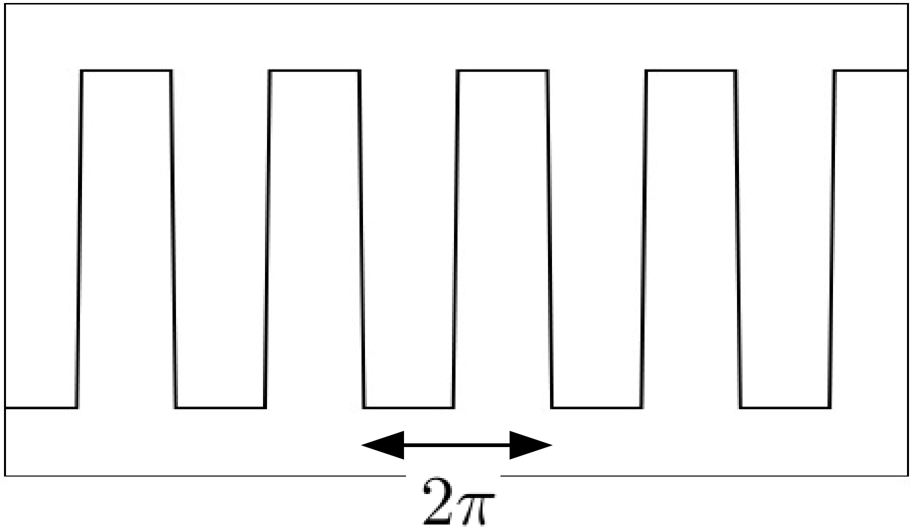
Fourier Analysis

Joseph Fourier
1768-1830

Fourier Series

Periodic function $f(x)$

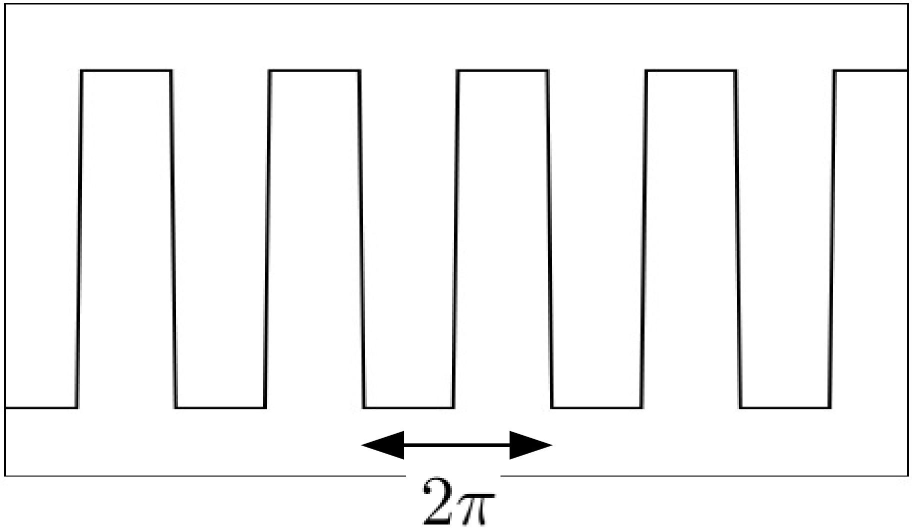


$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(nx) + b_n \sin(nx)]$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx \qquad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx$$

Fourier Series

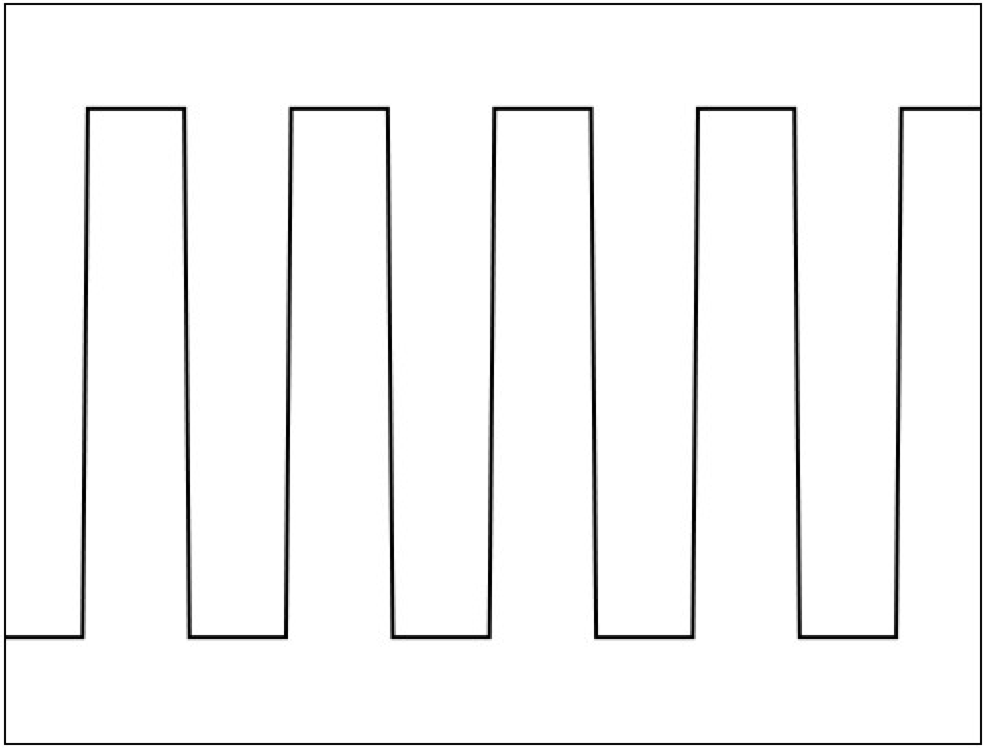
Periodic function $f(x)$



$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{jnx}$$

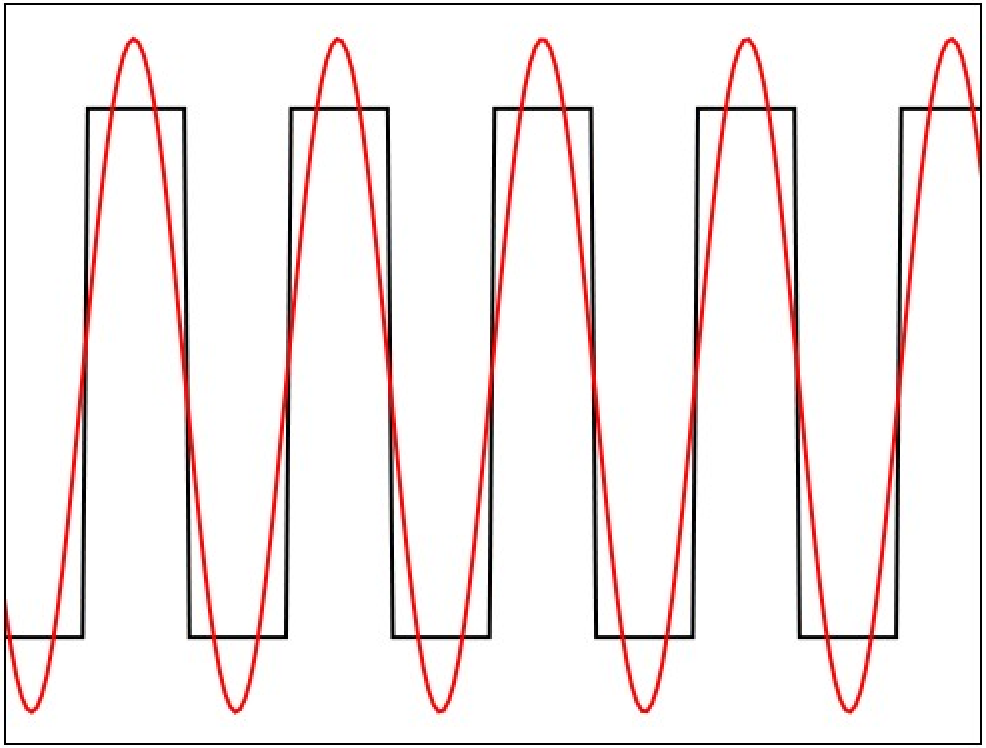
$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-jnx} dx$$

Fourier Series: Rectangular wave



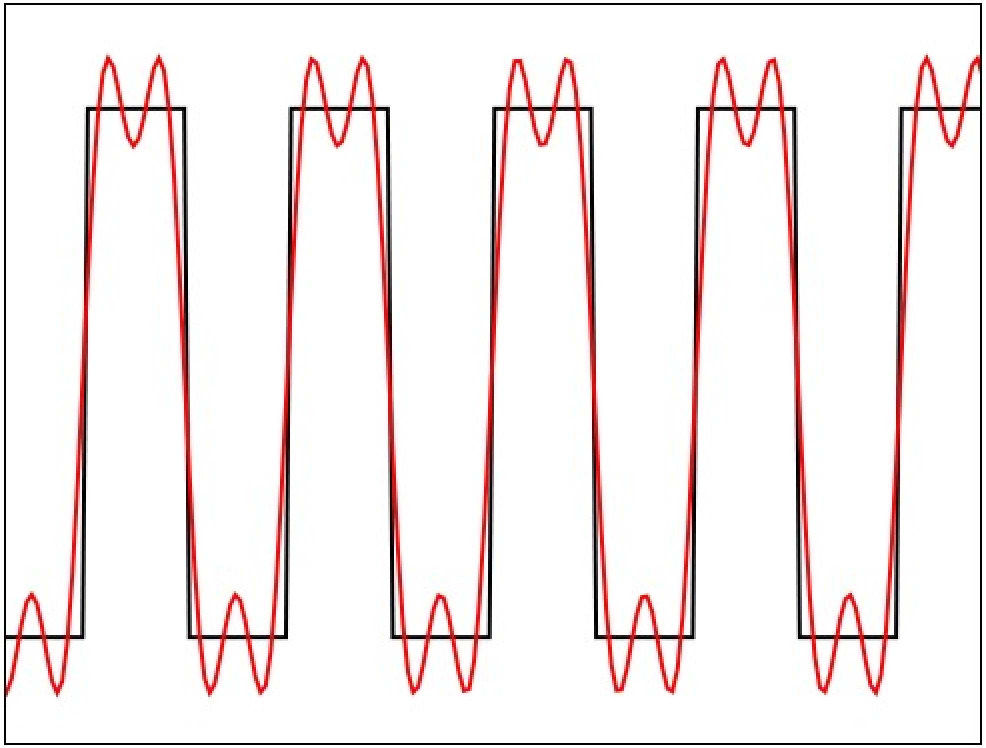
Fourier Series: Rectangular wave

1st order component



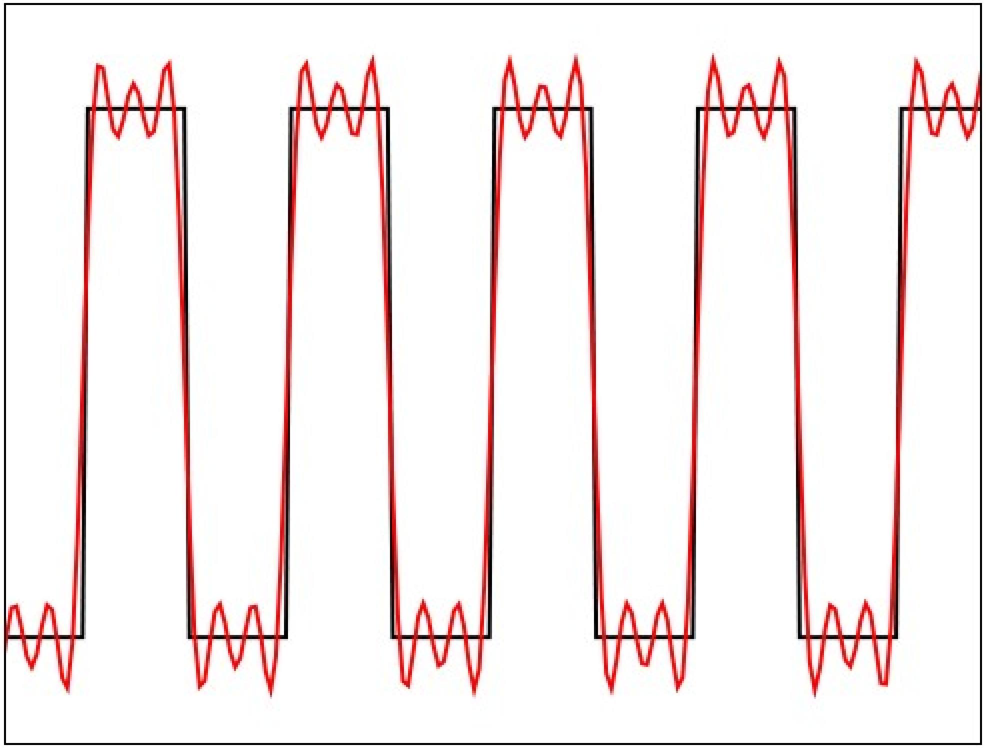
Fourier Series: Rectangular wave

1st + 2nd order components



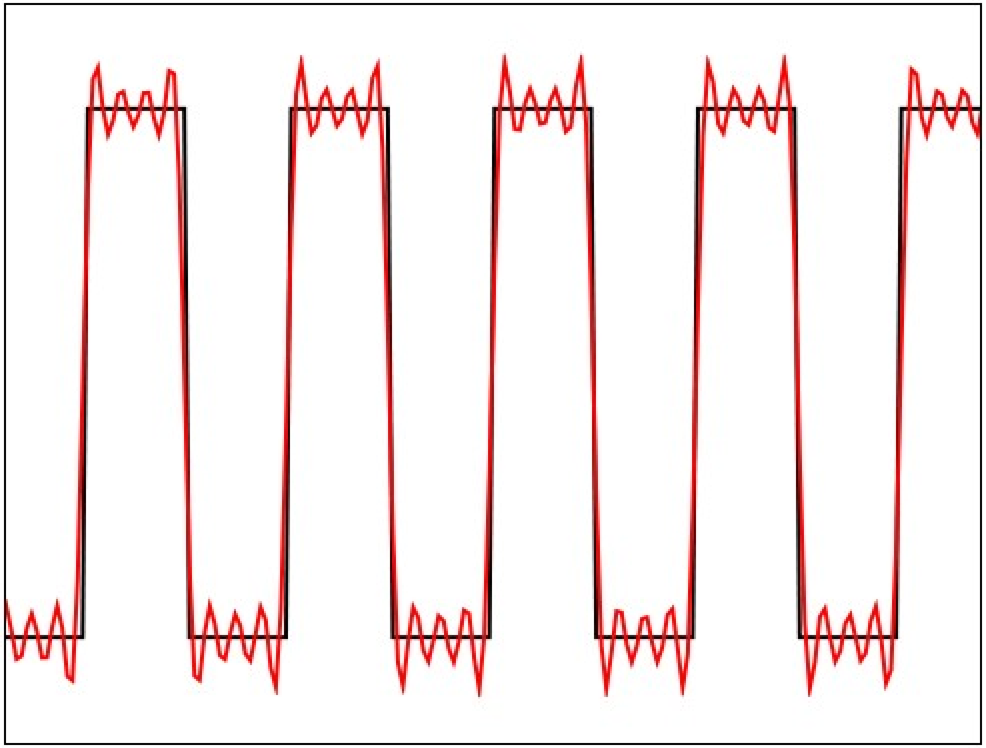
Fourier Series: Rectangular wave

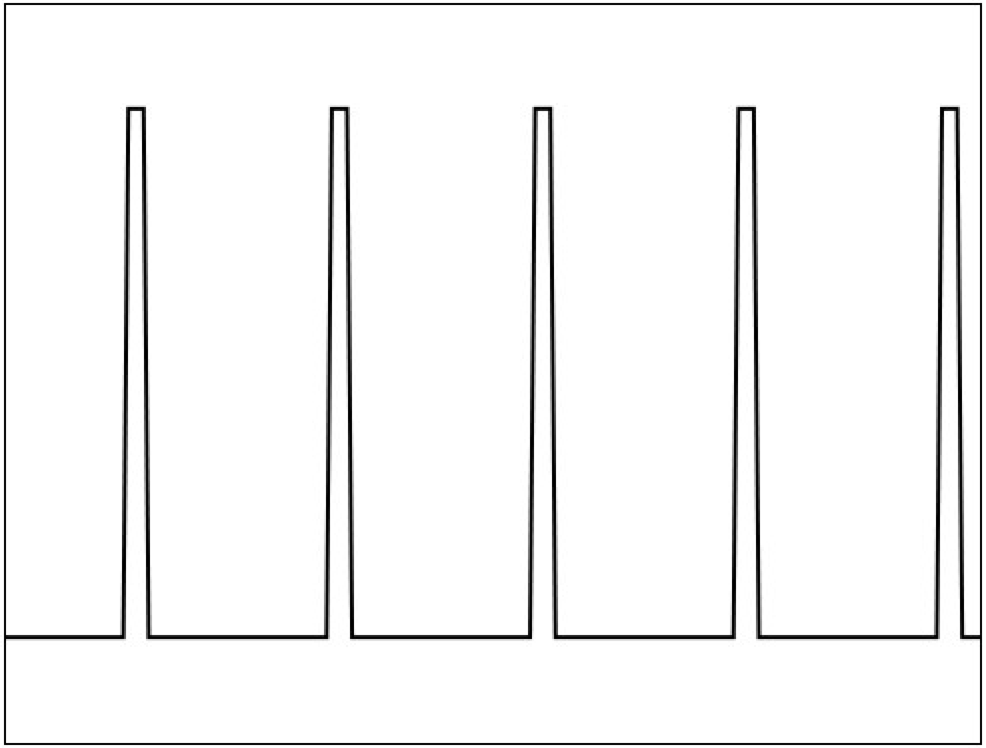
1st + 2nd + 3rd order components



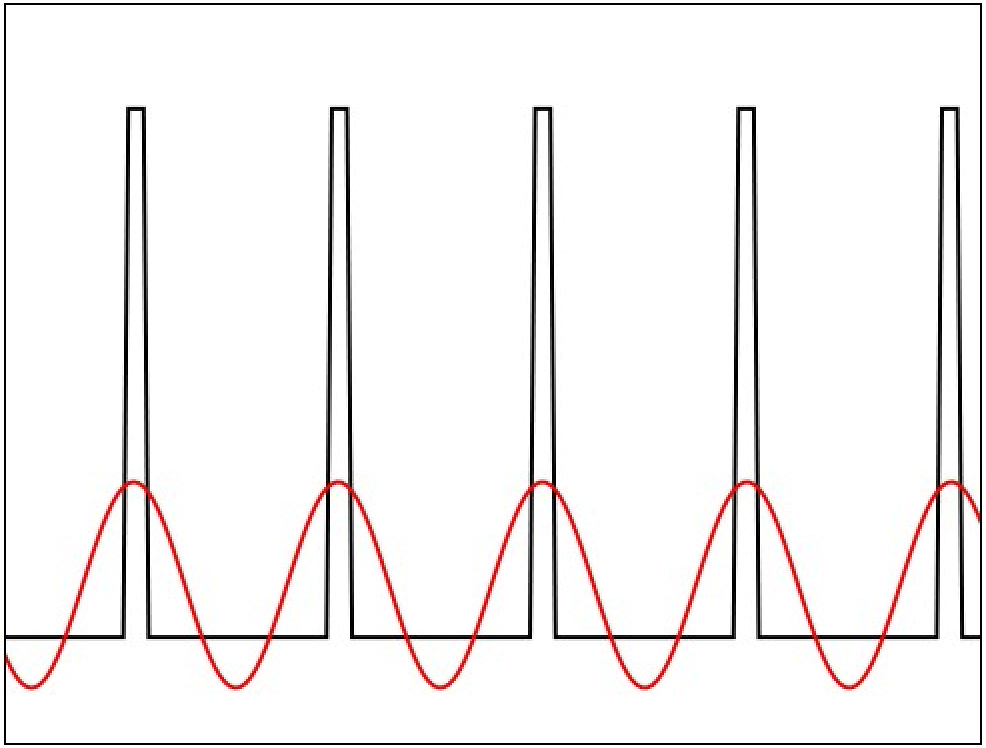
Fourier Series: Rectangular wave

1st + 2nd + 3rd + 4th order components

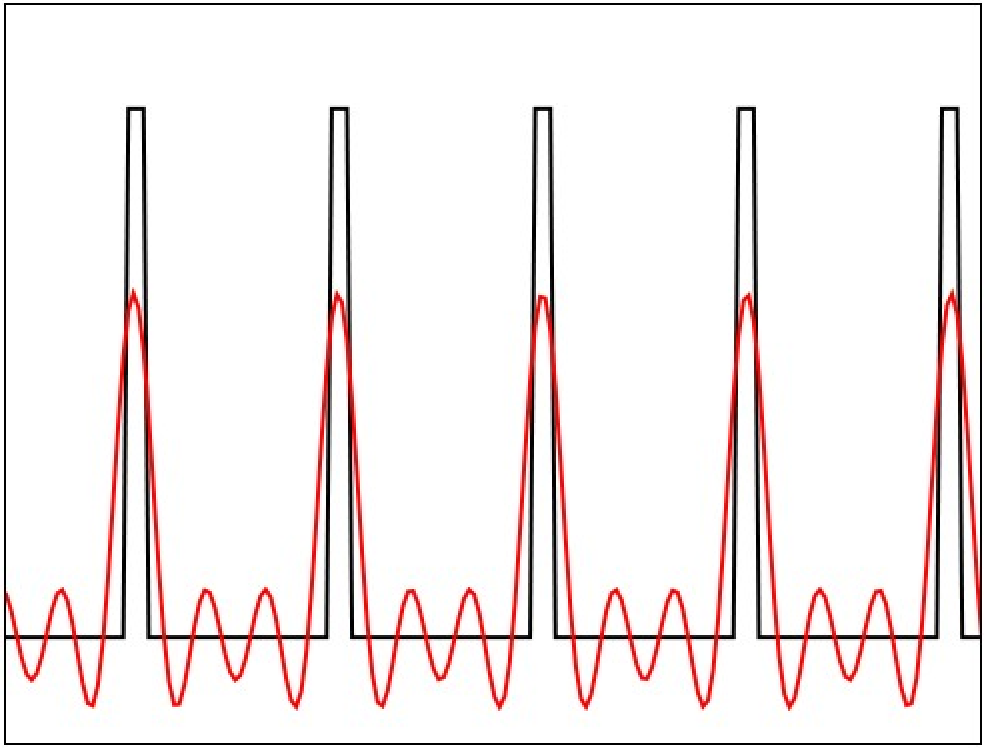




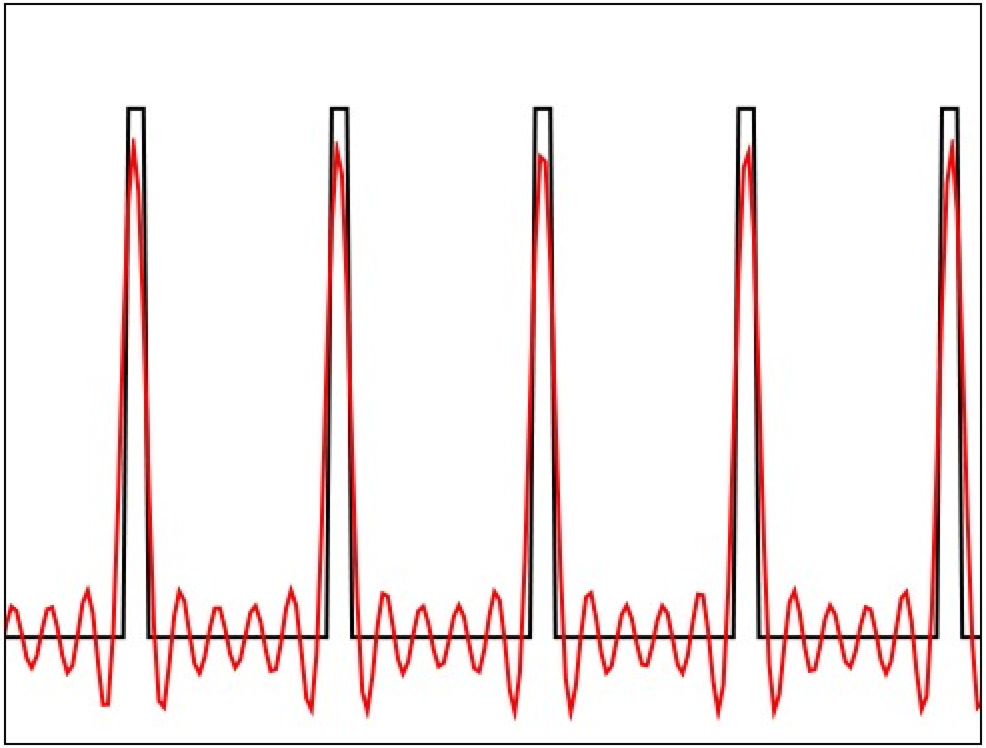
1st order



1st + 2nd order



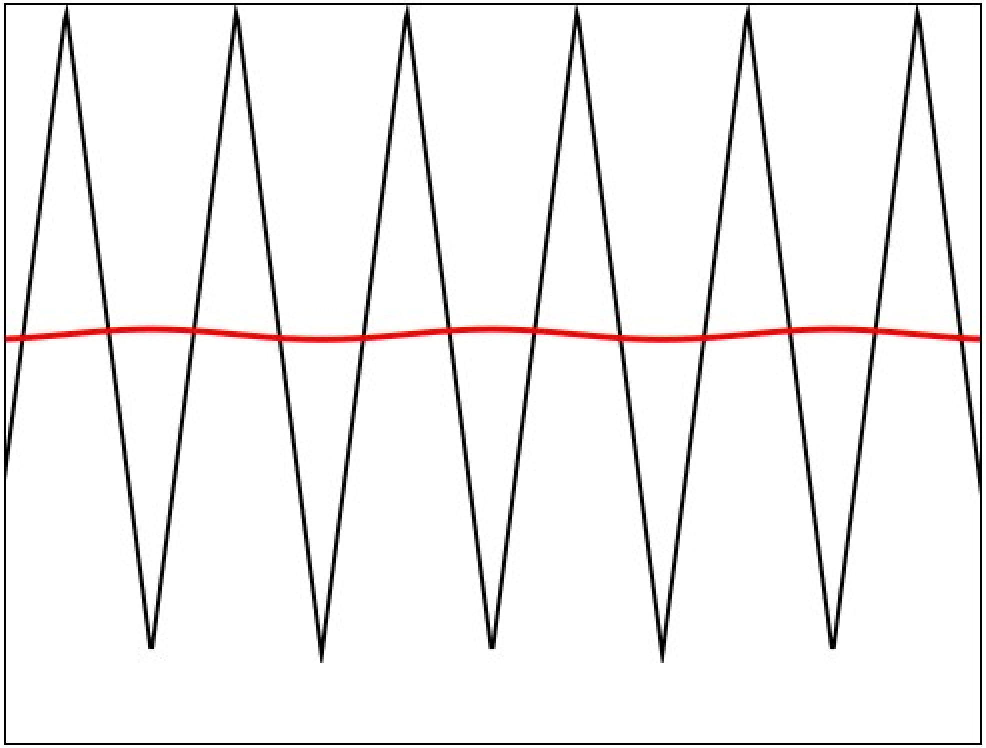
1st + 2nd + 3rd order



1st + 2nd + 3rd + 4th order components

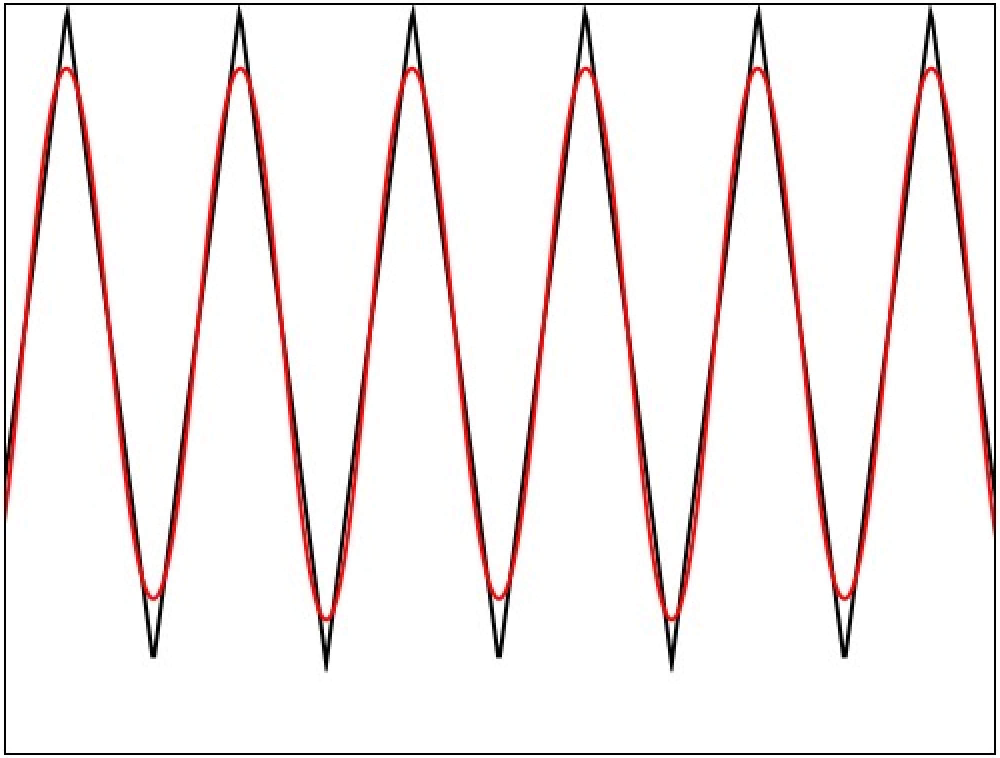
Triangle (Sawtooth) Wave

1st order



Triangle (Sawtooth) Wave

1st + 2nd order



Fourier Series ⇒ Fourier-Transform

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n x \frac{1}{T}} \qquad c_n = \frac{1}{T} \int_{-T/2}^{T/2} f(\hat{x}) e^{-j2\pi n \hat{x} \frac{1}{T}} d\hat{x}$$

$$f(x) = \frac{1}{T} \sum_{n=-\infty}^{\infty} e^{j2\pi n x \frac{1}{T}} \left[\int_{-T/2}^{T/2} f(\hat{x}) e^{-j2\pi n \hat{x} \frac{1}{T}} d\hat{x} \right]$$

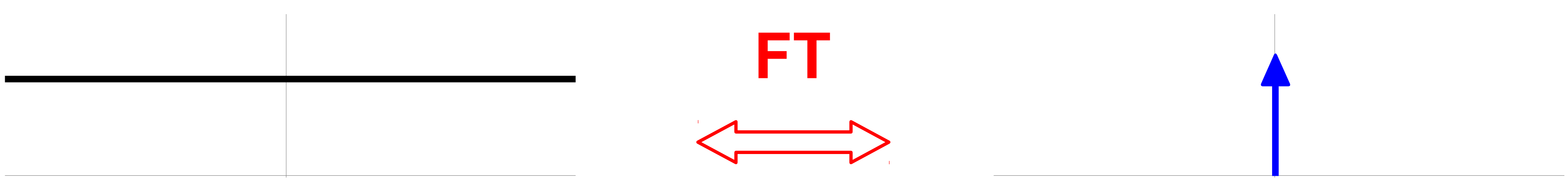
$$\nu = \frac{n}{T} \qquad f(x) = \int_{-\infty}^{\infty} d\nu \; e^{j2\pi \nu x} \left[\int_{-\infty}^{\infty} f(\hat{x}) e^{-j2\pi \nu \hat{x}} d\hat{x} \right]$$

Fourier-Transform: $$f(x) = \int_{-\infty}^{\infty} d\nu \; e^{j2\pi \nu x} F(\nu)$$

Fourier-Transform

$$f(x) = \int_{-\infty}^{\infty} d\nu \; e^{j2\pi\nu x} F(\nu)$$

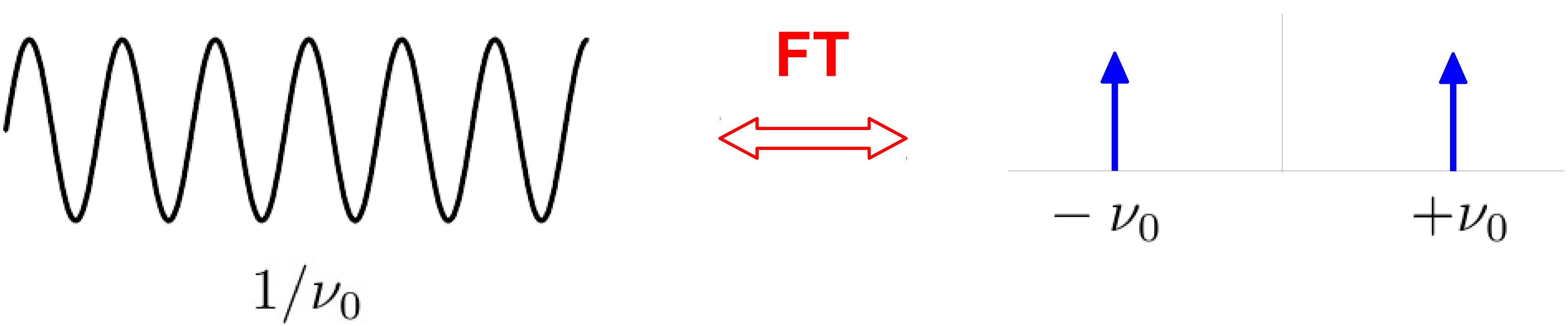
$$F(\nu) = \int_{-\infty}^{\infty} f(\hat{x}) e^{-j2\pi\nu\hat{x}} d\hat{x}$$



Fourier-Transform

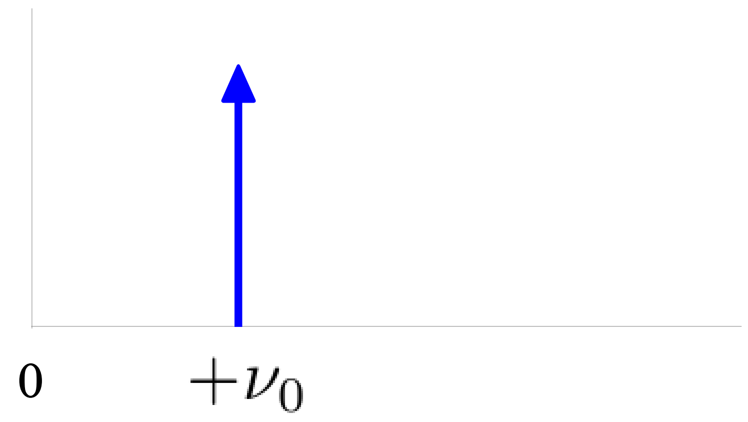
$$f(x) = \int_{-\infty}^{\infty} d\nu \; e^{j2\pi\nu x} F(\nu)$$

$$F(\nu) = \int_{-\infty}^{\infty} f(\hat{x}) e^{-j2\pi\nu\hat{x}} d\hat{x}$$



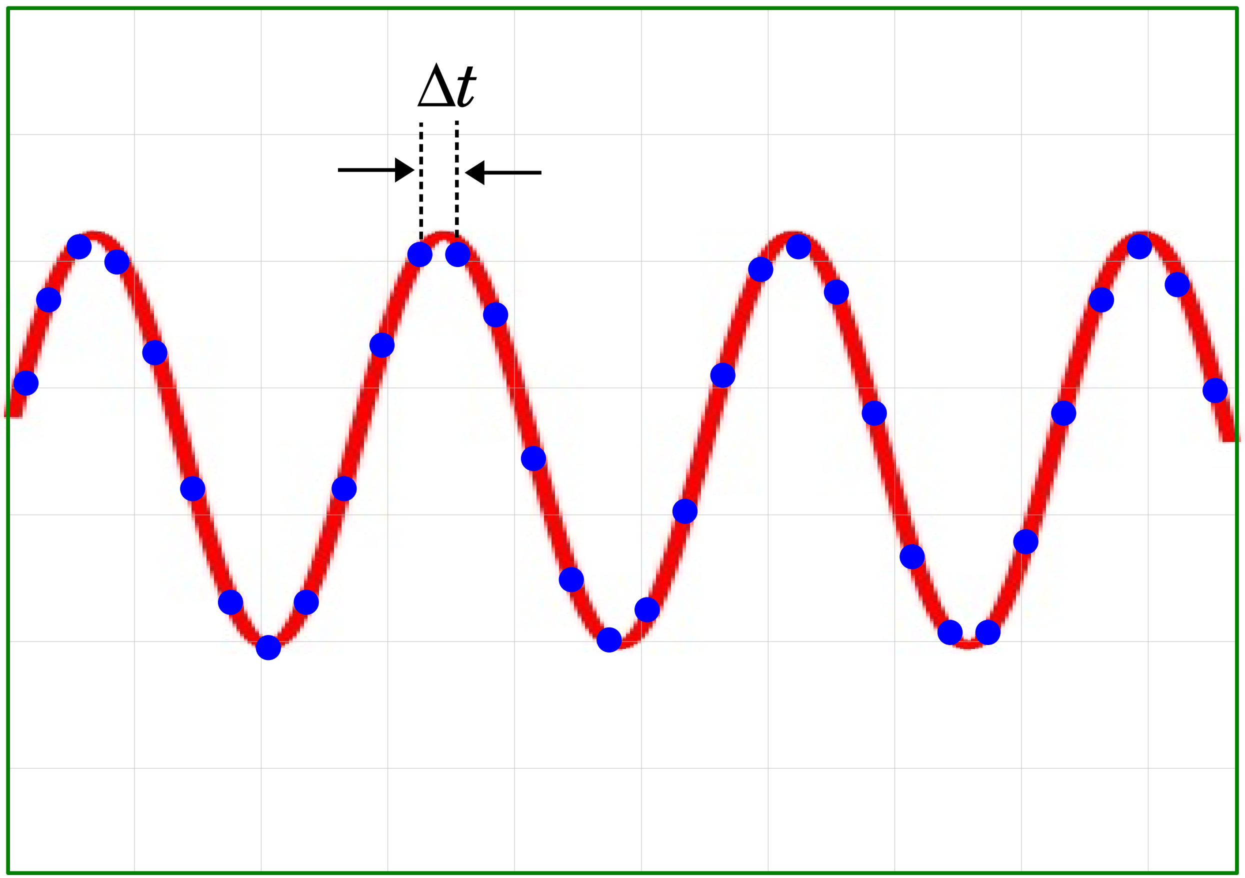
Power Spectrum

$$|F(\nu)|^2 = F(\nu)F^*(\nu)$$



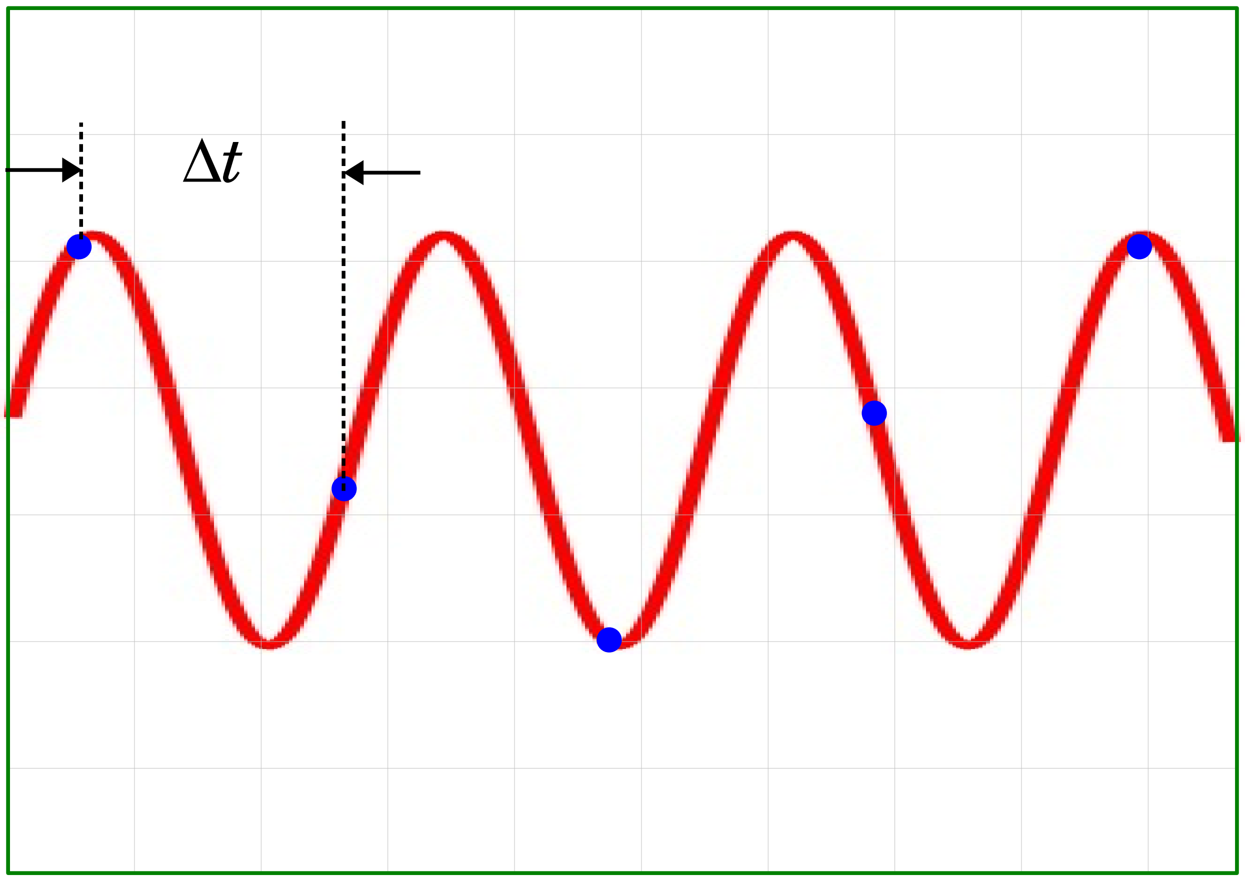
Nyquist theorem
Sampling theorem

Temporal spacing of signal sampling $\Delta t \leq \frac{1}{2\nu}$



Nyquist theorem
Sampling theorem

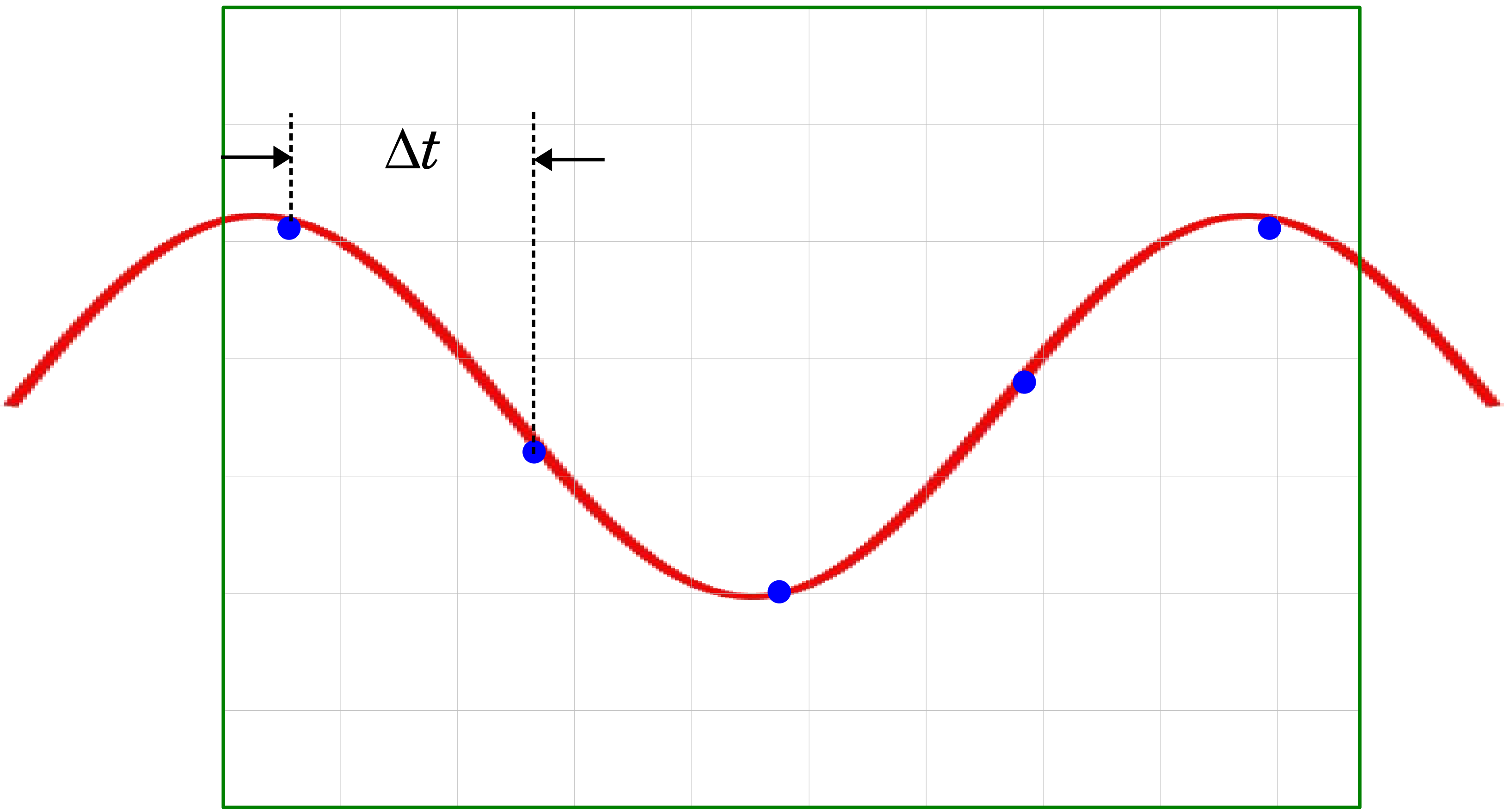
Temporal spacing of signal sampling $$\Delta t > \frac{1}{2\nu}$$



Nyquist theorem
Sampling theorem

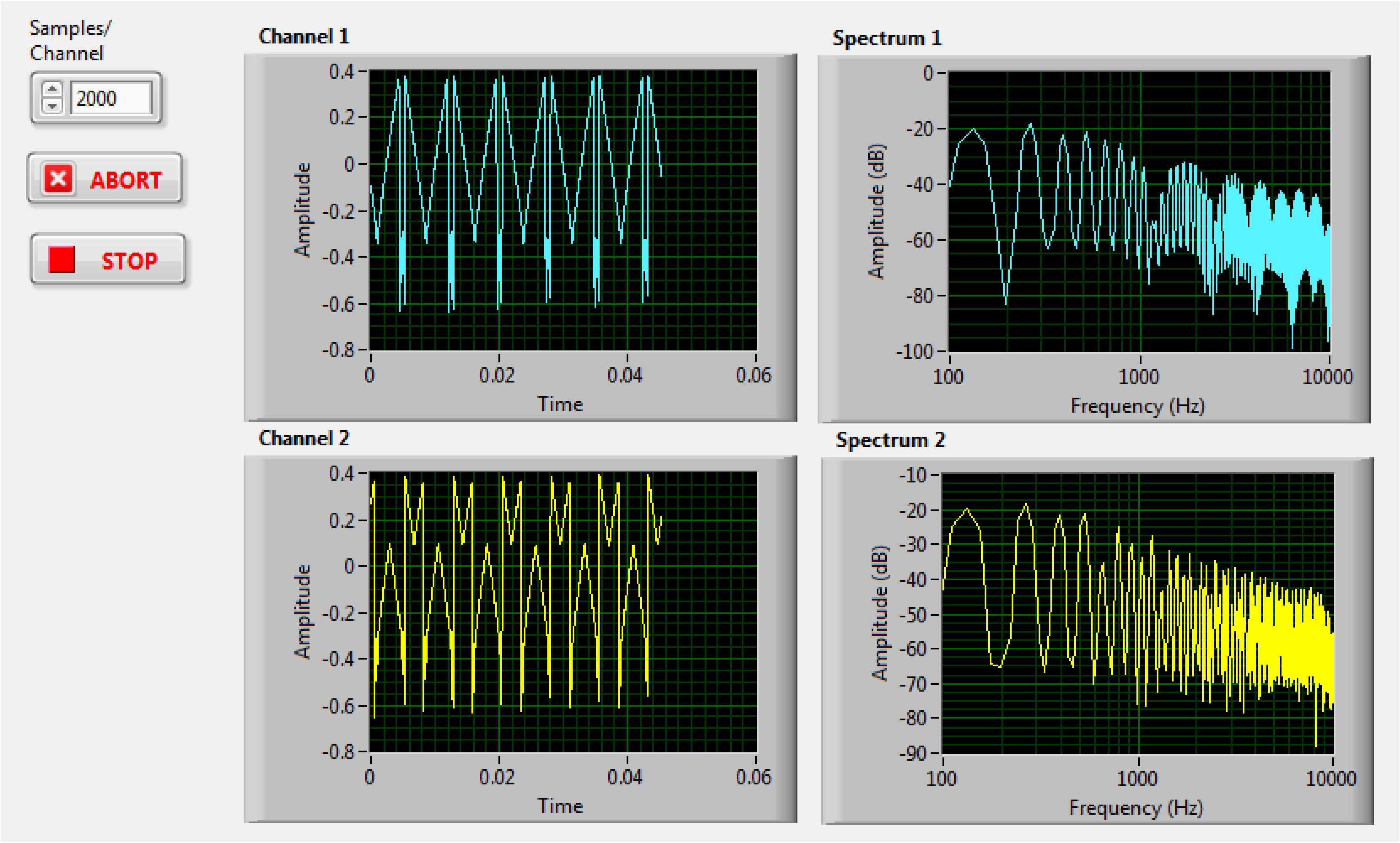
Temporal spacing of signal sampling

$$\Delta t > \frac{1}{2\nu}$$



ALIASING

LabVIEW Assignment 8: Audio Spectrum Analyzer



Lab 8: Operational Amplifiers

Often better alternative to simple transistor amplifiers

Stablity – circuits nearly immune to temperature drift

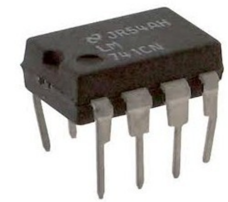
Versatile especially with use of feedback

Electrical implementation of mathematical operations

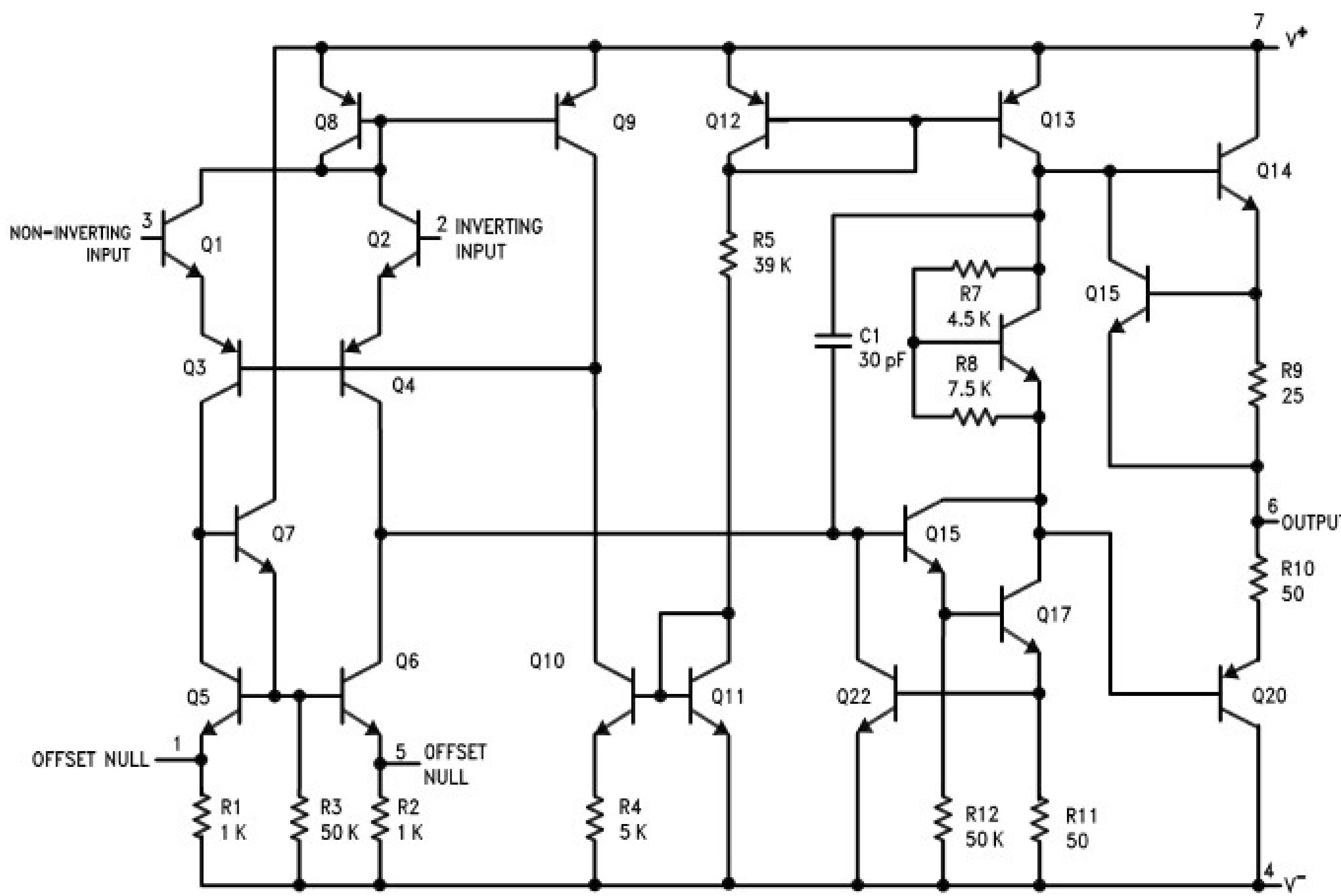
Individual transistors: highest frequency operation, high power

Op amps packaged as an integrated circuit (IC)

LM741 Operational amplifier

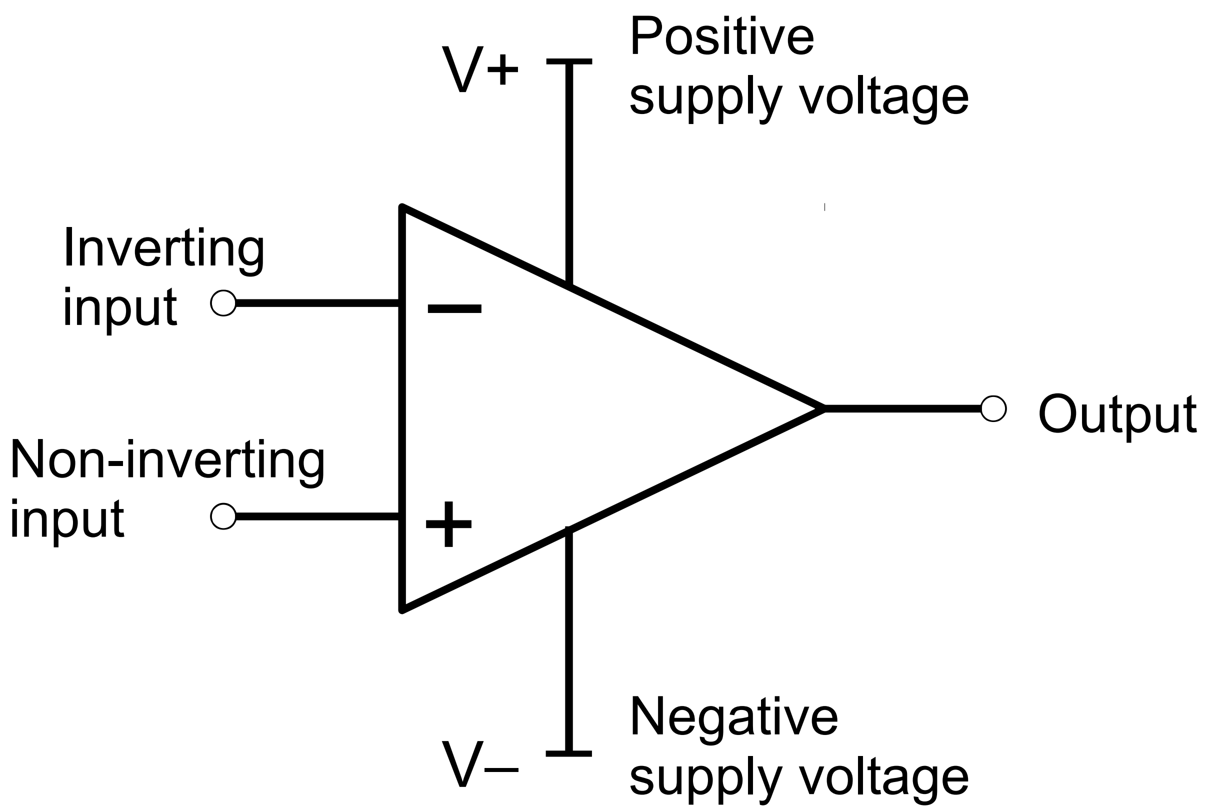


SCHEMATIC DIAGRAM

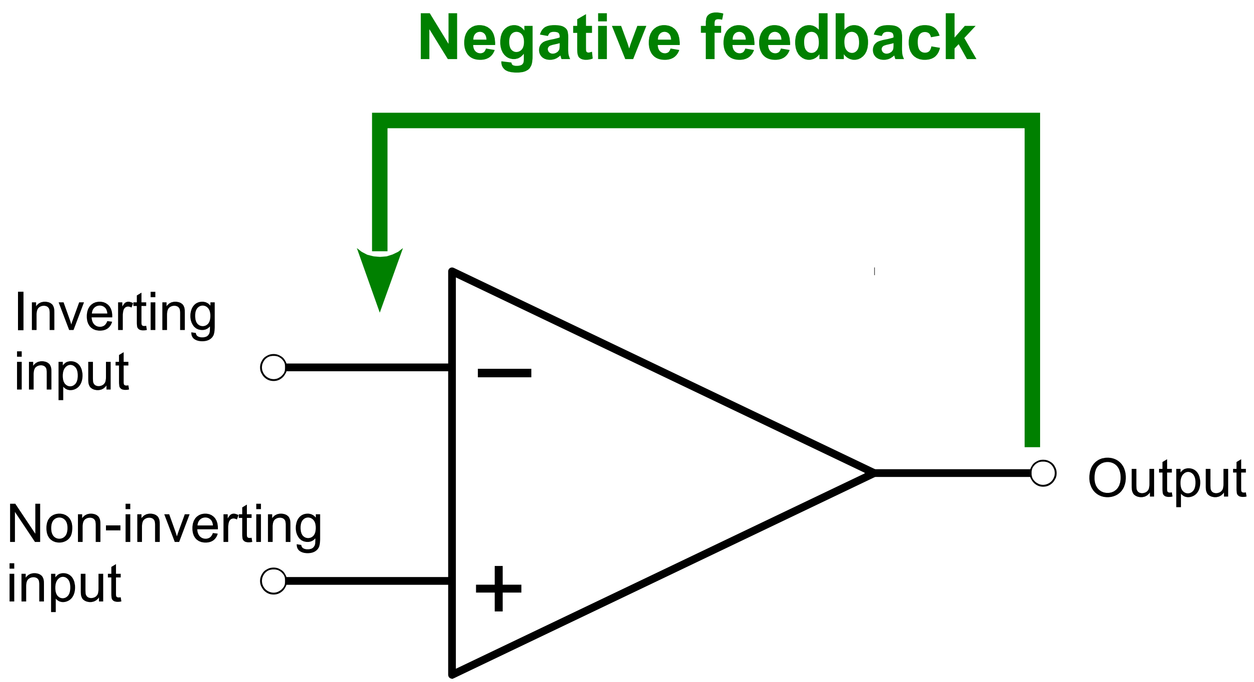


IDEAL OP-AMP

* Infinite gain
* Infinite input impedance/resistance
* Output current can go to infinity if needed



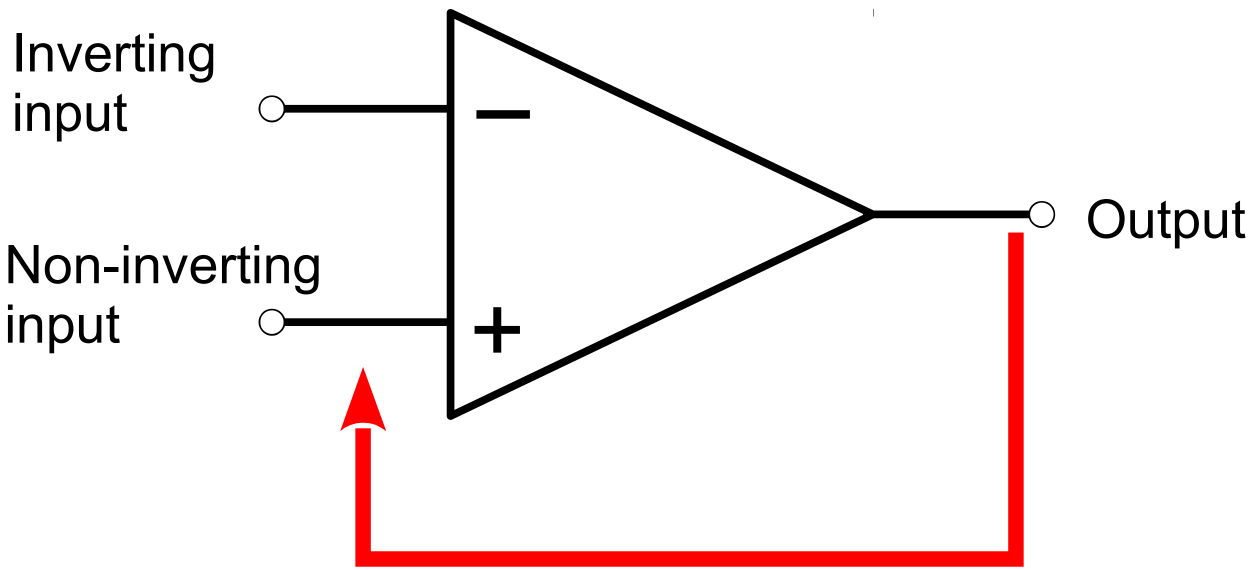
FEEDBACK: Sending a portion of the output back to the input



Stabilizes the output of an amplifier

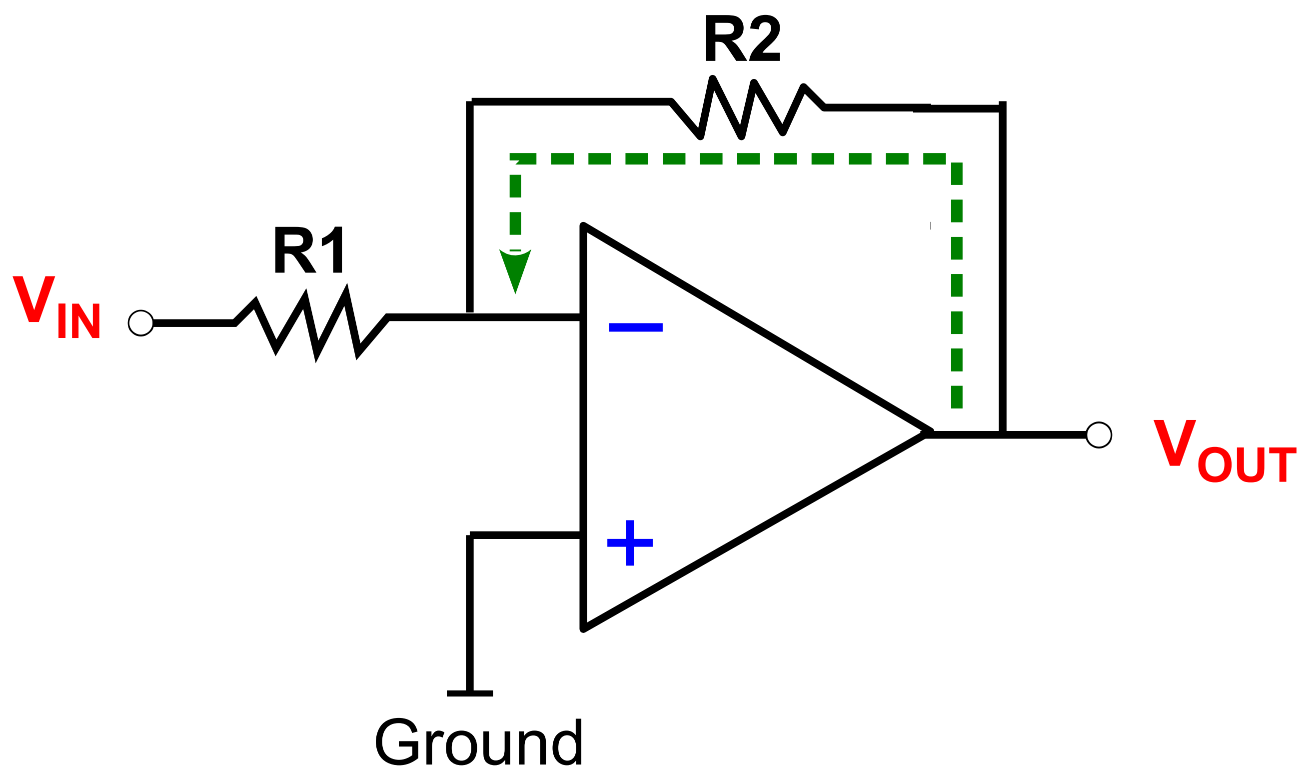
FEEDBACK: Sending a portion of the output back to the input

Positive feedback

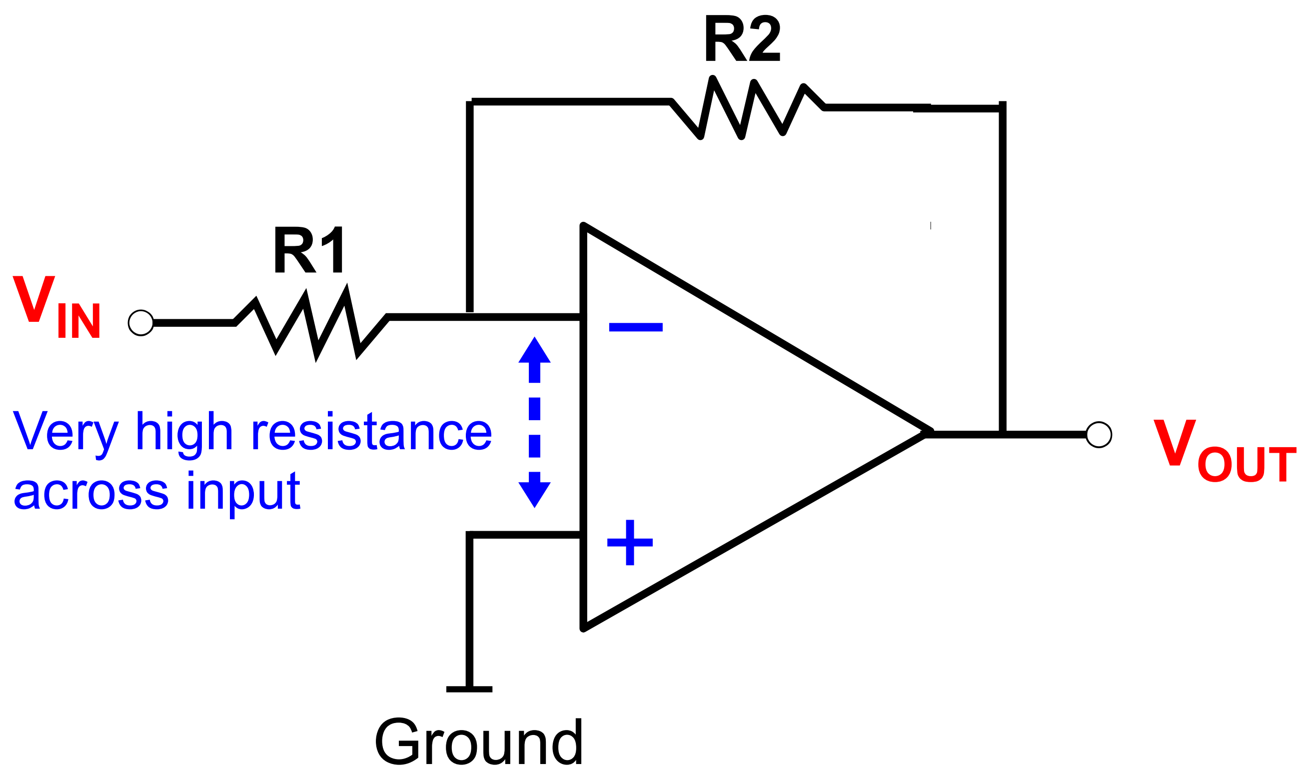


Runaway amplification – Oscillation

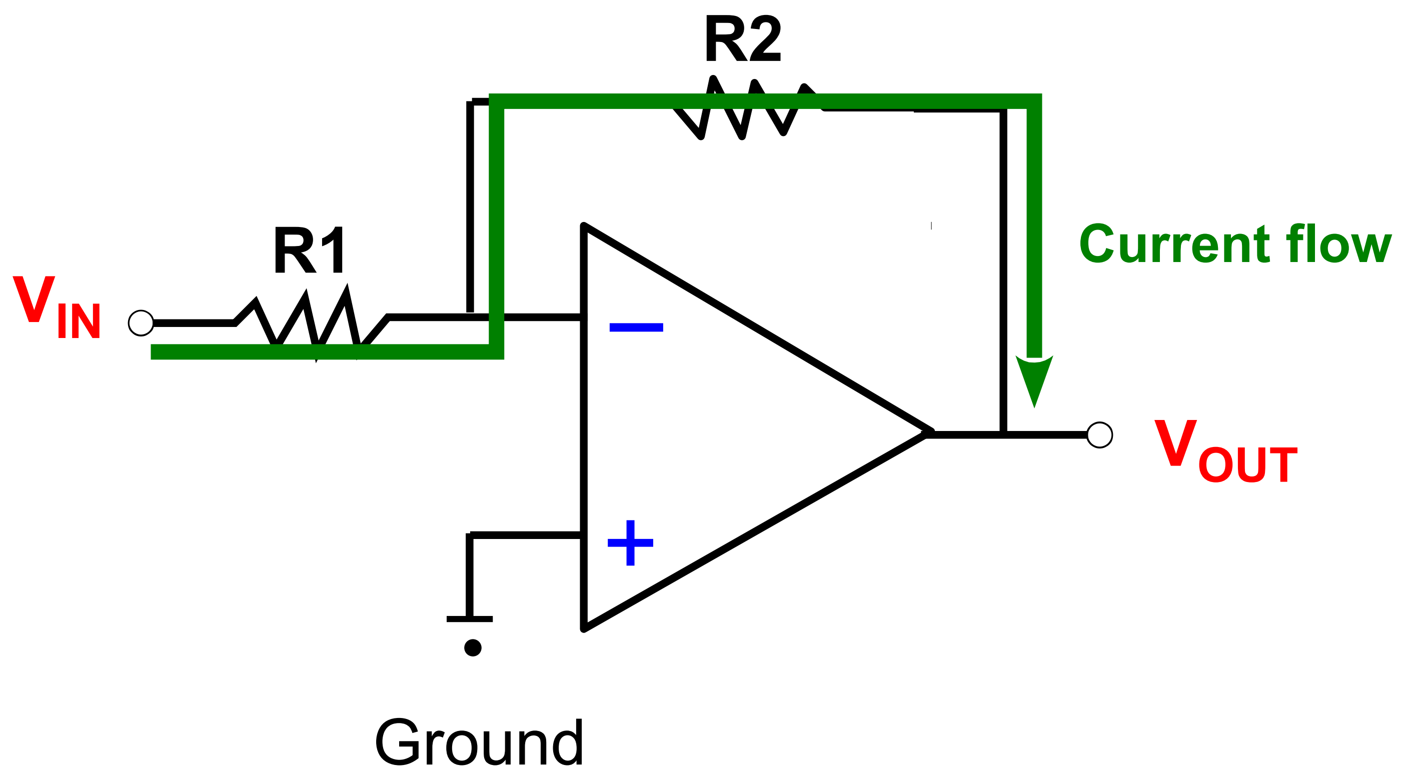
Op Amps use negative feedback to produce stable amplification



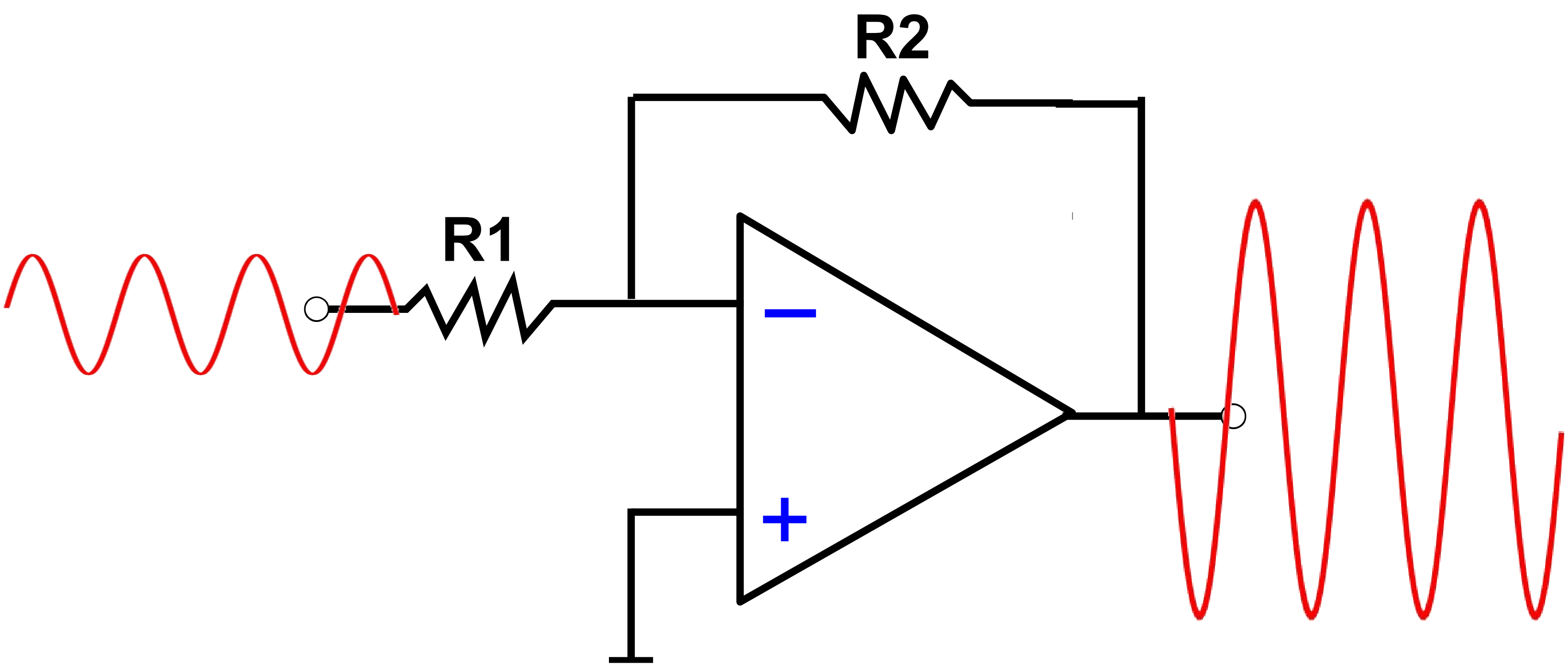
Analysis



Analysis

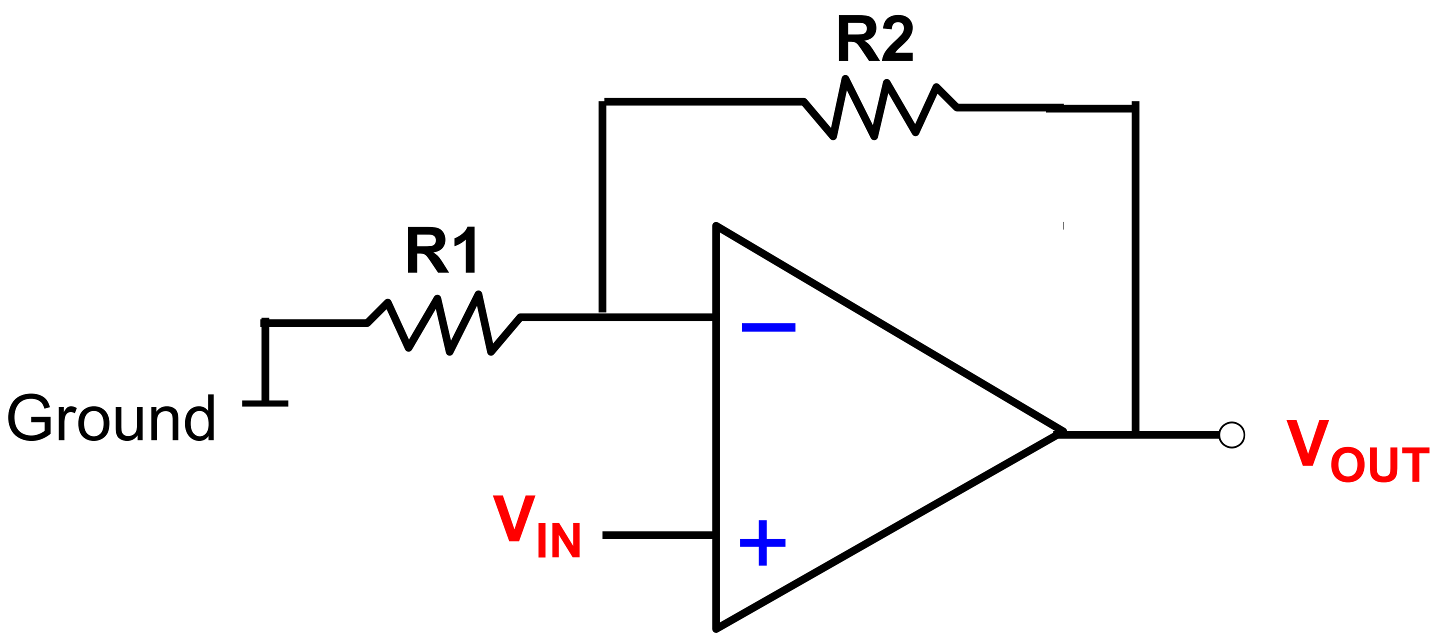


Analysis

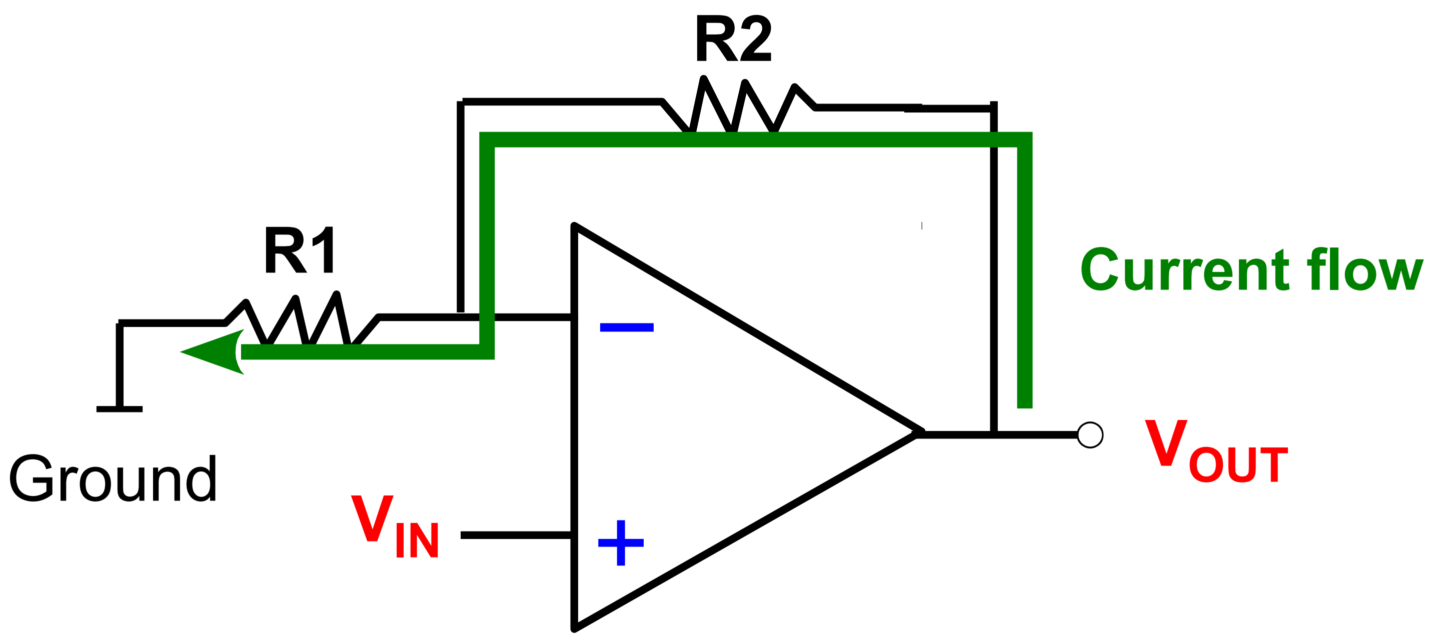


Inverting amplifier: GAIN $= -\dfrac{R2}{R1}$

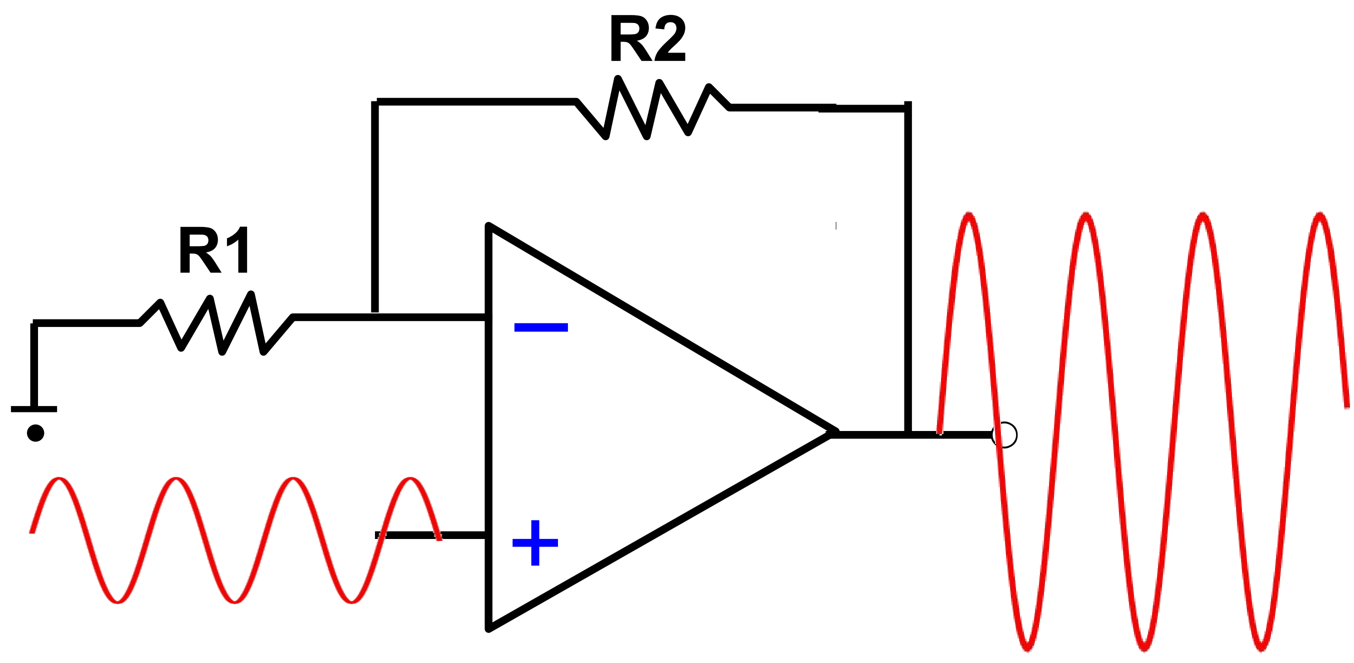
Op Amps use negative feedback to produce stable amplification



Op Amps use negative feedback to produce stable amplification

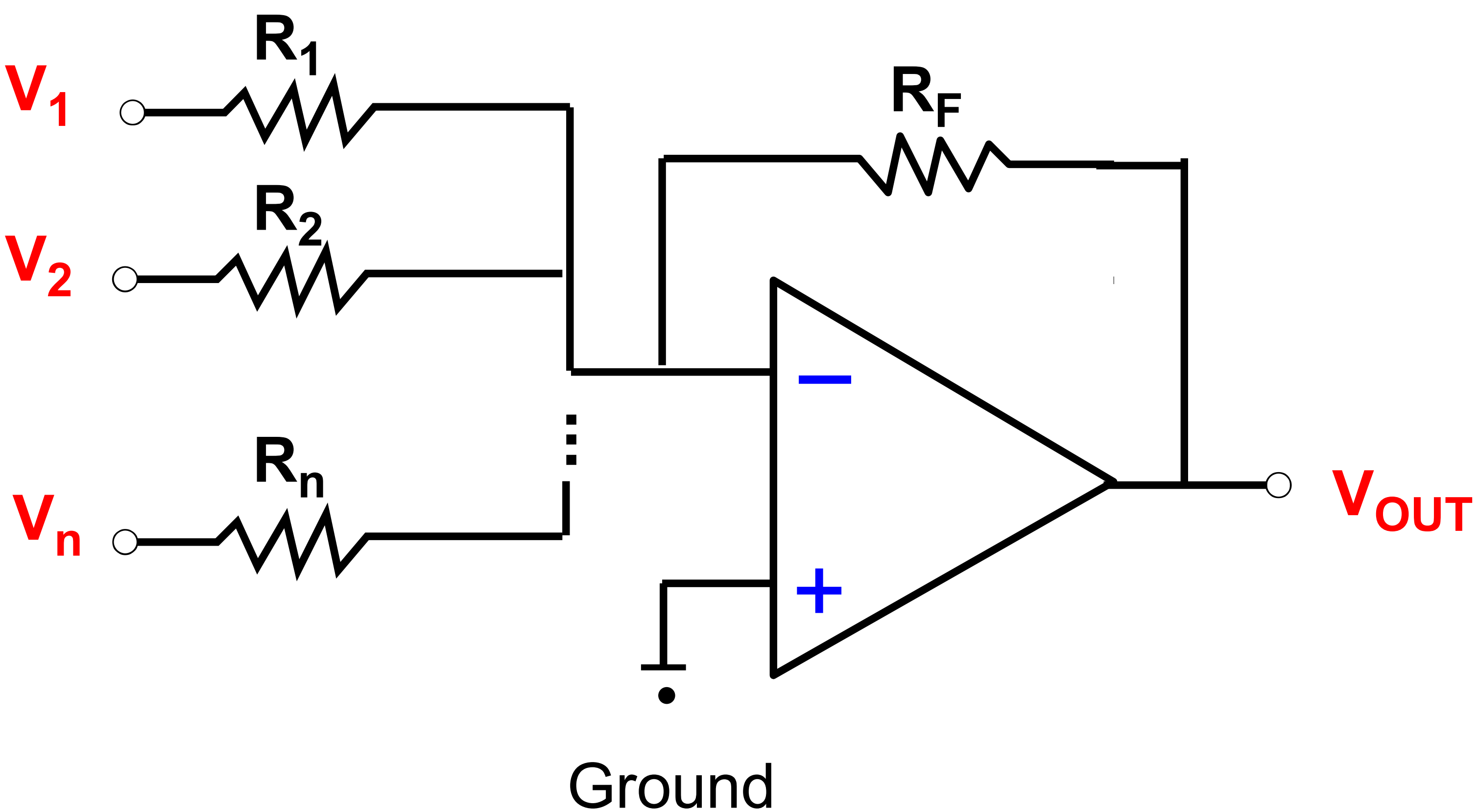


Op Amps use negative feedback to produce stable amplification



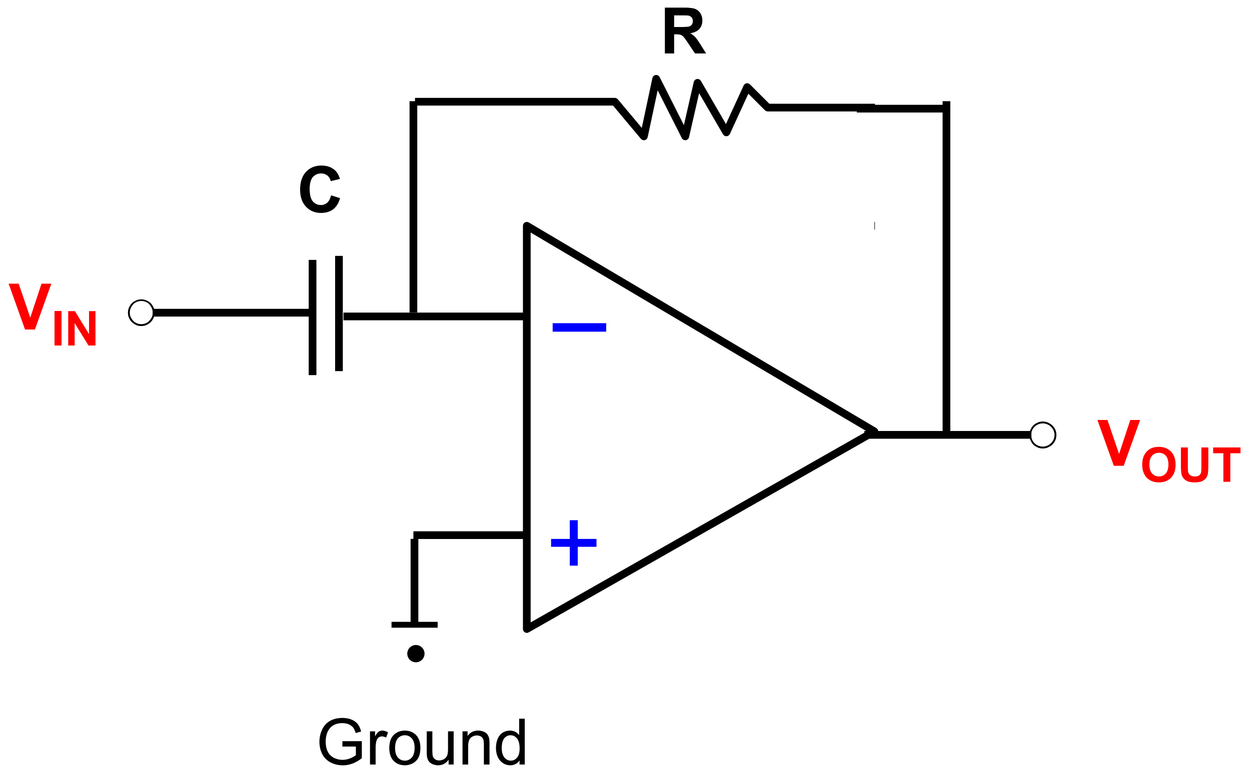
Non-inverting amplifier: GAIN = $1 + \frac{R2}{R1}$

Summing Amplifier



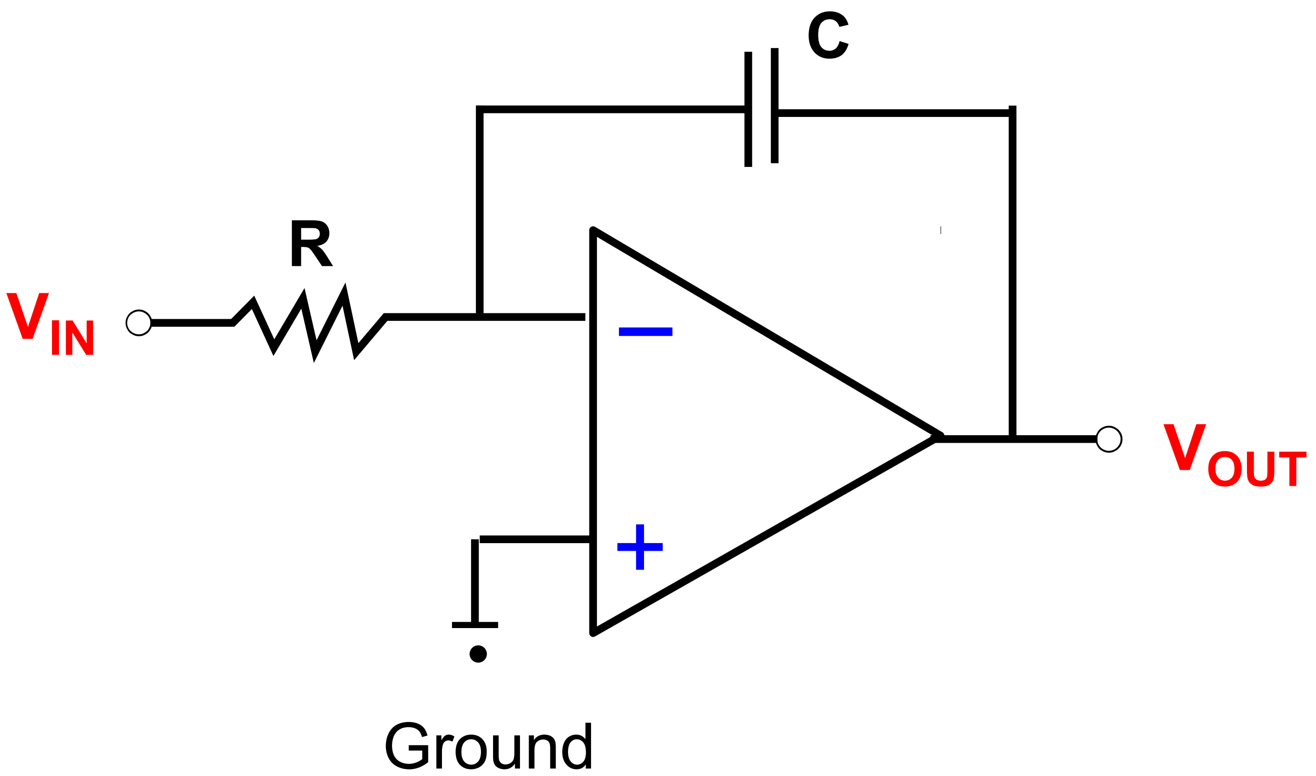
$$V_{OUT} = -V_1 \frac{R_F}{R_1} - V_2 \frac{R_F}{R_2} \ldots - V_n \frac{R_F}{R_n}$$

Differentiating Amplifier



$$V_{OUT} = -RC\frac{dV_{IN}}{dt}$$

Integrating Amplifier



$$V_{OUT} = -\frac{1}{RC}\int V_{IN}dt$$