LECTURE 2: Introduction to Error Analysis

Why do we need ERROR ANALYSIS?

Experimental results are only *ESTIMATES*

This is due to:

- **Uncertainties**
- **Randomness**
- **Limits of precision**
- **Equipment limitations**
- **Incomplete physical model**

Dictionary definition of ***ERROR***:

Difference between True Value and Measurement or Calculation

Truth is usually not known – the reason for doing experiments

In scientific analysis, the difference is a ***DISCREPANCY***

What are ERRORS?

1) Illegitimate. Mistake in setup, assumptions, calculations, etc

2) Uncertainties, randomness, statistical fluctuations

3) Systematic

Accuracy ***vs*** **Precision**

Accuracy: How close to the truth?

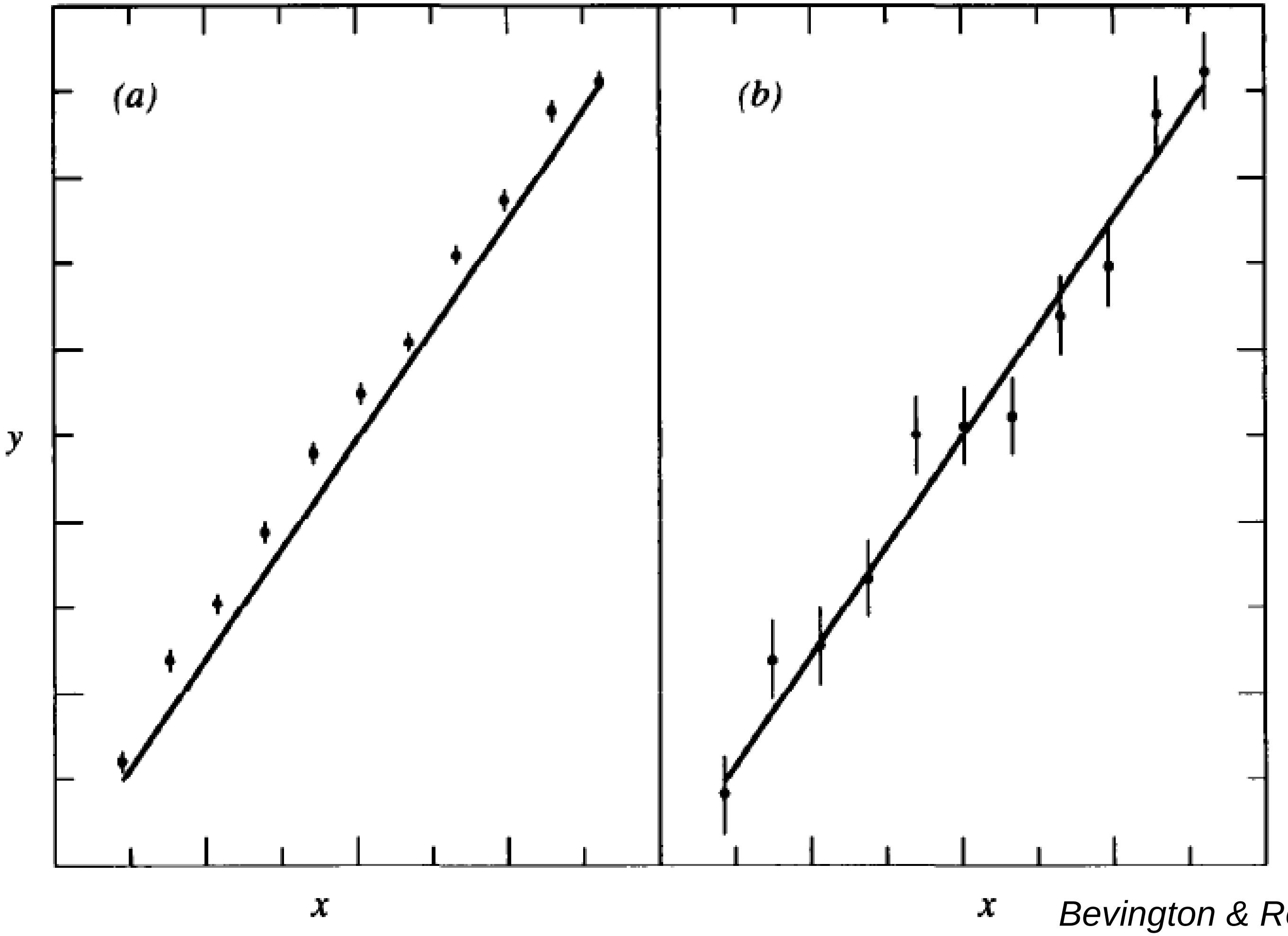
Precision: How well is the result known?

Accuracy = abs(Truth – Measurement)

Precision = Number of significant figures in Measurement

Precision can be high even if **Accuracy** is poor

High precision, inaccurate **Lower precision, more accurate**



Bevington & Robinson, 3rd ed.

Line: True behavior of $y = f(x)$
Experiment: Data points with error bars

Error bars indicate precision

SYSTEMATIC ERRORS

Systematic Errors harder to identify than random fluctuations

Statistical analysis is usually ineffective

Examples:

- Poor calibration of equipment
- Lack of familiarity with equipment
- Human bias – knowing expected result ahead of time

Avoiding systematic errors: Careful setup, not rushing, experience

RANDOM ERRORS DETERMINE PRECISION

Reduced by improving/refining experimental technique

Better equipment, less noisy

Statistics: Take more data
(although some experiments prevent this)

SIGNIFICANT FIGURES and ERROR BARS

$$1.60217662 \pm 0.1 \times 10^{-19} \text{ coulombs}$$

very precise

not very accurate

Probably should be written this way:

$$1.6 \pm 0.1 \times 10^{-19} \text{ coulombs}$$

STATISTICS AND RANDOM ERRORS

Variation between multiple measurements of same quantity

As number of measurements increase, pattern emerges from data

Pattern distributed around the correct value (assuming no systematic error)

Average value of x after N measurements:

$$\overline{x} = \frac{1}{N} \sum_i x_i$$

Median value of a data ensemble $\mu_{1/2}$

Half of all data > $\mu_{1/2}$

Half of all data < $\mu_{1/2}$

Deviation of a data point about the **mean**: $d_i = x_i - \overline{x}$

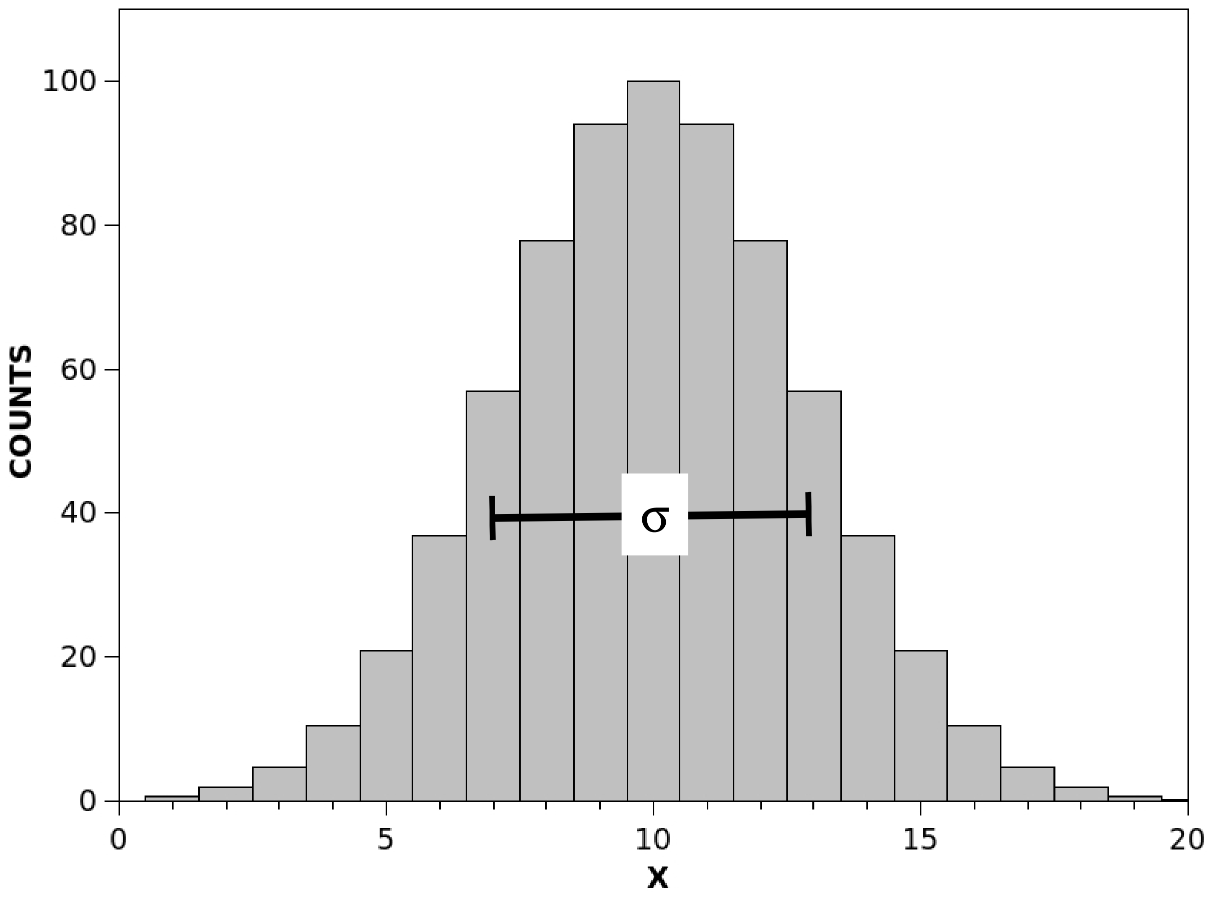
Average deviation: $\overline{d} = \overline{x} - \overline{x} = 0$ *Not useful*

Variance: $\sigma^2 = \frac{1}{N-1}\sum_i^N d_i^2 = \frac{1}{N-1}\sum_i^N (x_i - \overline{x})^2$

Standard deviation: $\sqrt{\sigma}$

PROBABILITY DISTRIBUTION

If the value of x is random: **GAUSSIAN** distribution



EXAMPLE

Most probable value: x = 10 (Mean)

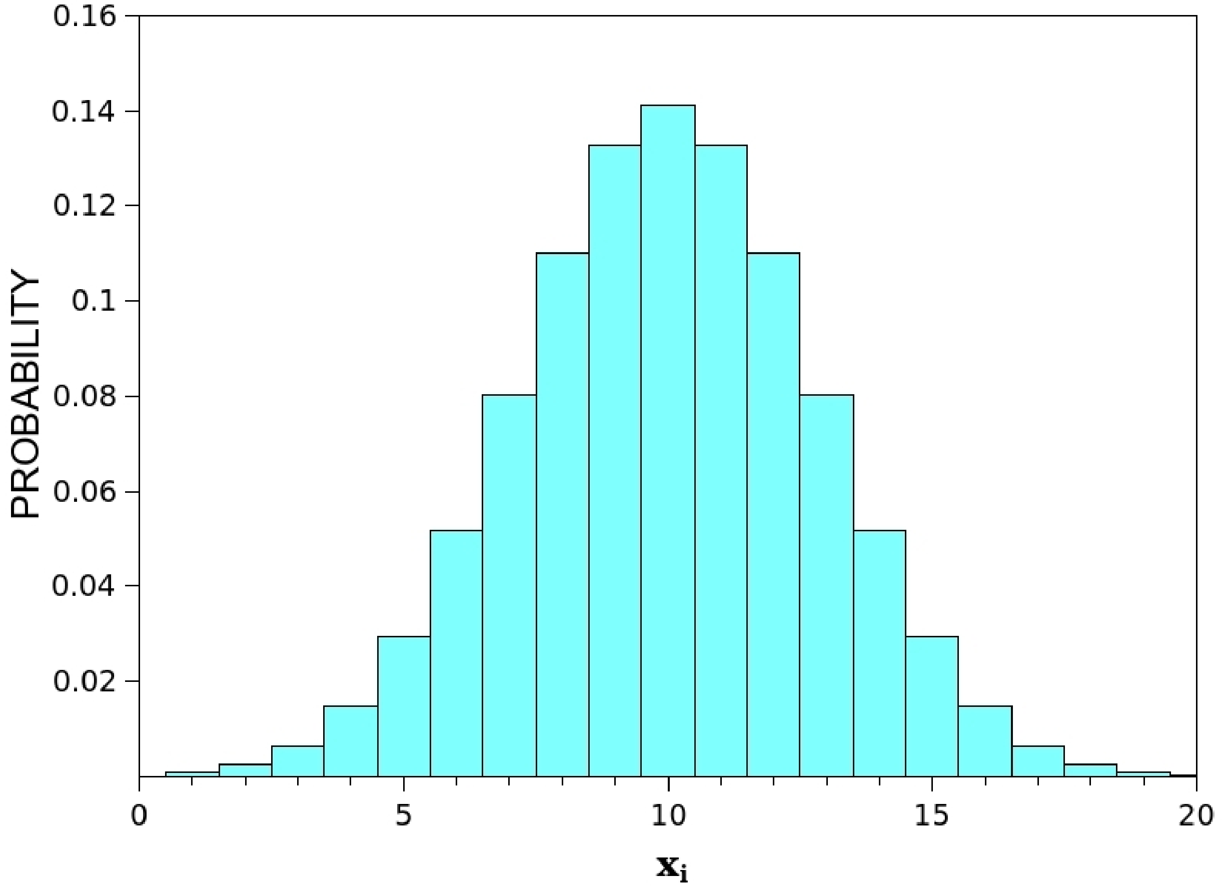
Variance: $\sigma^2 = 8$

Standard deviation: $\sigma = 2.82$

Probability p_i that x will have a specific value x_i

Probabilities must sum to 1: $\sum_i^N p_i = 1$

Expectation value: $\langle x \rangle = \sum_i^N x_i p_i = \overline{x}$



LAB NOTEBOOK

Leave a blank page at beginning for **Table of Contents**

Use a pen. Write neatly and clearly.

Date every page.

Start each new experiment on an odd numbered page. Record **Title** and **Objectives**.

Mistakes are common and expected. Just cross them out, don't erase or hide.

External printouts, plots, charts, etc should be taped into the notebook.

Record everything: each step, problems, explanations, etc.

Get each page initialed by instructor at end of session.

See class website for more details.

HOW NOTEBOOKS WILL BE GRADED

Was a pen used?

Is each page dated and initialed?

Table of Contents?

Experiments started on an odd page?

Title and objectives?

Legible handwriting?

Mistakes handled properly with cross-outs?

Data present? Informative and descriptive?

Notebooks count for 25% of Final Grade. Submitted at end of semester.