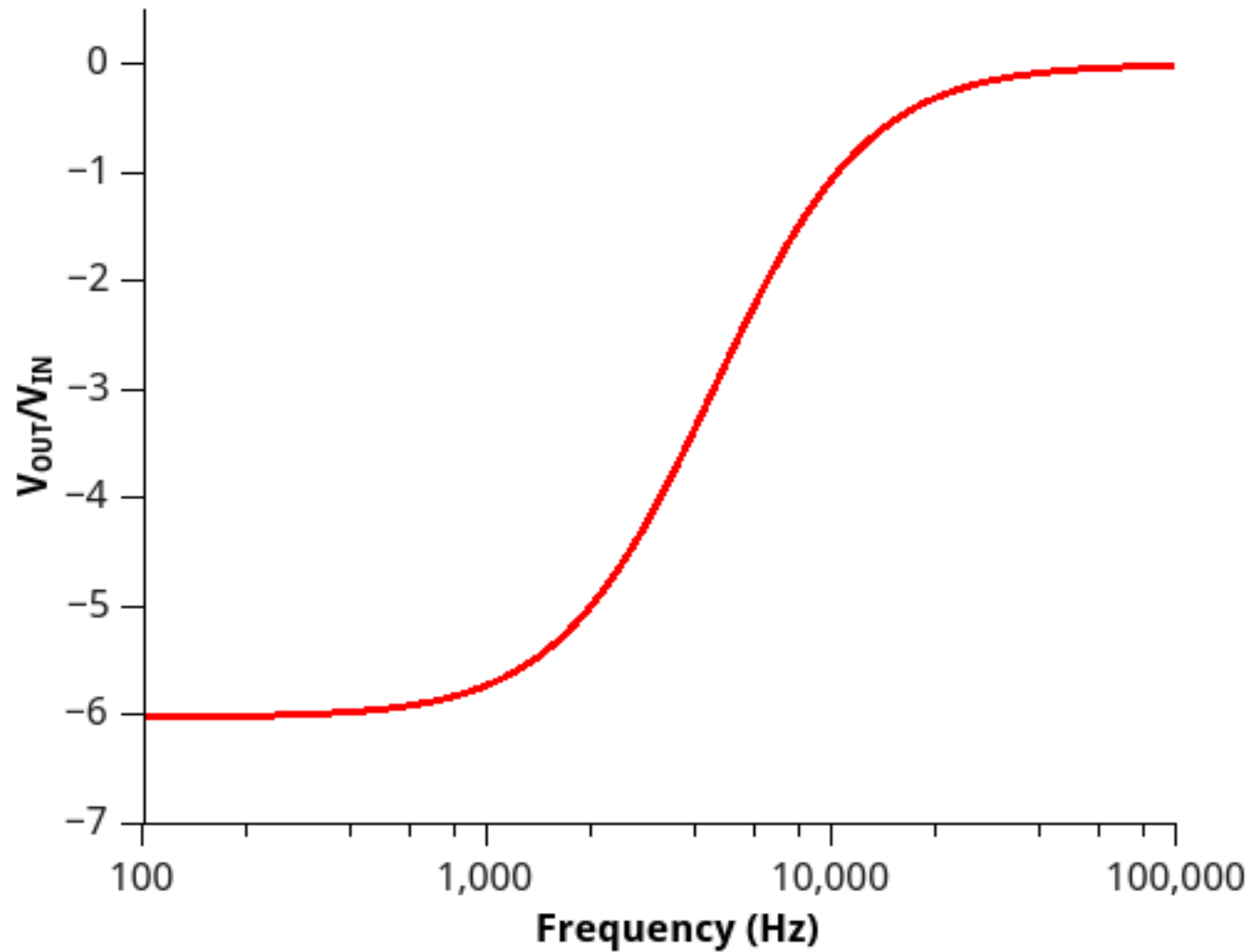


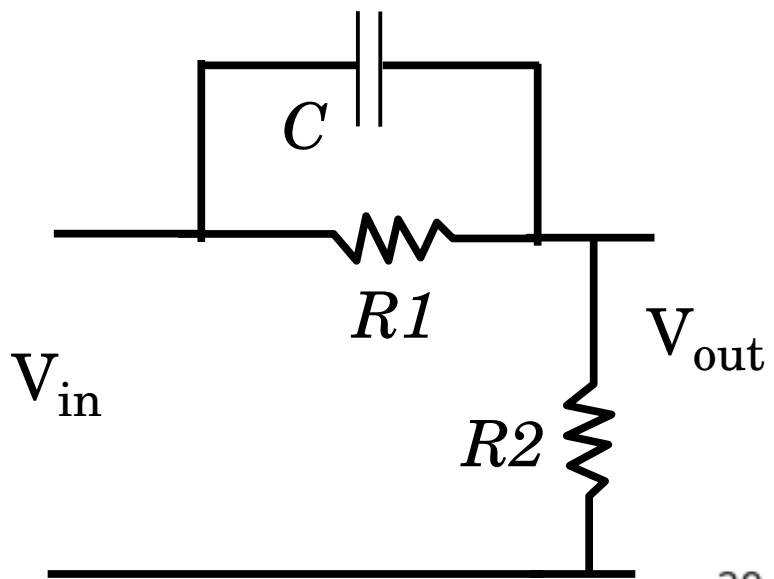
$$R1=R2 = 500 \Omega$$

$$C = 100 \text{ nF}$$

$$f_{ZERO} = 3.2 \text{ kHz}$$

$$f_{POLE} = 6.4 \text{ kHz}$$





$$R1=R2 = 500 \Omega$$

$$C = 100 \text{ nF}$$

$$f_{ZERO} = 3.2 \text{ kHz}$$

$$f_{POLE} = 6.4 \text{ kHz}$$

