Electron Diffraction v2.1


Davisson and Germer in 1927

Background. This experiment demonstrates the wave-like nature of an electron as first postulated by Louis de Brogile in 1924. All particles including electrons have a wavelength inversely related to momentum:

$$\lambda = h/p. \tag{1}$$

It was well known at the time that the light could be diffracted by a periodic grating, where the grating period is of the order of the electromagnetic wavelength. If an electron has wave properties, then there might be a suitable physical grating to diffract it. Max von Laue had first suggested that periodic planes of atoms in a solid could serve as a grating. X-ray reflection experiments of Lawrence Bragg confirmed this and gave an estimate of the inter-atomic spacing. Planes of atoms in a solid can also provide the appropriate grating spacing to cause electron diffraction. In a Nobel Prize winning experiment at Bell Labs in New Jersey, Clinton Davisson and Lester Germer used crystalline nickel to diffract electrons and prove their existence as waves. An independent experiment was done around the same time by George Thomson in Scotland. This was foundational work for the transmission electron microscope (TEM).

The present experiment uses an ultra-thin polycrystalline graphite layer in place of nickel as the diffraction grating. Graphite forms as single planes of carbon atoms in a hexagonal arrangement shown in Figure 1. Vertical planes are not chemically bonded to adjacent planes. Instead they are more loosely held by the electrostatic van der Waals force but the atoms are remain aligned vertically.

Diffraction occurs if the electron wave encounters aligned planes of atoms that repeat periodically across the structure. This periodicity is essential for the creation of a grating. There are several such gratings in the hexagonal structure graphite. The dashed green lines in Figure 1 show two orientations of the d_{10} grating. A different spacing exists for the d_{11} grating depicted with dashed blue lines in Figure 2. These two grating periods (d_{10} = 0.123 nm and d_{11} = 0.213 nm) are used to diffract an electron beam at distinctly different angles.

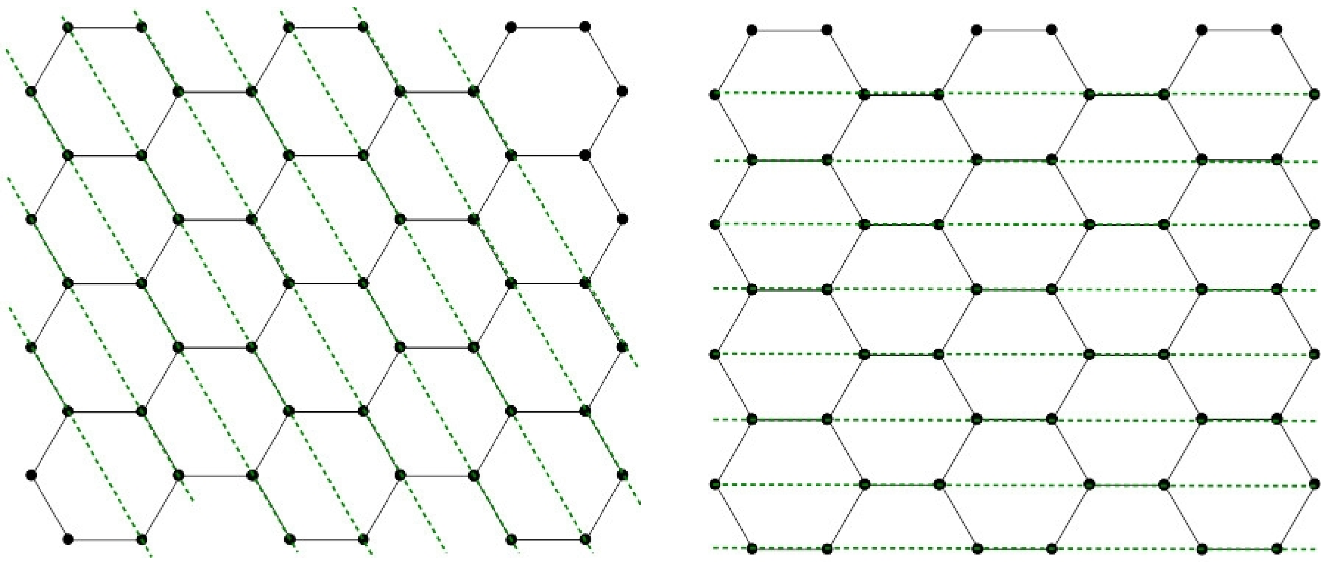

Figure 1: This is the hexagonal atomic structure of graphite encountered by the electron beam. The d_{10} grating planes are indicated by the dashed green lines.

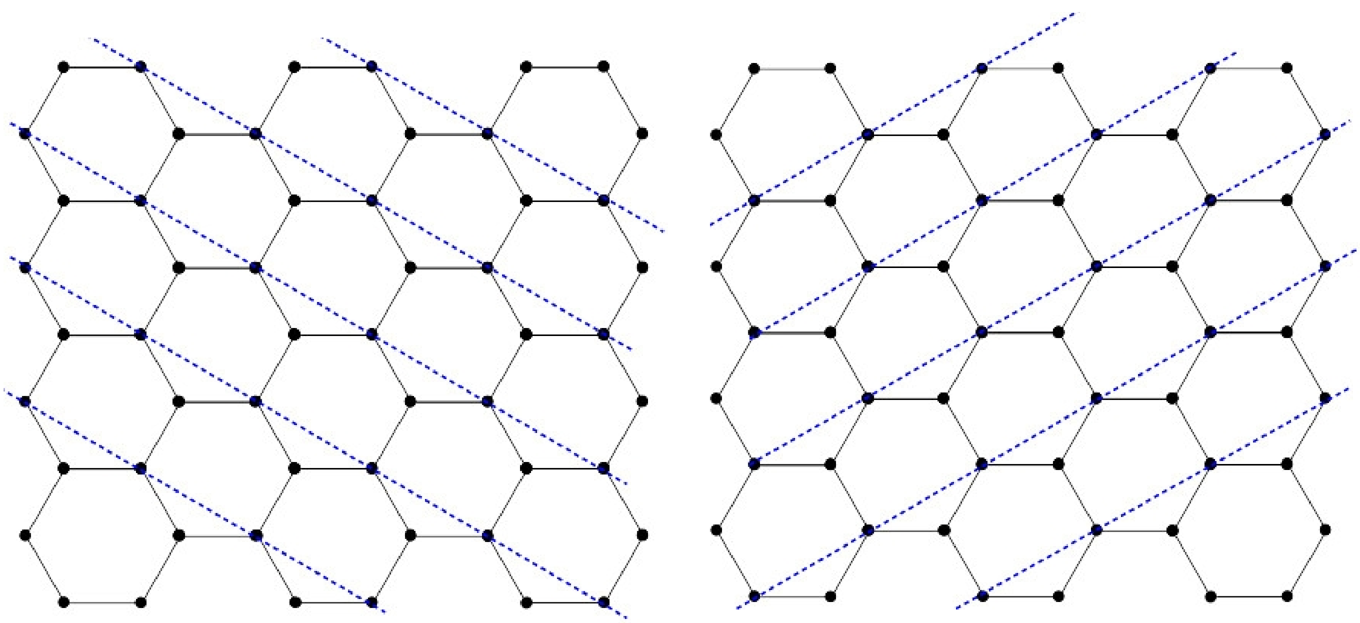

Figure 2: Same hexagonal atomic structure of Figure 1 with the d_{11} grating planes depicted by the dashed blue lines.

Transmission grating. When coherent light such as a laser of wavelength λ illuminates an optical transmission grating having vertical lines spaced at a distance d, the light is diffracted in the horizontal plane. This is illustrated in Figure 3. An infinite plane wave of wavelength λ is incident on a line grating; the lines block transmission at a periodic intervals. Transmission occurs in the space between the lines. The emerging diffracted light can be approximated with cylindrical waves or Huygens wavelets centered in the spaces.

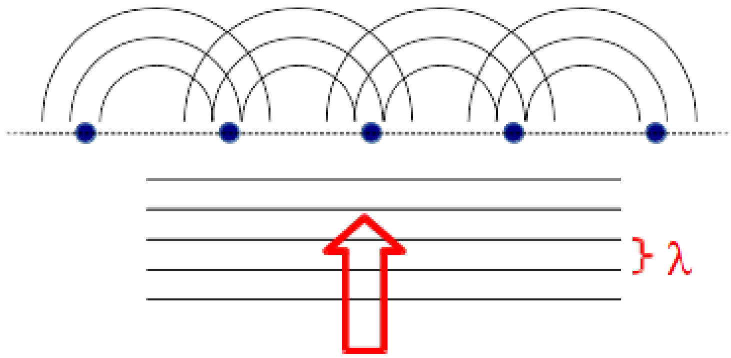

Figure 3: Top view of a transmission line grating illustrating diffraction of a plane wave. Black lines depict the points of constant phase.

The electric field vectors of the wavelets can interfere with each other constructively or destructively. To assess the interference, the points of constant phase must be tracked in position. This is illustrated in Figure 4.

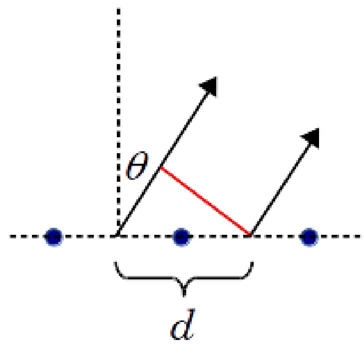

Figure 4. Interference occurs at planes of constant phase (red line).

Two rays are shown emerging from the grating at an angle θ. A plane of constant phase is depicted in red; it must be perpendicular to the direction of propagation. The point of constant phase on the left-hand ray has traveled a distance $d\sin\theta$ farther than the right-hand ray. For constructive interference, this distance must be an integer multiple (m) of the wavelength relative to the left-hand ray, which means:

$$d\sin\theta = m\,\lambda. \tag{2}$$

The lowest order for constructive interference occurs when m=1, defining an angle:

$$\sin\theta = \lambda/d. \tag{3}$$

Rotating the grating by a polar angle $\phi = 90^\circ$ causes the beam diffraction to rotate into the vertical plane.

The same principles apply to electron diffraction, except the blue dots in Figures 3 and 4 represent individual atoms. If the graphite layer used in the present experiment was a perfect crystal, we would expect electron diffraction in directions perpendicular to the grating lines shown in Figures 1 and 2. The electron beam will be displaced from the incident beam path at two different angles θ, corresponding to the two different spacings d_{10} and d_{11}. The diffraction planes of d_{10} and d_{11} will also be separated by various polar rotation offsets $\Delta\phi = 30^{\circ}$ as indicated by the relative orientation of the green and blue dashed lines in Figures 1 and 2. The graphite layer is not a perfect crystal – it is *polycrystalline*, which means there are small islands or domains of graphite crystals randomly oriented and randomly distributed across the thin target layer. The random arrangement of domains in the path of the electron beam produces two diffraction patterns with circular symmetry, i.e. two rings. One ring is diffracted by the d_{10} grating and the second originates from the d_{11} grating. These two rings are distinguished by their diffraction angle θ. Any dependence on the polar angle ϕ is washed out by the polycrystalline nature of the graphite layer.

Equipment:
Evacuated electron diffraction tube
Control unit (U33010)
Vernier calipers

Experiment. Refer to Figure 5. A glass cathode ray tube is held at high vacuum. Current is passed through the filament to create electrons at the adjacent oxide-coated cathode by thermionic emission. These electrons are accelerated toward the positive potential ring anode. The accelerating potential is adjustable in the range 0–5 kV. A nickel mesh electrode is also held at anode potential. Some of the accelerated electrons pass through the mesh and strike the graphite film that has been deposited on the other side. The graphite is only a few molecular layers thick. Most of the electrons pass through the graphite without being diffracted and are partially obscured by a black spot on the beam axis at the opposite end of the bulb. A minority of electrons are diffracted by the two graphite gratings and appear as faint rings on the phosphor screen.

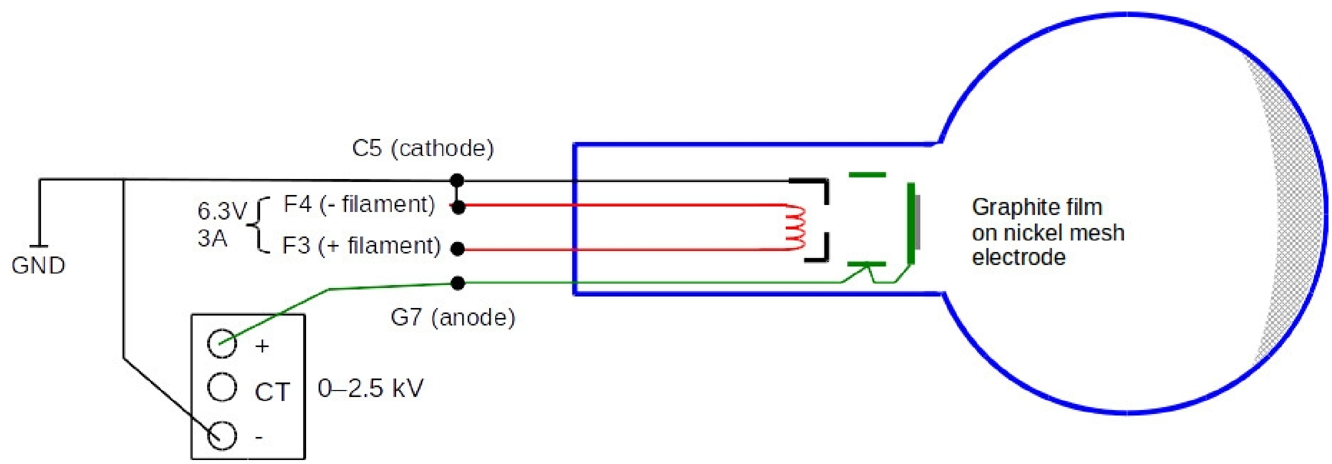

Figure 5: Experimental layout.

Procedure. Referring to Figures 1 and 2, use elementary trigonometry to derive the ratio d_{11}/d_{10}. Record the complete derivation in your notebook.

Derive a formula that predicts the angular deviation induced by the two gratings in graphite. You will need to relate the electron wavelength to the Bragg angle θ by linking Equations (1) and (3) through λ. It is reasonable to ignore relativity, but justify this assumption. Show all steps in your notebook.

Build the experiment as shown in Figure 5. Be sure to ground the filament and cathode at terminals C5 and F4. Before turning the controller on, have the wiring checked by an instructor. Place the high voltage control slider in the zero position and turn the unit on. Allow ~ 60 seconds for the cathode to warm up.

The diffraction rings can be hard to see, so make your workspace as dark as possible. They should be visible at an accelerating voltage > 2.5 kV, but do not exceed 5 kV. Try to center the rings in the tube; a small permanent magnet on a paper collar that can be placed around the neck of the tube to help center the rings.

Use a Vernier calipers to measure the diameter S of the diffraction rings on the surface of the glass bulb at different accelerating voltages (V). Be careful to center the rings; offsets will introduce error. Take multiple measurements at multiple voltages, but do not let the accelerating voltage sit for any length of time above 3 kV. Record the values of V and S in a table.

Analysis. Use the experimental geometry in Figure 6 to relate the measured ring diameters S to the diffraction angle. Take the distance from the graphite film to the edge of the bulb as L = 130 ± 2 mm and the radius of the bulb to be R = 66 mm. You must account for R in the analysis, so use trigonometry to establish a relation between S and D. You need to express δ in terms of R. Note that $L \neq 2R$. The starting point is:

$$\tan\theta = \frac{D}{2L} = \frac{S}{2(L-\delta)} \tag{4}$$

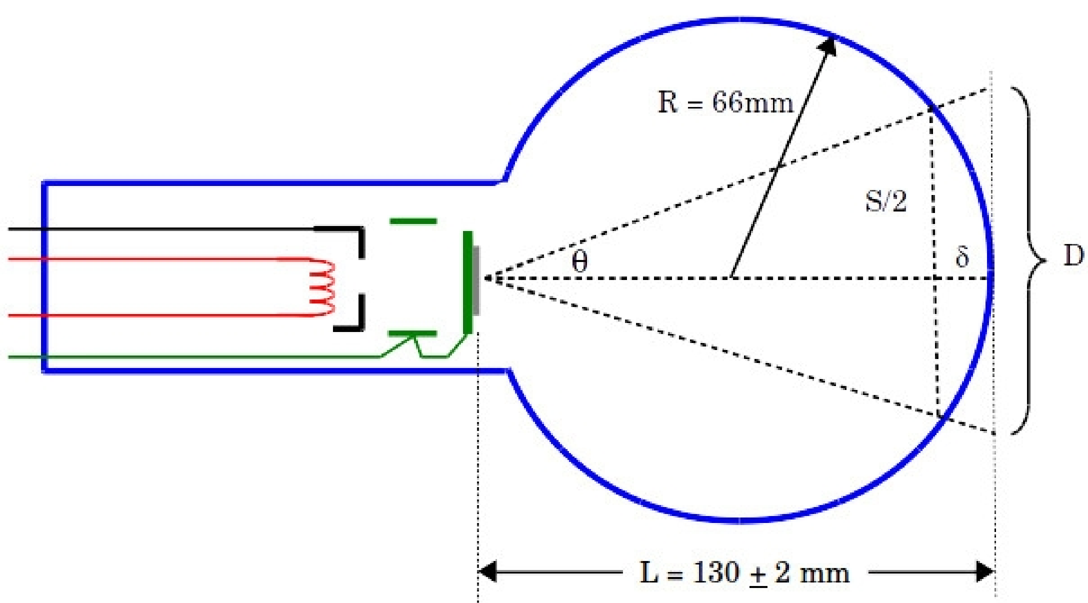

Figure 6: Geometry of the electron diffraction tube

Add a column to your data table that shows the calculated D at each accelerating voltage V. Equation (4) can also be manipulated to link D to the diffraction angle:

$$\sin\theta = \frac{1}{\sqrt{1+(2L/D)^2}}$$

This leads to an equation for D as a function of V:

$$D = \frac{2L}{\sqrt{2meV(d/h)^2 - 1}}$$

where h is Planck's constant, m and e are the mass and charge of the electron, respectively. Can the denominator be simplified given the values of the various terms? Plot D as a function of $V^{-1/2}$ to determine if there is a straight line. The slope of this line can be used to extract d. A second method is to calculate d directly for each V and D:

$$d = h\sqrt{\frac{1+(2L/D)^2}{2meV}}$$

Enter the data, your calculated results, and complete derivations in your notebook. There are many sources of error in this experiment that must be accounted for in the final reported values of d.