LECTURE 3:

- Error Bars
- Introduction to Probability and Statistics

MORE ABOUT *ERRORS*

Instrumental Uncertainties:

Lack of perfect precision

Typically: ½ the smallest scale division

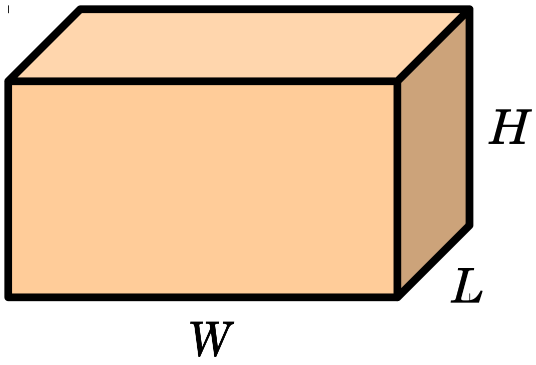
Equipment manufacturers often specify ***TOLERANCE*** as a percentage (eg ± 1%)

Statistical Uncertainties:

Unrelated to precision

Fluctuations due to finite data counts in a finite time (Poisson Distribution)

PROPAGATION OF ERRORS



Volume: $V = LWH$

Measurement of each dimension will have uncertainty:

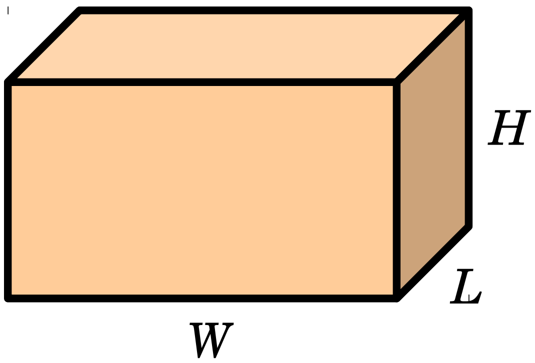
$$\Delta L, \Delta W, \Delta H$$

Partial derivatives or Taylor Series expansion:

$$\Delta V = \left(\frac{\partial V}{\partial L}\right)\Delta L + \left(\frac{\partial V}{\partial W}\right)\Delta W + \left(\frac{\partial V}{\partial H}\right)\Delta H$$

$$\Delta V = WH\Delta L + LH\Delta W + LW\Delta H$$

EXAMPLE:



$$L = 8 \text{ cm}; \quad W = 10 \text{ cm}; \quad H = 6 \text{ cm}$$

$$\Delta L = +2 \text{ mm}; \quad \Delta W = -1 \text{ mm}; \quad \Delta H = +1 \text{ mm}$$

$$V = LWH = 480 \text{ cm}^3$$

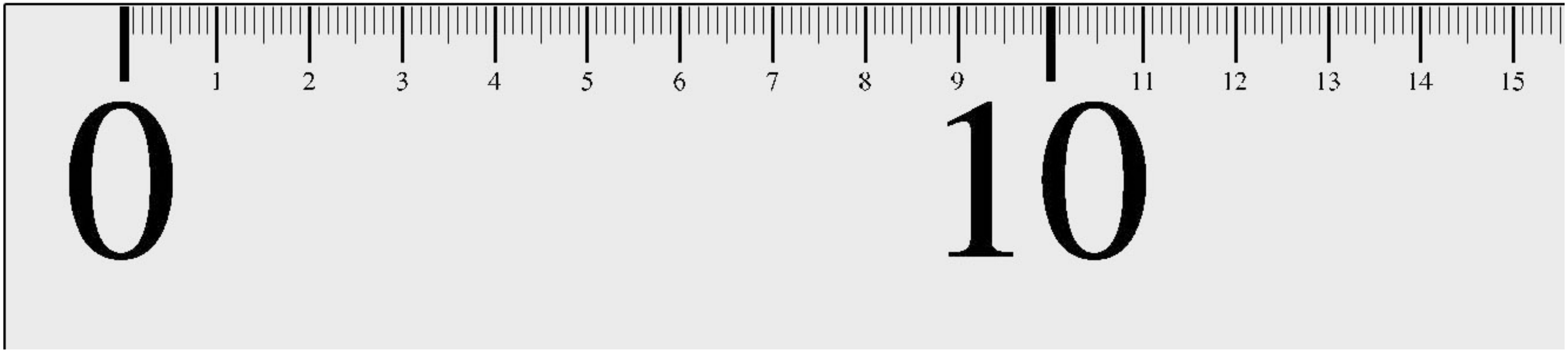
$$\Delta V = WH\Delta L + LH\Delta W + LW\Delta H$$

$$\Delta V = 12 - 4.8 + 8 = 15.2 \text{ cm}^3$$

Measured Volume: $480 + 15.2 \text{ cm}^3 = 480 - 495.2 \text{ cm}^3$

Measurement Uncertainty is usually an *ESTIMATE*

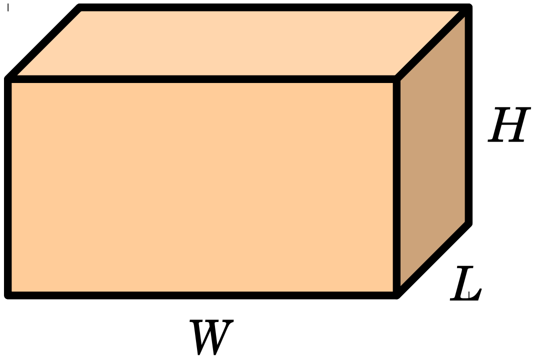
Standard deviation of each measurement: $\Delta L = \Delta W = \Delta H = 1 \text{ mm}$



ERROR PROPAGATION EQUATION:

$$\Delta V = \sqrt{\left[\left(\frac{\partial V}{\partial L}\right)\Delta L\right]^2 + \left[\left(\frac{\partial V}{\partial W}\right)\Delta W\right]^2 + \left[\left(\frac{\partial V}{\partial H}\right)\Delta H\right]^2}$$

EXAMPLE:



$$L = 8 \text{ cm}; \quad W = 10 \text{ cm}; \quad H = 6 \text{ cm}$$

$$\Delta L = \Delta W = \Delta H = 1 \text{ mm}$$

$$V = LWH = 480 \text{ cm}^3$$

$$\Delta V = \sqrt{(WH\Delta L)^2 + (LH\Delta W)^2 + (LW\Delta H)^2}$$

$$= 0.1\sqrt{60^2 + 48^2 + 80^2} = 11.09 \text{ cm}^3$$

Measured Volume: $480 \pm 11.09 \text{ cm}^3$

EXAMPLE:

Measuring Velocity

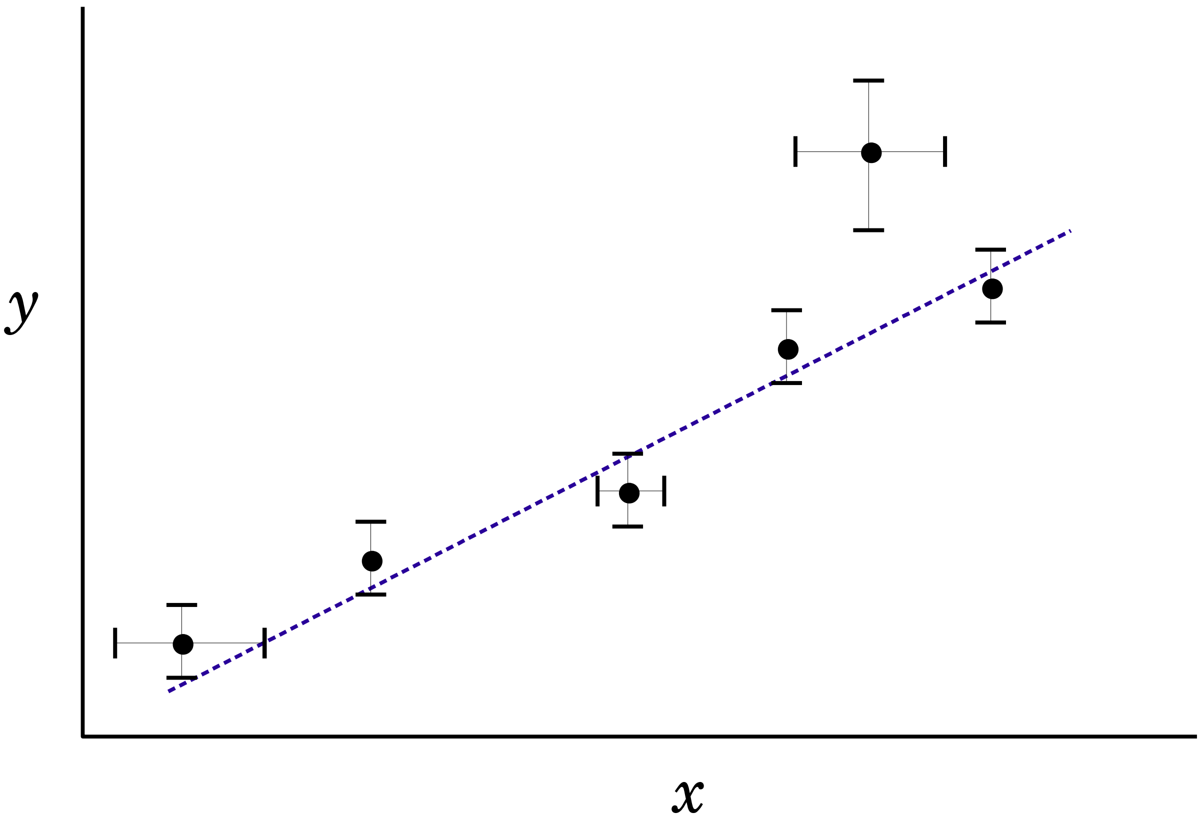
$$v = \frac{x}{t}$$

Independent measurements of x and t

$$\Delta v = \sqrt{\left[\left(\frac{\partial v}{\partial x}\right)\Delta x\right]^2 + \left[\left(\frac{\partial v}{\partial t}\right)\Delta t\right]^2}$$

$$= \sqrt{\left(\frac{\Delta x}{t}\right)^2 + \left(\frac{x\Delta t}{t^2}\right)^2}$$

UNCERTAINTIES are expressed graphically with ***ERROR BARS***



- Error bars may be shown on both axes
- Lengths may be different for different data points

PROBABILITY DISTRIBUTION FUNCTIONS

- **GAUSSIAN:** Random data, experimental parameters uncertain
- **POISSON:** Number of counts in a specified time interval
- **BINOMIAL:** Small number of possible outcomes (eg. heads or tails)

BINOMIAL DISTRIBUTION: 1 COIN

OR

p=50%

p=50%

BINOMIAL DISTRIBUTION: 2 COINS

***p*=25%**

***p*=25%**

***p*=25%**

***p*=25%**

BINOMIAL DISTRIBUTION: 3 COINS

8 DIFFERENT OUTCOMES: p = 12.5%

SAME COIN TOSSED 3 TIMES

8 DIFFERENT OUTCOMES: p = 12.5%

Possible states for the coin: $S = 2$

Number of coins flipped once: N

– or –

Number of times single coin is flipped: N

Possible outcomes = $S^N = 2^3 = 8$

Possible states for single die: $S = 6$

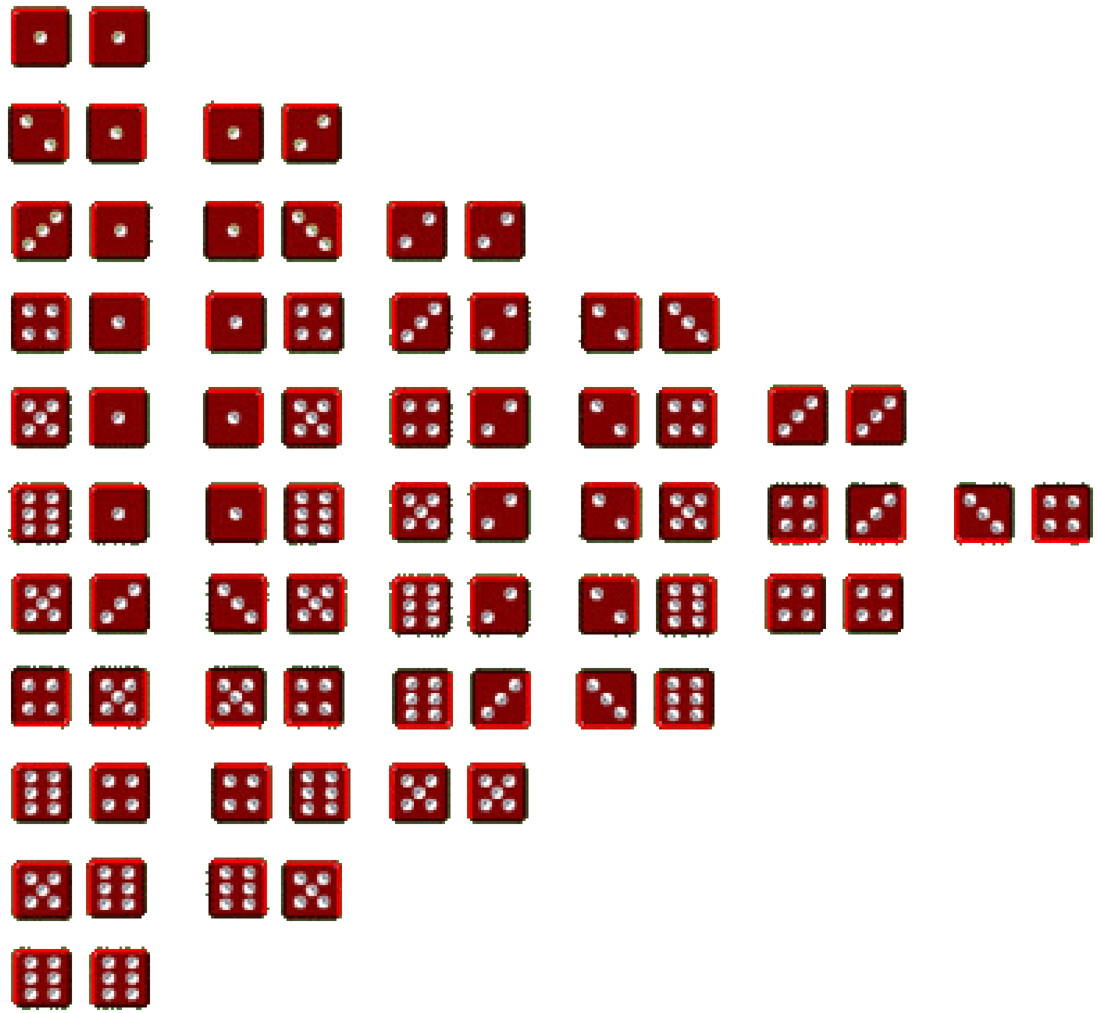
Number of times a single die is thrown: $N = 1$

Possible outcomes = $S^N = 6^1 = 6$

Possible states for single die: $S = 6$

Number of dice thrown: $N = 2$

Possible outcomes = $S^N = 6^2 = 36$



PERMUTATIONS

Starters: $N = 6$

6 different winners are possible

EXACTA: Pick the correct order of finish 1-2

Starters: $N = 6$

6 different winners are possible
Once winner is specified only five 2nd places possible
Number of different 1-2 sequences = 6 x 5 = **30**

TRIFECTA: Pick the correct order of finish 1-2-3

Starters: $N = 6$

6 different winners are possible
Once winner is specified only five 2^{nd} places possible; then four 3^{rd} place finishes possible
Number of different 1-2-3 sequences = 6 x 5 x 4 = **120**

Number of different race outcomes: 1-2-3-4-5-6

Starters: $N = 6$

6 x 5 x 4 x 3 x 2 x 1 = **720** different outcomes = $N!$

P: Number of possible **PERMUTATIONS**

N: Number of trials, events, participants, etc

x: Sequence of outcomes

$$P = \frac{N!}{(N-x)!}$$

Winner: $P = \frac{6!}{(6-1)!} = 6$

Exacta: $P = \frac{6!}{(6-2)!} = 30$

Trifecta: $P = \frac{6!}{(6-3)!} = 120$

6 horses in order: $P = \frac{6!}{(6-6)!} = 720$

PERMUTATIONS FOR SINGLE DICE TOSS

$$P = \frac{6!}{(6-2)!} = 30 \neq 36$$

Does not allow for doubles: 1-1 2-2 3-3 4-4 5-5 6-6

P = 6 x 6 = 36

COMBINATIONS:

Possible outcomes *irrespective* of order

Assume $N = 6$ horses

First place: $C = 6$ possible winners

Places 1-2: $C = 15$

1-2	2-3	3-4	4-5	5-6
1-3	2-4	3-5	4-6	
1-4	2-5	3-6		
1-5	2-6			
1-6				

Places 1-2-3: $C = 20$

1-2-3	1-4-5	2-4-6
1-2-4	1-4-6	2-5-6
1-2-5	1-5-6	3-4-5
1-2-6	2-3-4	3-4-6
1-3-4	2-3-5	3-5-6
1-3-5	2-3-6	4-5-6
1-3-6	2-4-5	

C: Number of possible **COMBINATIONS**

N: Number of trials, events, participants, etc

x: Number of outcomes, order does not matter

$$C = \frac{N!}{(N-x)!\ x!} = \binom{N}{x}$$

Possible winners: $C = \frac{6!}{(6-1)!\ 1!} = 6$

Possible top-2 finishers: $C = \frac{6!}{(6-2)!\ 2!} = 15$

Possible top-3 finishers: $C = \frac{6!}{(6-3)!\ 3!} = 20$

Possible top-6 finishers: $C = \frac{6!}{(6-6)!\ 6!} = 1$

5 of 69 numbers:

$$C = \frac{69!}{(69-5)!\ 5!} = \frac{69 \times 68 \times 67 \times 66 \times 65}{5 \times 4 \times 3 \times 2 \times 1} = 11,238,513$$

Prize: $1,000,000

POWERBALL®
POWERPLAY®

5 of 69 numbers:

$$C = \frac{69!}{(69-5)!\,5!} = \frac{69 \times 68 \times 67 \times 66 \times 65}{5 \times 4 \times 3 \times 2 \times 1} = 11,238,513$$

1 of 26 numbers:

26 x 11,238,513 = 292,201,338

COMBINATIONS: Same coin tossed 3 times

COMBINATIONS: Same coin tossed 3 times

All 3 tosses are heads:

$$C = \frac{3!}{(3-3)!\ 3!} = 1$$

COMBINATIONS: Same coin tossed 3 times

2 of 3 tosses are heads:

$$C = \frac{3!}{(3-2)!\ 2!} = 3$$

COMBINATIONS: Same coin tossed 3 times

1 of 3 tosses are heads: $$C = \frac{3!}{(3-1)!\ 1!} = 3$$

COMBINATIONS: Same coin tossed 3 times

0 of 3 tosses are heads: $C = \frac{3!}{(3-0)!\ 0!} = 1$

PROBABILITIES: Same coin tossed 3 times

$$P_B = \frac{N!}{(N-x)!\ x!}\ p^x (1-p)^{N-x}$$

Heads: p = 1/2;
Tails: 1 – p = 1/2

P_B: BINOMIAL DISTRIBUTION

3 of 3 tosses are heads: $P_B(x=3) = 1 \times\ (1/2)^3(1/2)^{3-3} = \frac{1}{8}$

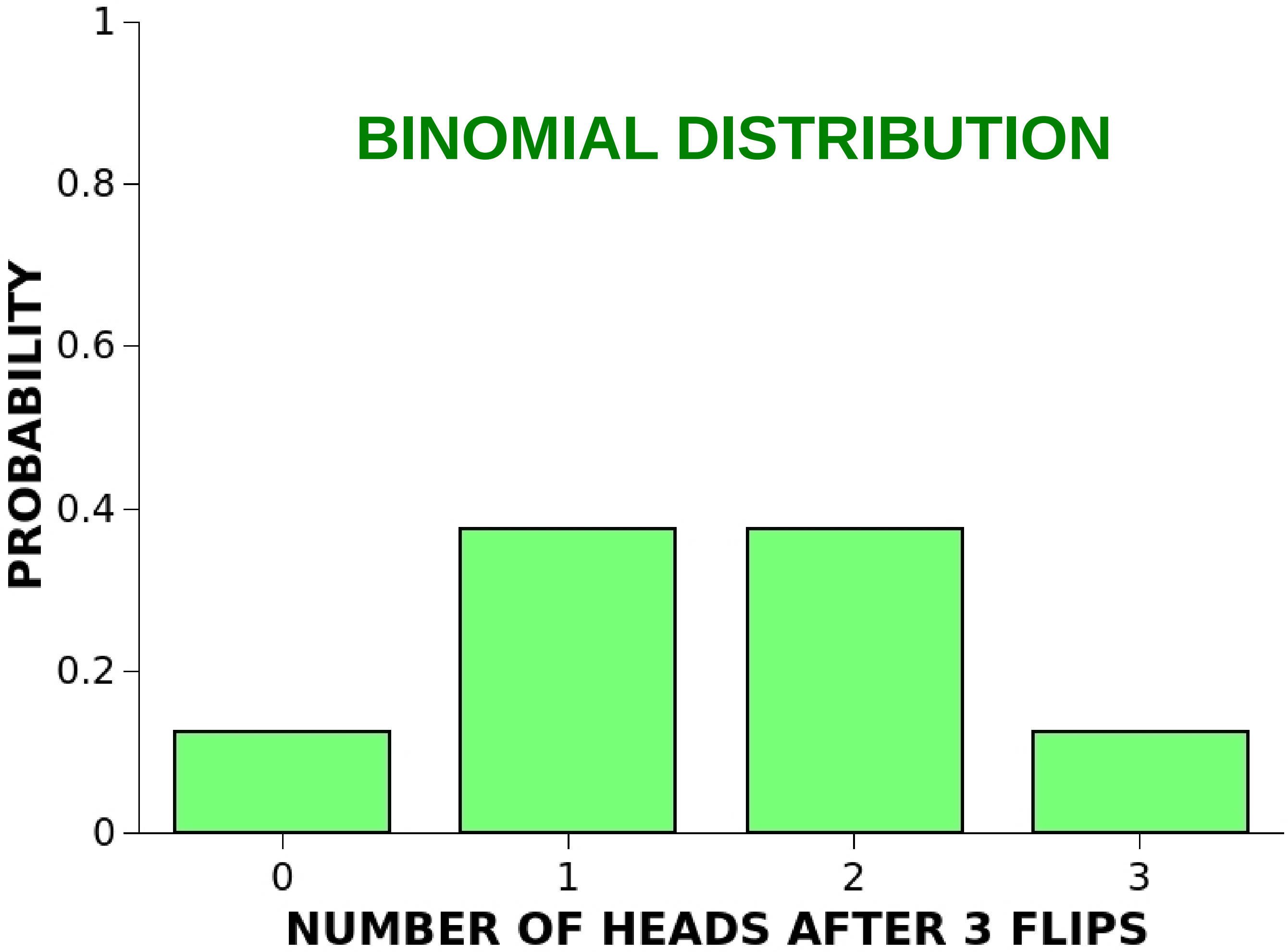
2 of 3 tosses are heads: $P_B(x=2) = 3 \times\ (1/2)^2(1/2)^{3-2} = \frac{3}{8}$

1 of 3 tosses are heads: $P_B(x=1) = 3 \times\ (1/2)^1(1/2)^{3-1} = \frac{3}{8}$

0 of 3 tosses are heads: $P_B(x=0) = 1 \times\ (1/2)^0(1/2)^{3-0} = \frac{1}{8}$

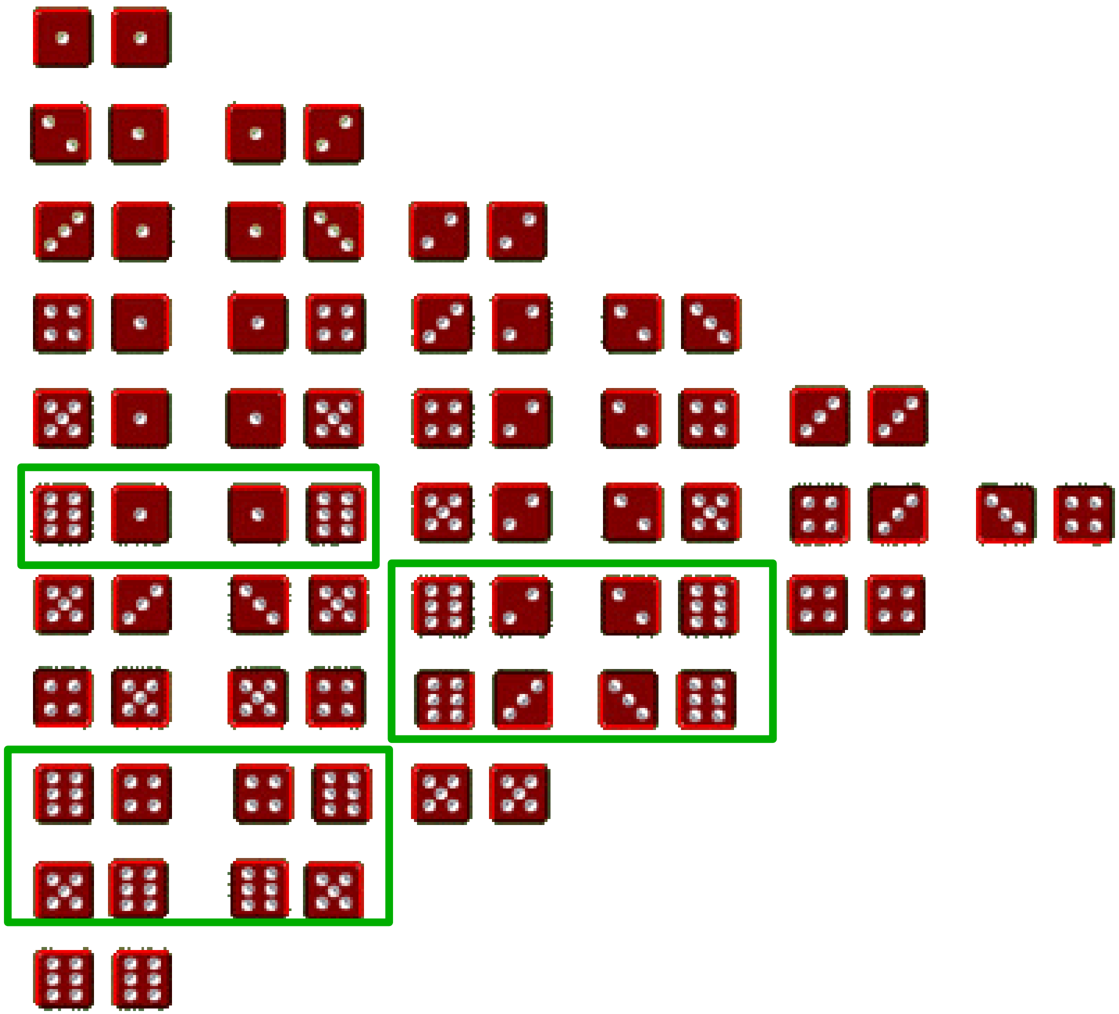
Probabilities sum to 1

PROBABILITY DISTRIBUTION:
Number of HEADS occurring on 3 consecutive coin flips



PROBABILITY that exactly x=1 **SIX** appears in N=2 rolls of the die [or one roll of two dice]:

$$P_B = \frac{2!}{1!(2-1)!} \times \left(\frac{1}{6}\right)^1 \left(1-\frac{1}{6}\right)^{2-1} = \frac{10}{36}$$

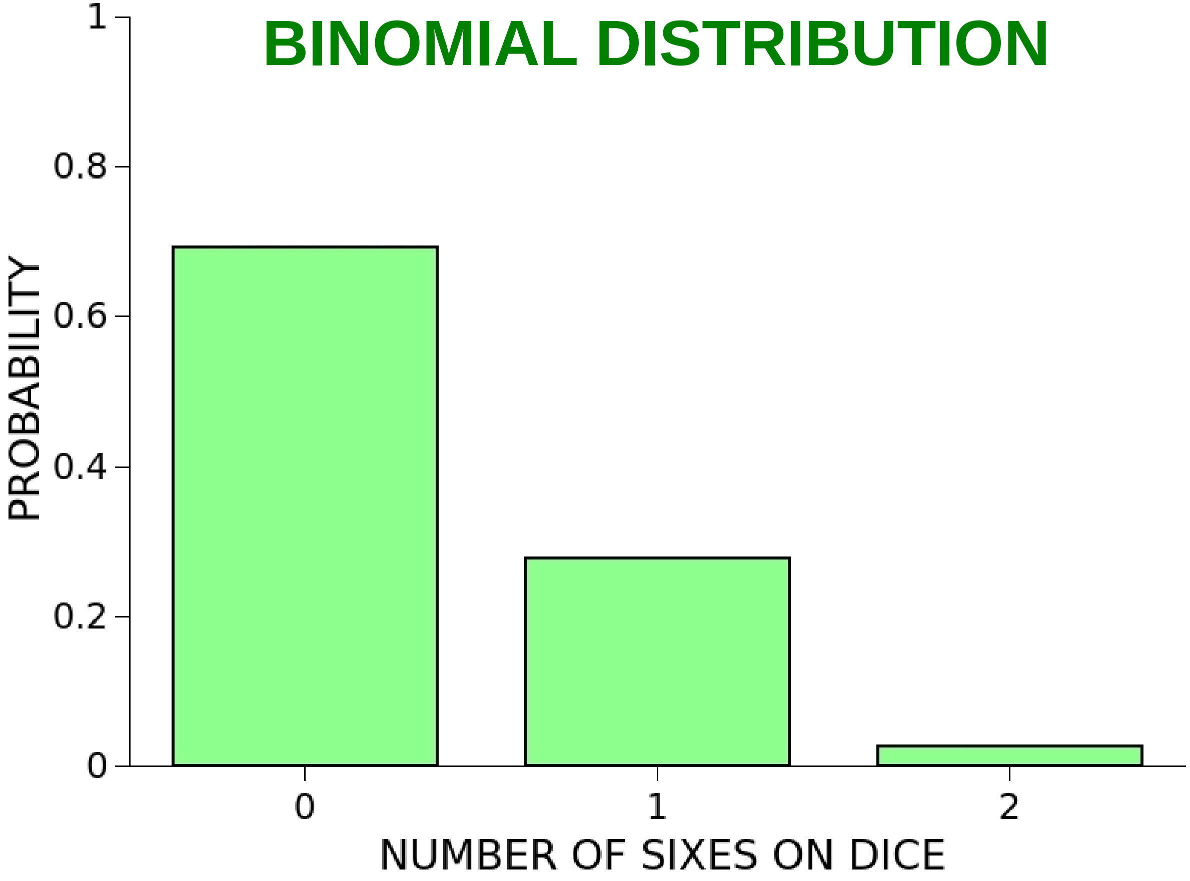


PROBABILITY DISTRIBUTION:
SIX appearing on pair of dice

Probability of zero SIXES: 25/36

Probability of one SIX: 10/36

Probability of two SIXES: 1/36



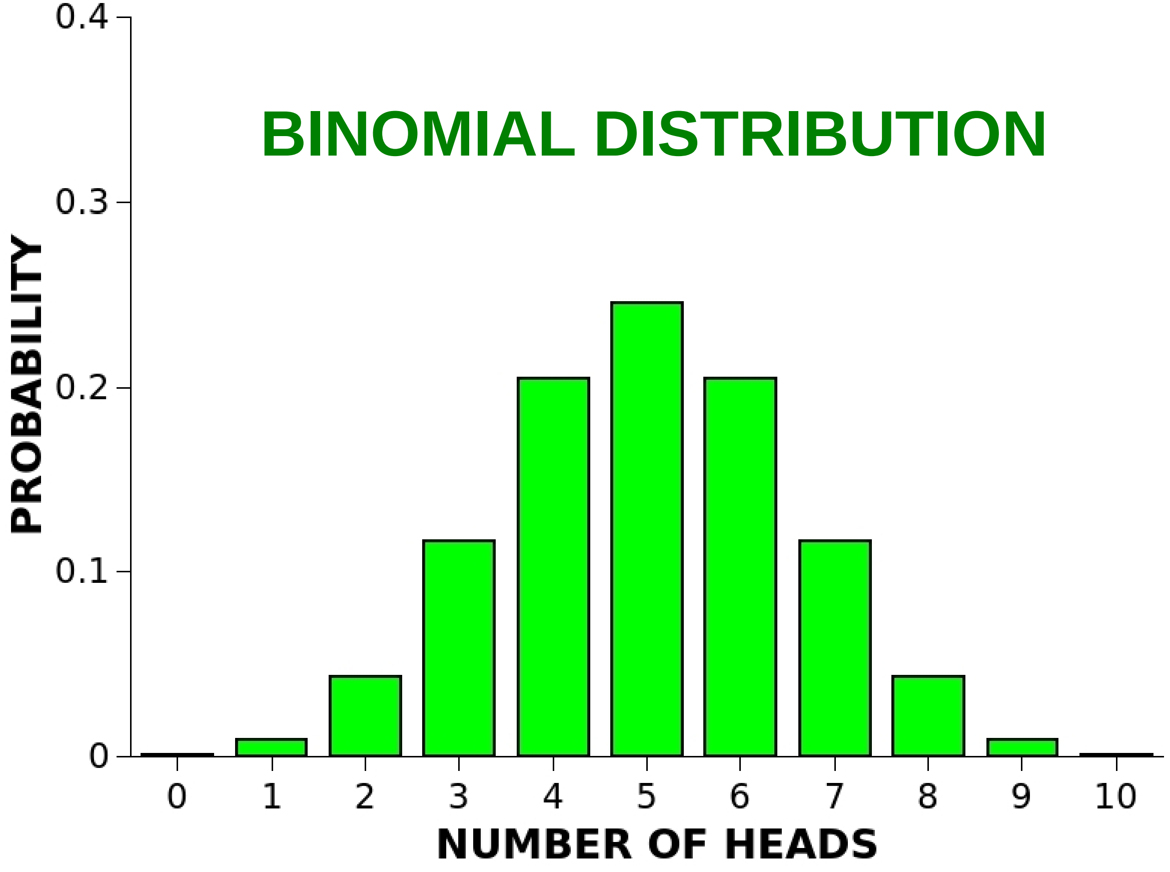
Toss same coin tossed N = 10 times

x: Number of times HEADS appears

$$P_B = \frac{N!}{(N-x)!\,x!}\;p^x(1-p)^{N-x}$$

Heads: p = 1/2;
Tails: $1 - p$ = 1/2

PROBABILITY DISTRIBUTION:
Number of HEADS occurring on 10 consecutive coin flips

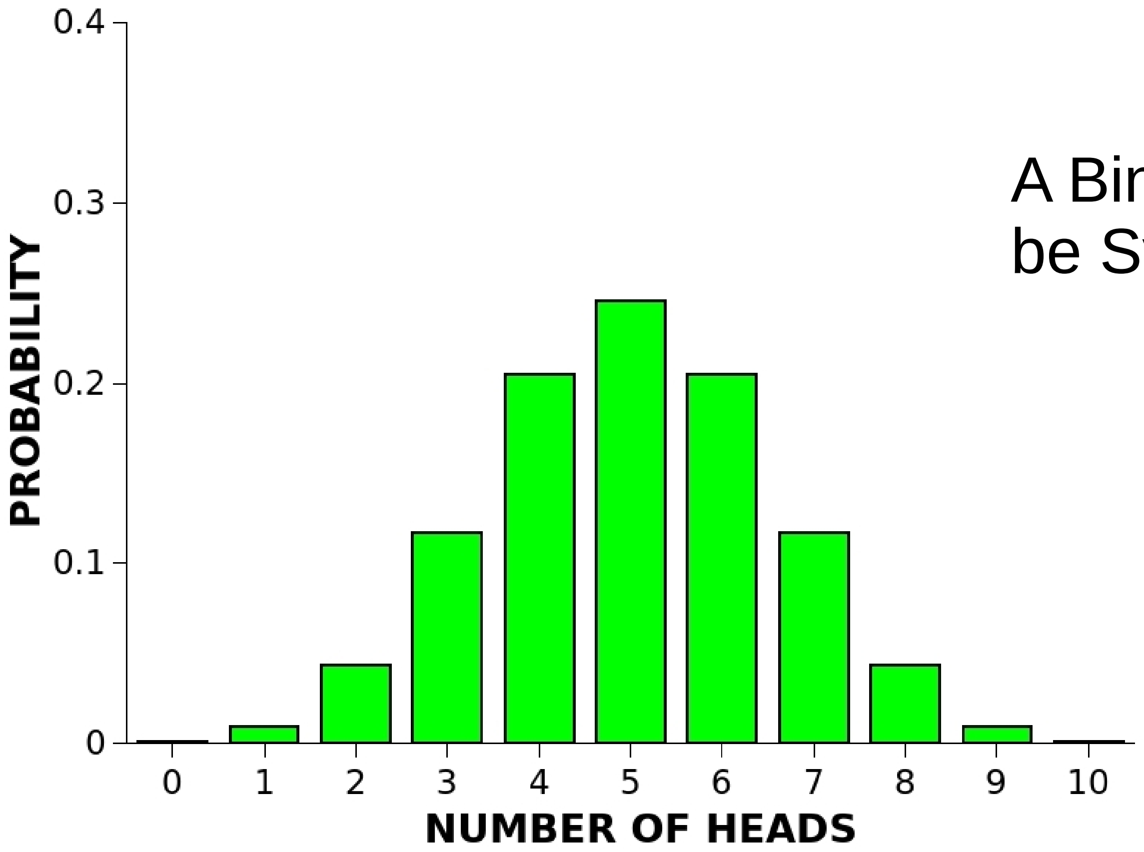


BINOMIAL DISTRIBUTION

Mean: $Np = 5$

Variance: $\sigma^2 = Np(1-p) = 2.5$

Standard Deviation: $\sqrt{\sigma} = \sqrt{Np(1-p)} = 1.58$



A Binomial Distribution may be Symmetric or Asymmetric

Writeups: Technical journal format

Abstract: Brief statement of methodology and results. If a quantitative result was found, report its value and uncertainty (eg. $\lambda = 633 \pm 8$ nm).

Introduction: Background material, motivation for the experiment, general description of your experimental approach. Relevant equations and most references are found here.

Experiment: Describe your experimental setup here. You will need at least one diagram. Provide enough information that a physics professional could reproduce the experiment.

Results/Analysis: Here is where the data gets presented, usually involving tables and/or graphs (best). This is a good place to describe the experimental errors and how they affect the uncertainty of the measurements. Do the results support theory? What are the limitations of the experiment? How could it be improved?

Summary/Conclusion: Concisely summarize the experiment here: what you did, what you found, what went right, what went wrong. This section is similar to the Abstract, but includes more information

References

Writeups: Technical journal format (continued)

Label all figures/diagrams and include a caption. Figures must be referenced in the text. Copying figures/pictures from other sources is discouraged, but if you do this include a reference to that source.

Be consistent with your referencing methodology. The APS citation scheme looks as follows:

S.H. Neddermeyer and C.D. Anderson, Phys. Rev., 884 (1937).

Use a template from a research journal (eg. APS, OSA). Look online or in hallways for examples.

No page limit, but write clearly and concisely.

Reports are due no later than 1 week after conclusion of a module. Files in .pdf format are strongly preferred.