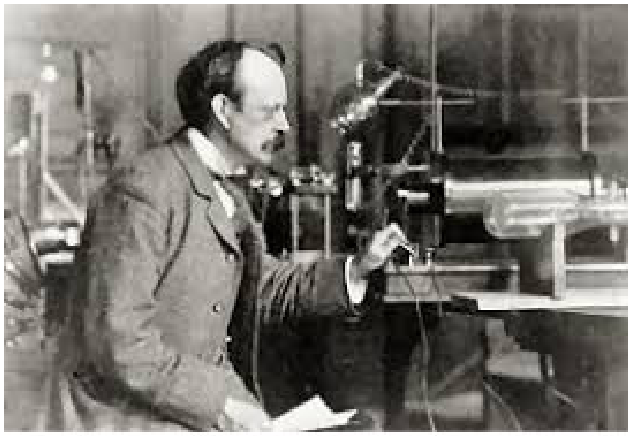
Ratio e/m for electrons v2.0



J.J. Thomson.

Background. In 1897, English physicist J.J. Thomson showed that cathode rays were composed of individual charged particles. These sub-atomic sized particles eventually became known as electrons. Thomson determined the charge-to-mass (e/m) ratio of the "corpuscles" in cathode rays by measuring how much they were deflected by a magnetic field and comparing this with the deflection by an electric field.

Experiment. The present experiment differs from Thomson's original setup, but applies the same principles. An electron gun is enclosed in a glass bulb filled with $\sim$ 10 mTorr of helium gas. A filament is heated to liberate free electrons by the process of thermionic emission. The gun is biased at a potential V that accelerates these electrons into the partial vacuum with kinetic energy eV. Because they have velocity $\mathbf{v}$ and charge e, the electron motion can be deflected by a magnetic field via the Lorentz force: $e(\mathbf{v} \times \mathbf{B})$, where the vectors $\mathbf{v}$ and $\mathbf{B}$ are perpendicular. If the path of the electron beam is made to form a circle of radius r, the centrifugal force balances the Lorentz force. This produces a simple algebraic expression for the ratio e/m.

A uniform magnetic field is generated by a pair of Helmholtz coils of radius R, separated by a distance $z = R$, with N turns per coil. The magnetic field at the center of the coils along the axial direction z is:

$$B(z) = \frac{\mu_0 N I R^2}{(z^2 + R^2)^{3/2}} \tag{1}$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m and I is the current. For the equipment used here, $N = 130$, and $R =$ 15 cm. By locating the electron beam at the midway point between the two coils, i.e. at $z = R/2$, Eq.(1) simplifies to:

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N I}{R} \tag{2}$$

The gun accelerating voltage V and Helmholtz coil current I are adjusted to force the electron beam into a circular trajectory of radius r. With the three measured values (V, I, r), the ratio e/m can calculated.

Setup. The experiment uses the following equipment:

- e/m apparatus in Figure 1.
- Split-mode ±12V DC power supply for the coils and filament

- 300V power supply for the electron gun
- 2 digital multi-meters
- Hookup wires

Figure 1: Photo of the e/m measurement apparatus similar to used here. A ruler behind the glass bulb helps measure the electron beam radius (r).

The split-mode DC power supply has two outputs that produce low-voltage and relatively high current. These are connected to the bulb filament and Helmholtz coils. A third power supply has high-voltage and very low current that sets the electron kinetic energy eV.

You may need to configure the split mode supplies for independent operation; slide switches to left. The filament can be driven by either AC or DC and should not exceed 6.3V. This voltage is not critical; its function is heat the filament sufficiently to induce thermionic emission of electrons. It also illuminates small lamps on the coils.

The second low-voltage supply provides the current I to the Helmholtz coils. This current must be carefully measured and requires that a precision ammeter be placed inline, i.e. in series. Coarse adjustment is with the supply voltage and fine control is with the Current Adjust knob on the console. Observe the indicated polarities. It is wise to use red and black hookup wires for this purpose. Never exceed 2A in the Helmholtz coils or they will be permanently damaged.

The third high-voltage DC supply connects to the electrode terminals of the electron gun. Observe the correct polarity. This potential V accelerates the electrons that are being released from the heated filament. Without any B-field present, the beam travels in a straight line directly into wall of the glass bulb. Use the second digital multi-meter to monitor this voltage at the terminals indicated.

When connections are complete, set Current Adjust to its minimum position and the panel switch to e/m. Do not connect anything to the Deflection Plates. Turn on the heater supply and let the filament warm up for a few minutes. Next apply a voltage of $\sim$ 200V to the electrodes. You should observe the fluorescence of the beam as it travels in a horizontal, straight path. The lab will need to be darkened and a cloth hood placed over the coils.

The next step is to deflect the beam with a magnetic field. Turn up the current in the Helmholtz coils, making certain not to exceed 2A. As the current increases, the electron beam should bend, eventually tracing out a circular path of radius r. This is sketched in Figure 2. The bulb can be twisted as needed to keep the path in a plane.

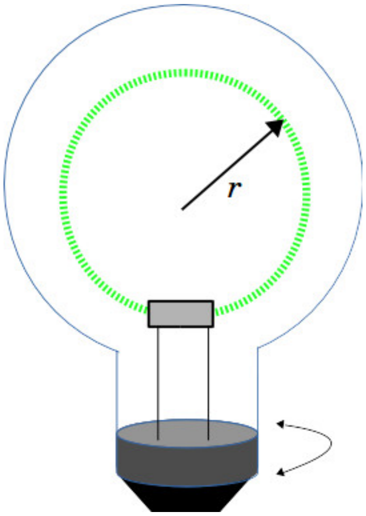


Figure 2: The electron beam inside the bulb should emit a fluorescent, approximately circular path of radius r as illustrated. The beam track can be altered by gently but firmly twisting the bulb in its socket.

If the bulb is twisted sufficiently, a spiral electron beam path will occur. Explain this behavior in your notebook. Turn off the current and reverse the polarity of the potential driving the coils. Turn the current back on, observe how the spiral changes, and provide an explanation.

Application of an additional electric field can also deflect the beam. A pair of parallel plates are located at the output of the electron gun to place a uniform electric field perpendicular to the beam path. Turn off the current to the Helmholtz coils to make B=0 and also disable the gun voltage. Leave the filament on. Make a parallel connection between the gun electrode terminals and deflection plates. Move the panel switch to “Electrical Deflection” and re-activate the high voltage. Note the amount of beam deflection as the voltage changes. Turn off the high voltage, switch the polarity applied to the deflection plates (not the gun electrodes) and re-apply the high voltage. Explain how this determines the electron charge.

Apply a magnetic field to cancel out the electric field deflection. The Lorentz force is now exactly balancing the Coulomb force, so that:

$$\frac{e}{m} = \frac{1}{2V}\left(\frac{E}{B}\right)^2 \tag{3}$$

This is the essence of the experiment performed by Thomson. It cannot be analyzed quantitatively here because of the difficulty in determining the deflection field E. The circular trajectory approach does not require a deflecting electric field and removes this uncertainty.

Turn off all power except to the filament. Disconnect the deflecting coils and switch the panel back to the e/m setting. Re-activate the accelerating voltage and B-field and establish a circular trajectory of the electron beam in the vertical plane, i.e. no spirals.

For up to 10 different settings of V and I, measure and record r. The radius is difficult to measure because the glass bulb introduces parallax distortion. A ruler is located behind the bulb that compensates for this. Adjust your observation point to place the reflected light of the beam directly behind it. If you can also measure the radius on the left side, average the two values. The ratio e/m can be computed for each measurement, allowing for an average and standard deviation for the data ensemble. Uncertainty in r is the largest source of error in this experiment.

A second scheme fixes V and varies r for different Helmholtz coil currents I. Generate some data points, plot r vs I^{-1}, and perform a least-squares fit to extract e/m.

A third scheme fixes I and varies r for different accelerating voltages V. Plot data of r^2 vs V and perform a least-squares fit to extract e/m.

Compare all the above to the accepted value $e/m = 1.76 \times 10^{11}$ coul/kg.

The most significant *systematic* error in the experiment is related to the electron velocity. It will be less than expected based on its kinetic energy eV. First, the accelerating field is non-uniform and reduced due to the hole in the anode that lets electrons escape. Second, there are inelastic collisions with helium atoms that dissipate energy. These collisions have a greater effect at low energies, so expect the results to be more accurate at the highest accelerating voltages. If the electron energy is too high, however, the beam trajectory will be distorted by its proximity to the glass enclosure.

Writeup. In addition to providing derivations for all relevant equations, address the following questions in your report:

1. Why is the electron beam visible?

2. Estimate how much error is introduced by ignoring the earth's magnetic field.

3. What if the charged particles were protons instead of electrons?

4. In a constant magnetic field, the time for an electron to make a complete circuit of the path is independent of V. The inverse of this period is the cyclotron frequency. Provide a derivation.

5. Comment on the need for a relativistic correction for the electron velocity.