

## Chapter 7:

### Formalism and Force: The Many Worlds of Badiou

As we have seen (above, chapter 1), early in *Being and Event* Badiou makes a *fundamental* methodological and thematic decision on the consequences of formalism which determines the entire subsequent trajectory of his conception of language, politics, and the event. This is Badiou's decision to conjoin an axiomatic affirmation of the One of consistency with a denial of the All of completeness, and hence to pursue the consistency of formalism within a universe (or, better, a multi-verse) conceived as radically incomplete. Badiou does not mark this decision as such, since he misleadingly portrays it simply as a decision, prompted by the appearance of formal paradoxes such as Russell's paradox, against what he terms the "One-All." However, as we have seen, the effect of Russell's paradox and related semantic paradoxes is not in fact to demand or even suggest such a simple decision between the "One" and the "Many" of traditional thought; it is, rather, effectively to *disjoin* the sovereign One into the two aspects of consistency and completeness and demand a decision between the two: *either* consistency with incompleteness (Badiou's decision) *or* completeness and totality with fundamental contradictions and paradoxes attaching to the limits of thought and signification. The second is, of course, the decision taken by the representatives of the paradoxico-critical orientation.

As we have also seen, this decision between consistency (with incompleteness) and completeness (with inconsistency) is *intimately* tied to another one on the very status of formalism and its philosophical uses. This is the decision on the relationship of language to formalism, which may very well determine the legacy of the twentieth-century linguistic turn itself. In particular, in deciding firmly and foundationally against the linguistic turn, Badiou also identifies in *mathematics* the possibility of a formalism that is, for him, wholly disjoint from language and capable of a grasping of truth beyond its limits. Thus, he decides for a position from which it is (apparently) possible to discern the *incompleteness* of any existing language. By contrast, the fundamental gesture of paradoxico-criticism is to insist upon the impossibility of any such exterior position, and hence faithfully to trace the radical consequences of the necessary formal reflection of the totality of language within itself. For paradoxico-criticism, it is *language alone* that introduces this reflexivity into the world, and hence is the basis for the reflexive structure of the subject as such. This means that there is no meta-language position from which it is possible to treat the totality of the linguistically sayable as incomplete, or to entertain the possibility of a "higher truth" to be demonstrated by formal means. The structure of language must be wholly reflected within itself, leading to the paradoxes at the limits that are the most important critical resource of the paradoxico-critical account of imaginary power and its formal resistance.

We can now summarize some of these points of difference by means of a table of contrasts:

<u>Generic Orientation</u>	<u>Paradoxico-Critical Orientation</u>
<p>Mathematics as an <i>extra-linguistic formalism</i></p> <p>Formalism as an <i>extra-linguistic position...</i></p> <p>from which it is possible to treat any existing language as <i>incomplete...</i></p> <p>and hence <i>preserve consistency</i>.</p>	<p>Mathematics as a (<i>linguistic</i>) <i>technique...</i></p> <p>and so <i>no position</i> outside language-as-such (no metalanguage)...</p> <p>and so formalism of language is necessarily a writing that paradoxically <i>reflects itself...</i></p> <p>and hence <i>inscribes limit-paradoxes</i>.</p>

## Formalization: Impasse of the Real or “Forced Pass” of Truth?

In his twentieth seminar (1972-73) in the course of a discussion of the relationship between knowledge and truth, Lacan writes, “The real can only be inscribed on the basis of an impasse of formalization.”<sup>1</sup> He suggests that it is just such an “impasse” that his own use of mathematical formalization models, by producing an excess of the function of signification – or “signifierness” – over meaning or sense, amounting almost to a *contre-sens*” direction of pure signification “without meaning.” It is, in fact, in just such an excess of the signifying function, evincing the real, that the true structure of the subject and the object of its desire, the object *a*, is ultimately to be found.<sup>2</sup> The ultimate goal of psychoanalysis is thus a pure, mathematical formalization, a writing that, although it is capable of being “integrally transmitted” without transformation or loss, is nevertheless essentially dependent on “the use of language itself” and thus cannot be separated from the “act of speaking,” whereby “I speak without knowing it...” “with my body” and “unbeknownst to myself.”<sup>3</sup> This function of speech that is simultaneously beyond the limit of knowledge defines the very meaning of the subject for psychoanalysis. Arising from “some relationship of being that cannot be known,” the structure of speaking-without-knowing “has nothing to do with what I am forced to put in being;” its knowledge is “enough ... for it to hold up, but not one drop more.” Indeed, this excessive relationship of speaking to knowledge, excessive as well with respect to what can be measured of being, manifests the specific impasse of the formal:

This is what was hitherto called form. In Plato’s work, form is the knowledge that fills being. Form doesn’t know any more about it than it says. It is real in the sense that it holds being in its glass, but it is filled right to the brim. Form is the knowledge of being. The discourse of being presumes that being is, and that is what holds it.<sup>4</sup>

The task of psychoanalysis is henceforth to investigate the structure of excess and impasse which underlies this “relationship of being that cannot be known” and which is therefore the subject of an “impossible” knowledge that is “censored or forbidden” but as *inderdicted*, “said between the words,” or written “between the lines.” It is in this excessive, prohibited relation of the subject to being that the impasse of formalism demonstrates, as an inherent consequence of the excessive function of signification

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<sup>1</sup> Lacan (1973), p. 93.

<sup>2</sup> “Lastly, the symbolic, directing itself toward the real, shows us the true nature of object *a*.” Lacan (1973), p. 95.

<sup>3</sup> Lacan (1973), p. 119.

<sup>4</sup> Lacan (1973), p. 119.

itself, that psychoanalysis must “expose the kind of real to which it grants us access” and the possibility of the “only truth that is accessible to us.”<sup>5</sup>

Early in his own career, Badiou repeats Lacan’s formulation but supplements it with a kind of programmatic reversal, writing as the title of his own seminar session of February 4, 1975: “The real is the impasse of formalization; formalization is the place of the forced pass of the real.”<sup>6</sup> He explains:

If, as Lacan says, the real is the impasse of formalization, as we saw when we ran up against the limit as return, we must venture from this point that formalization is the im-passe of the real.

The algorithm scission-determination-limit, with its deviations to the right and to the left, is the truth of the structural dialectical sequence but only up to the point where this impeccable formalism is summed up in the ‘do not trespass’ that orders a return.

We need a theory of the pass of the real, in the breach opened up by formalization. Here, the real is no longer only what can be lacking from its place, but what passes through by force.

And there is no other way of grasping this excess than to return to the Two.<sup>7</sup>

Badiou’s immediate concern, in this context, is to draw from Hegel a “dialectics” that, in its founding assertion that “there is a Two,” distinguishes itself from a “metaphysics” which only ever (and through whatever displacements and ruses, all the way up to the apparent positing of the Multiple in Spinoza and Deleuze) posits the One, and therefore “forever gets tangled up in deriving from it the Two.” Such a dialectics, Badiou suggests, is alone capable of thinking the dynamic and problematic relationship between the element and its place within a larger structure, what is indeed involved in a basic way in every inscription of a linguistic symbol and makes structurally possible its infinite iteration. But even such a dialectics is in eminent danger of winding up trapped in an eternal circle of repetition whereby the iteration of the dialectic process of “scission-determination-limit” can only ultimately yield a “theological circularity” which “presupposing the absolute in the seeds of the beginning,” returns to this beginning ever again.<sup>8</sup> For such a circular dialectic, the real “runs in circles” rather than “periodizing;” in its

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<sup>5</sup> Lacan (1973), p. 120.

<sup>6</sup> Badiou (1982), p. 22.

<sup>7</sup> Badiou (1982), p. 23.

<sup>8</sup> Badiou (1982), p. 19.

positive movement it only ever encounters a structural “impasse of the return to self.”<sup>9</sup> In order to see our way past such an infinitely repetitive, circular movement, it is necessary to conceive on the basis of formalization itself a “pass of the real” that introduces novelty and progress by producing a “redoubling of the place by that which is no longer of its order and which is no longer spatially figurable.”<sup>10</sup> The possibility of such a “pure passage” is what Badiou, already by 1978, finds modeled in the mathematical schematism of the multiplicity of pure infinities and in Cohen’s demonstrative technique of “forcing.”<sup>11</sup>

In the difference between Lacan’s formulation and Badiou’s, we can see very clearly exactly what is at stake in the *two* orientations toward formalism (the paradoxico-critical and the generic) that, as I have suggested, are coming to define the possibilities open to critical political thought in our time. “The real can only be inscribed on the basis of an impasse of formalization:” this is Lacan’s statement of the absolute necessity and unavoidability of the specific effects of reflexive language, in excess of sense and of what can be known of being, in defining the constitutively out-of-place formal structure of the subject that speaks. The claim that formalization can be, by contrast, “the place of the forced pass of the real,” is Badiou’s first articulation of his ongoing attempt to locate the transformative *force* of the action of a subject at a formal position ostensibly outside of language and immune to its inherent structural repetition.

So we have, between Lacan and Badiou, the following oppositions: a mathematical formalism of language *vs.* an extra-linguistic formal position of the mathematical; a Real which escapes signification in the gap between saying and being *vs.* a Real that is forced to appear as the new in being by the very formalism that encodes its own limits; a de-centered subject that, having “nothing to do with what I am forced to put into being,” is a structural effect of language (and appears, as such, only in its relation to its inherent “impossible-Real” lack) *vs.* an agentic subject capable of forcing the new (from a formal position beyond the effects of this lack). The choice between the two alternatives is intimately related, on

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<sup>9</sup> “How is it that the real passes beyond? How is it that it periodizes, rather than running in circles? ‘To encircle’ is said of barrels, and before it was said of suitcases. The voyage of the real is sometimes without baggage, and, according to Saint Luke, the old cask does not exclude the new wine that must be poured into it.” Badiou (1982), p. 23. Badiou’s choice of metaphors here is worth noting, and interestingly reminiscent, in fact, of Lacan’s own metaphorical description of “form” as “holding being in its glass.”

<sup>10</sup> Badiou (1982), p. 21.

<sup>11</sup> Badiou (1982), pp. 265-74. For more on *Theory of the Subject*, including an instructive comparison to the problematic of Derrida’s “Force of Law,” see Bosteels (2008b).

the one hand, to the question of the status of formalism *vis a vis* language and its structure, and on the other hand, to the question of the status of the unifying One itself.<sup>12</sup>

For whereas Badiou, in *Theory of the Subject*, already insists upon the dialectical position of the “there is a Two” from which, as he says, the One can only be subsequently inferred, Lacan’s whole conception of the relationship of the subject to the Real of being depends on the problematic structural effects of the One of language, figured in Lacan’s recurrent motto, *Ya d’l’Un* : (“there is some One” or “there is something of the One”) and demonstrated in the impasses of formal writing itself.<sup>13</sup> These impasses are structurally correlative with signification as such and constitutively linked to the complex One of the world that is assumed to exist as a whole. Thus, according to Lacan, “we know of no other basis by which the One may have been introduced into the world if not by the signifier as such, that is, the signifier insofar as we learn to separate it from its meaning effects”<sup>14</sup> and “it is at the level of language (*langue*) that we must investigate this One”.<sup>15</sup> Accordingly, “nothing seems to better constitute the horizon of analytic discourse than the use made of the letter by mathematics”<sup>16</sup> and “it is at the very point at which paradoxes spring up regarding everything that manages to be formulated as the effect of writing (*effet d’ecrit*) that being presents itself ...”<sup>17</sup> For Badiou in *Theory of the Subject*, by contrast, the Lacanian affirmation of the One is the basis only of a weak “structural” dialectic “without leverage;”<sup>18</sup> its yield is only that of “force in the position of the state, or of the symbolic,” which he already rigorously opposes to the transformative potential of “force in the position of the revolution, or the real.”<sup>19</sup>

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<sup>12</sup> This opposition between a “Lacanian” position on the impasse of the Real (one also shared by Žižek) and Badiou’s very different perspective is also helpfully discussed (with reference to the same juxtaposition of formulations I develop here) by Bruno Bosteels in Bosteels (2002), pp. 182-3 and pp.195-99.

<sup>13</sup> Thus: “It is insofar as something brutal is played out in writing (l’ecrit) – namely, the taking as ones of as many ones as we like – that the impasses that are revealed thereby are, by themselves, a possible means of access to being for us...” (Lacan (1973), p. 49); these are none other than the impasses of set theory. It should be noted, as well, that there are changes and complications in Badiou’s sense of the dialectic and its application to these formal problematics over the course of his career. For instance, in *Being and Event* he appears to reject the relevance of dialectic reasoning, especially with respect to the central disjunction between the realm of Being and the event itself. However, in *Logics of Worlds*, he once more characterizes his position (rehabilitating a term employed briefly by Althusser) as a “materialist dialectic” (see below).

<sup>14</sup> Lacan (1973), p. 50.

<sup>15</sup> Lacan (1973), p. 63/67.

<sup>16</sup> Lacan (1973), p. 44/44.

<sup>17</sup> Lacan (1973), p. 44/45.

<sup>18</sup> Badiou (1982), p. 30.

<sup>19</sup> Badiou (1982), p. 37.

The terms of this divergence already suffice to show that what is at stake in the question of the upsurge of the Real of being is not to be determined by any distinction of perspectives simply internal or external to the project of “formalization” itself. That is, following Lacan and Badiou, we can no longer purport to think the Real as a reserve of truth or being simply external to the formal as such, or attempt to hold it there on the basis of a (perhaps aesthetically or theologically defined) privilege of the “unformalizable.” Rather, although both recognize that the real of being resists symbolic expression in structurally important ways, both Lacan and Badiou pursue, in exemplary fashion, the place of the real itself through its formal writing, or mathematization. Nor, in adjudicating this disagreement, can we rely (as philosophical discourse in the wake of the linguistic turn still tends to do) on any straightforwardly conceived opposition or distinction between a position simply internal to, and one simply external to, formal thought. In particular, we cannot rely on the distinction between the position of “mathematics” on one hand, and “literature” or a kind of language thought to be radically disjoined from mathematics and its structures, on the other. For as we have seen, Lacan as well as Badiou sees in set theory and the formal *matheme* the possibility of discerning, beyond the “meaning-effects” of the Imaginary, the very structure which manifests the Real. Here, the endless twentieth century *arcana* of the “scientific” vs. the “literary,” of formal structures and their “poetic,” extra-formal resistance, are accordingly of no use, and can be left behind.

There is thus no possibility of deciding this debate on the basis of a simple *exterior* interpretation of the “meaning” or “structure” of formalization as such. Instead, if there is a decision to be made here, it will have to be made, as I have argued, on the basis of a rigorously *internal* formal reflection on the status and bearing of forms and formalisms themselves. Indeed, as the great legacy of the twentieth-century drama of language and its structure already amply witnesses, there is no way to continue with its most important results without taking radical account, as Lacan and Badiou both do, of the ongoing implications of a formal thinking that has its model in the historically radical (if *ultimately* unsuccessful) attempt to join mathematics and language in a unified and self-consistent logical structure.<sup>20</sup>

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<sup>20</sup> The continuity of this project with one of the main projects definitive of twentieth century analytic philosophy, which Badiou tends to underestimate, may already provide sufficient reason to doubt Badiou’s conviction, expressed on the last page of *Being and Event*, that “A close analysis of logico-mathematical procedures since Cantor and Frege can ... enable the thinking of what this intellectual revolution ... conditions in contemporary rationality” and thereby “permit the undoing, in this matter, of the monopoly of Anglo-Saxon positivism.” (*B&E*, p. 435). This is wildly inaccurate in any case, since there has not been a recognizable “monopoly of ... positivism” in Anglo-American academic philosophy since at least the 1960s; but additionally we may well dispute, on the basis of the evidently vast and enduring consequences of the meta-logical project inaugurated by Cantor and Frege and continued in at least some sectors of contemporary analytic philosophy, Badiou’s sense of its imminent demise.

## ***Being and Event: Forcing and the Generic***

As we saw in chapter 1, the most central burden of Badiou's argument in *Being and Event* is to demonstrate the genuine *possibility* of the advent of radical novelty beyond being itself in what he calls the "event," given the ontological theory of being as codified in the axioms of standard, ZF set theory. The undertaking involves Badiou's exposition in the rarified and complex results of set theory's investigation of the nature and relations of the immense variety of infinite sets, the "paradise" of infinities to which Cantor first showed the way. An infinite set, according to the definition Cantor drew from Dedekind, is any set whose elements can be put into a one-to-one correspondence with the elements of a proper subset of itself. Thus, for instance, the set of all natural numbers is an infinite one, since it bears a one-to-one relationship to the set of all *even* natural numbers (to match them up, we just pair each number with its double). The set containing all of the natural numbers,  $\omega$  (or, as it is also sometimes called,  $\omega_0$ ), is then the first infinite set. Its size or "cardinality" is designated  $\aleph_0$ .<sup>21</sup> As Cantor already argued, however, there are *many* more. Recall that, by Cantor's theorem, the power set of a set is always cardinally bigger (that is, it contains more elements) than the set itself. Thus it is certain that the power set of  $\omega_0$  is strictly "bigger" than  $\omega_0$  itself; this power set essentially exceeds the cardinality of  $\omega_0$  and cannot be put into one-to-one correspondence with it. The power set of  $\omega_0$  can also be identified with the set of all *real* numbers, or points on a continuous line. The question that then leads to the most complex developments of set theory is one that Cantor also already posed: *how much* bigger is this power set, the set of points on a continuum, than  $\omega_0$  itself?

Cantor formulated the question as a hypothesis, the so-called "continuum hypothesis" which he struggled in vain through the last years of his life to prove or disprove. The hypothesis asserts that the cardinality or size of the power set of  $\omega_0$  is equal to  $\aleph_1$ , the *first* cardinal larger than  $\aleph_0$ .<sup>22</sup> If the hypothesis holds, there is no *third* cardinal between the size of  $\omega_0$  and the size of its power set; if it fails to hold, there may

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<sup>21</sup> Two sets have the same cardinality if and only if their elements can be put into one-to-one correspondence. The best way to understand the distinction between ordinals and cardinals is as follows: whereas ordinals answer the question "which one?" in an *order* (e.g. the *seventh* or the *one-thousandth*), cardinals answer the question "how many?" (*seven* or *one thousand*). For finite numbers, ordinals and cardinals correspond, but in the transfinite case, although all cardinals are ordinals, it often happens that two distinct ordinals have the same cardinality. This is because, in the transfinite case, it often happens that two distinct sets can be put into one-to-one correspondence.

<sup>22</sup> *B&E*, pp. 275-80. More technically,  $\aleph_1$  is defined as the smallest *ordinal* that cannot be put into one-to-one correspondence with  $\aleph_0$  or anything smaller than it;  $\aleph_2$  is the smallest ordinal that cannot be put into one-to-one correspondence with  $\aleph_1$  or anything smaller, etc. All cardinals are thus identical with particular ordinals (although many ordinals are not cardinals). We thus symbolize these as  $\aleph_0, \aleph_1, \aleph_2$ , etc. when discussing these numbers in their "cardinal" aspect and  $\omega_0, \omega_1, \omega_2$ , etc. when discussing them in their "ordinal" aspect; bear in mind, though, that the aleph- and omega-notations actually refer to the *same* number for each index.



be one such, or infinitely many such cardinals. In its more general form, the hypothesis holds that the cardinality of the power set of *any* infinite set is equal to the very next cardinality (that, for instance, the cardinality of  $p(\omega_1)$  is  $\aleph_2$ , the cardinality of  $p(\omega_2)$  is  $\aleph_3$ , and so on.

The continuum hypothesis may at first seem to represent only a very specialized problem in the development of the peculiar theory of transfinite cardinals, but given Badiou's assumptions and terminology, it actually marks a question that is essential to the success of his doctrine of the event. Remember that the power set of any set is, for Badiou, the "state" representation of what is presented in the original set. Given this, and if, as seems plausible, the sets of interest to ontology are uniformly infinite, then the continuum hypothesis in its general form, *if it holds*, establishes that the gap between a situation and its state, in Badiou's sense, can always be regulated by a *uniform system of measure*. In particular, if the hypothesis holds, the size of the state is always greater than the size of the original set, but the extent to which it is greater is strictly measurable and controllable through the regular succession of cardinals:  $\aleph_0, \aleph_1, \aleph_2, \aleph_3$ , etc. If the continuum hypothesis turns out to be true, therefore, there will always be what Badiou terms a "measure of the state's excess"; it will always be possible to determine how much "more" a representation contains than what is initially presented, how *much* novelty it is possible to add to the situation.<sup>23</sup> If it does not, on the other hand, this "state excess" will be unmeasurable, allowing the event full range to "wander" and "err," introducing its radical consequences in an essentially unpredictable way throughout the situation in which it intervenes.

We now know that the continuum hypothesis is neither *provable* nor *refutable* from the standard ZF axioms of set theory. That one can neither demonstrate the continuum hypothesis nor its negation means, for Badiou, that although there is no way to *prove* the doctrine of the event within ontology, there is no way that ontology can rule it out either. Nothing in being necessitates the event, but nothing shows that it *cannot* take place. And the detailed derivation of this result, Badiou argues, shows a great deal about the conditions under which it is possible to think, or assert, the event. It is to the examination of these conditions that Badiou now turns.

Badiou thus takes the set-theoretical result that the continuum hypothesis is neither provable nor refutable from the axioms to have a profound ontological as well as political significance. It was Gödel himself who demonstrated the second half of this result, that it is impossible to *refute* (or prove the negation of) the continuum hypothesis within the standard axioms of set theory. His method was to exhibit a restricted

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<sup>23</sup> *B&E*, pp. 278-278, p. 293.

*model* of the standard axioms in which, as he demonstrated, the continuum hypothesis in fact holds.<sup>24</sup> In doing so, he made use of a formalized notion of *constructability*, which is in fact the formal basis of the “constructivism” that Badiou cites as the greatest threat to his own doctrine. The condition of constructability places a restriction, much in the spirit of Russell’s theory of types, on the sets that can exist. In particular, it holds that a set exists only if it can be “constructed” by taking all and only elements of some *already existing set* that have some particular (first-order specifiable) property, P, which is itself definable solely in terms of the already existing set. That is, P must be such that it is possible to determine its extension solely by considering the elements of this existing set and asking whether or not they belong to this extension; if P satisfies this condition it is said to *predicatively* define this extension.<sup>25</sup> For instance (to adapt an example given by Cohen), given the set of all finite integers,  $\omega$ , it is possible predicatively to define the set of all finite integers having a specific numerical property (such as being even or odd), but it is not possible predicatively to define the set P which contains all  $n$  such that there is a partition of  $\omega$  into  $n$  disjoint sets of a certain sort.<sup>26</sup> This is because, in considering whether a particular number, say 5, belongs to P or not, we must consider *all possible* partitions of  $\omega$  into 5 sets. The definition thus requires us implicitly to run through the entire set of all sets of integers (including, possibly, the set P itself), which cannot be said to “exist” yet, simply *given* the existence of  $\omega$ . It is thus termed an “impredicative” definition and the set P is said to be *non-constructible*.

The restriction to constructible sets yields a hierarchy of sets, the so-called “constructible universe”, that, although perhaps somewhat restricted with respect to the universe of sets overall, nevertheless contains many (if not all) of the transfinite cardinals and can, as Gödel showed, serve as a model for ZF (that is, all the axioms hold for this “restricted” universe).<sup>27</sup> Moreover, because of the restriction of constructability, the sets within the constructible universe are strictly orderable into a unified and unequivocal hierarchy. It follows that, as Gödel showed, *if we assume the constructible universe is the (whole) universe of sets*, the cardinality of  $p(\omega_x)$  = the cardinality of  $(\omega_{x+1})$ ; that is, *within the constructible universe*, the continuum hypothesis in its general form is provably true.<sup>28</sup> The limitation to the constructible universe formulates the natural-seeming thought that a new set can only be said to exist if we can define it predicatively: that

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<sup>24</sup> In general, a model is a domain of objects and relations for which the axioms all hold true. If a theorem, such as the continuum hypothesis, holds in at least one model, it is not provably false (since if it is provably false, it does not hold for any model).

<sup>26</sup> Cohen (1966), pp. 85-86.

<sup>27</sup> *B&E*, pp. 299-301.

<sup>28</sup> *B&E*, p. 309.

is, only if we can say, in terms of “already existing” sets, what defines it. The assumption of the constructible universe thus amounts to a restriction of the axiom schema of separation to allow only properties that are “predicative” in this sense to define a set. Introducing the limitation also introduces a strict measure for the “excess,” in Badiou’s terms, of the state over the situation. Other consequences of significance follow as well. For instance, if we stay within the constructible universe, the axiom of foundation does not have to be held as an axiom, since it now follows directly from the other axioms of set theory; the effect of the restriction to constructability is thus also to *require* that all sets be well-founded (that is, that their decomposition halts somewhere in a basic, founding element).<sup>29</sup>

By demonstrating one model of the ZF axioms (the constructible universe) in which the continuum hypothesis holds true, Gödel thus demonstrated that it is impossible, in the ZF axioms in general, to prove its negation; it is thus impossible to prove that the continuum hypothesis does *not* hold for ZF set theory in general. The other half of the result, that it is impossible to *prove* the continuum hypothesis in ZF, was demonstrated by P.J. Cohen in 1963. The complex technique of “forcing” that he used is robust in its formal apparatus and subtle in its conceptual implications. For Badiou, it is significant most of all in that the demonstration that it is impossible to prove the continuum hypothesis shows also that it is impossible to prohibit the event in ontology, and indeed helps to demonstrate how it might, paradoxically, appear there by “subtracting” itself from what ontology can discern.

Cohen’s general method, once again, was to construct a model; this time, however, the aim is to develop a model in which the continuum hypothesis is definitely *not* true. If there is such a model, it will follow that the hypothesis definitely cannot be proven in ZF. The details of the actual construction that Cohen used are complex. I shall therefore try to convey only a sense for the general strategy, pausing on the parts of it that are of particular interest to Badiou.

The intuitive idea is to construct a certain kind of model of ZF and show that *within this model*, we can make the cardinality of  $p(\omega_0)$  arbitrarily high (i.e., much higher than  $\aleph_1$  if we wish, making the continuum hypothesis false). In order to do so, we must begin with a certain kind of set of cardinality  $\aleph_0$ , the so-called “quasi-complete” set or situation.<sup>30</sup> The strategy will then be to add to such a set a “generic” or “indiscernible” extension; if we can do so, it will be possible to show that we can (essentially by stipulation) make the cardinality of the continuum, or  $p(\omega_0)$ , as high as we like. A set is called “discernible” if there is some property *specifiable only in terms of existing sets* that discerns it; in other

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<sup>29</sup> *B&E*, p. 304.

<sup>30</sup> *B&E*, pp. 358-62

words, if a set is discernible within a larger set  $S$ , then there is some property *definable in terms ranging only over members of  $S$*  that picks out all, and only, the things in  $S$  that are in that set.<sup>31</sup> In this sense, the discernible sets will be all the sets that an “inhabitant” of  $S$  (who is restricted to considering *only* elements of  $S$  in defining his terms) can talk about, or have any knowledge about.

Now, the demonstration that the continuum hypothesis can fail depends on our demonstrating the existence of an *indiscernible* (or non-constructible) set, a set that, although real, is definitely not nameable in a language thus restricted, or discerned by any property *it* can name (Badiou symbolizes the indiscernible set: ‘ $\varnothing$ ’). We can then add this indiscernible set to an existing quasi-complete situation to produce a “generic extension” of the original set and we will subsequently be able to demonstrate the falsehood of the continuum hypothesis with respect to the thus extended situation.<sup>32</sup> Cohen’s technique for generating the indiscernible set, and subsequently demonstrating its existence, is a complex piece of formalism. Intuitively, however, the idea behind it is this. We construct  $\varnothing$  by “running through” all the possible properties  $\lambda$  that discern sets. For each discernible property  $\lambda$ , however, we include in  $\varnothing$  one element that has that property. Once we’ve run through all the properties in this way, we know that the set we’ve created has “a little bit of everything”; since it has one element of *each* discernible property, there is *no* discernible property that discerns this set itself. (This is, yet again, an instance of the general “technique” of diagonalization).<sup>33</sup> Thus we definitely have an indiscernible set. This set will exist, but it will not have *any* possible determinant (for we have built it in such a way, by running through *all* the specific determinants, that no *one* specific determinant can determine it). It is in this sense that it is the “anonymous representative” of the whole range of discernible subsets of the original situation.<sup>34</sup> Its appearance in ontology, according to Badiou, marks the free and immanently indeterminable circulation of the errant consequences of the event.

Developing the implications of the formal argument, Badiou draws out the consequences he sees in it for the theory of the subject and the possibility of truth. Art, science, politics, and love are “generic procedures;” their pursuit, by analogy with the construction of a generic extension, progressively adds to the existing situation the indiscernible set of consequences of an event.<sup>35</sup> This addition is conceived as

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<sup>31</sup> *B&E*, p. 367. What Badiou calls “discernibility” is thus the same as what Gödel calls “constructability.”

<sup>32</sup> *B&E*, pp. 373-76.

<sup>33</sup> *B&E*, p. 337, p. 392.

<sup>34</sup> *B&E*, p. 371.

<sup>35</sup> *B&E*, p. 340.

connecting the generic set to the event by means of what Badiou calls an “enquiry”; each member of the existing situation which is “investigated” is indexed positively or negatively as belonging or not belonging to the generic extension, and it is of the essence of the inquiry that it can traverse an infinite number of elements.<sup>36</sup> Such progressive addition, at its infinite limit, constitutes the addition of a “truth” to the existing situation; it is to be strictly distinguished from the discernment within a situation, by means of properties, of what (is not necessarily true) but merely “veridical” in it.<sup>37</sup> In intervening, a subject “forces” a new situation which, like Cohen’s “generic extension,” adds to the original situation a set of consequences which are indiscernible by any concepts or properties formulable within it.<sup>38</sup>

Because they are collectively indiscernible, these consequences cannot be picked out by any term of an “encyclopedia” or schematization of possible knowledge accessible from within the situation; the consequences of an event are in this sense “subtracted” from positive knowledge.<sup>39</sup> Nevertheless, as Cohen demonstrated, it is possible to “force” them by successively considering conditional statements about the membership of certain elements in the generic set. Though it is not possible for the inhabitant of the initial situation to determine *whether* a given element is an element of the generic set, he can say (by means of forcing) that if the element is in the generic set, such-and-such statement about that set *will be* true (in the extended situation to be created). The elements that are considered as possible elements of the indiscernible generic set are thus treated as “conditions” and these conditions determine, by way of the forcing relation, statements that will be true of the new model created by adding the generic set to the existing one. It is possible in this way to build up a series of consistent conditions such that the entire infinite series of conditions, if thought of as complete, determines a set that cannot be specified by any positive predicate but, in that it contains at least one element discerned by *each* possible predicate, is “typical” or “generic” of the initial set as a whole.<sup>40</sup> By running through the series of conditions and their consequences, we are in a sense “reading out” the generic set, term by term, in such a way as to preserve its genericity, or its indiscernibility by any internally definable predicate.<sup>41</sup> In so doing, though we are

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<sup>36</sup> *B&E*, p. 333.

<sup>37</sup> *B&E* pp. 331-34.

<sup>38</sup> *B&E* p. 342.

<sup>39</sup> *B&E* pp. 335-38.

<sup>40</sup> *B&E* pp. 367-71.

<sup>41</sup> I draw the metaphor of “reading out,” as well as much of the exposition of this paragraph, from Peter Hallward’s immensely useful description of forcing in the “Appendix” of Hallward (2003). For Cohen’s own description, see Cohen (1966). Badiou himself relies heavily on Kunen (1980); it is from this treatment that much of Badiou’s own discussion of what is accessible or not to an “inhabitant” of the initial situation, including the identification of what is discernable to such an “inhabitant” with the constructible sets, appears to be drawn.

not directly in a position to determine which elements belong to the generic set, we are in a position to determine what statement will hold true of the generic set *if* a certain element belongs to it (and if it is indeed generic).

At the infinite end of the process, we will have the complete specification of a set that is indeed generic and cannot be determined by any internally definable predicate of the language. With the addition of the generic set, various statements that were not formerly accurate or “veridical” in the initial, unextended situation will be so in the new, extended one. It is thus that the subject, as Badiou says, “forces a veracity, according to the suspense of a truth.”<sup>42</sup> The centerpiece of Cohen’s own demonstration is the proof that it is possible with this method to force the truth of a (more or less) arbitrary statement about the cardinality of the power set of  $\omega_0$ ; for instance, we can force the truth of  $p(\omega_0) = \aleph_{35}$  or  $p(\omega_0) = \aleph_{117}$  or whatever we like. This is accomplished by considering an arbitrarily high cardinal ( $\aleph_{35}$  or  $\aleph_{117}$  or whatever) and showing by means of the construction of series of conditions that it is possible to distinguish, in the extended situation formed by adding the generic set, at least as many different subsets of  $\omega_0$  as there are elements of that (arbitrarily high) cardinal.<sup>43</sup> Thus, in the extended model formed by adding the generic set (which is itself composed entirely of *subsets* of the unextended situation, though these subsets were initially indiscernible in the unextended situation), these statements become true and the continuum hypothesis fails.

From the perspective of the initial situation and its state, these consequences of the addition of the generic set remain random; only the generic procedure itself, in “fidelity” to the event, picks them out. The subject is then definable as anything that can practice this fidelity; the result – and with it Badiou closes the book – is an updated, “post-Cartesian” and even “post-Lacanian” doctrine of the subject. On this doctrine, the subject is not a thinking substance; it is equally not (in the manner of Lacan) a void point, or (in the manner of Kant) a transcendental function.<sup>44</sup> It is the “faithful operator” of the connection between the event and its infinite consequences, the generic procedure of truth in its coming-to-be.

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<sup>42</sup> *B&E*, p. 407.

<sup>43</sup> *B&E*, pp. 421-23.

<sup>44</sup> *B&E* pp. 391-92.

## ***Logics of Worlds: Being, Appearance, and Truth***

In *Logics of Worlds*, Badiou supplements this earlier “ontological” account of eventual change with a comprehensive formal theory of *appearance*, what Badiou here terms a “phenomenology.” This is a formal theory of the structuration of the relations in virtue of which objects can appear as phenomena and the dynamic conditions for the possible transformation of these relations. But although the underlying apparatus is once again drawn from mathematical formalism, the sociopolitical implications of such possibilities of change are also, once again, very much to the fore. Indeed, in its “Preface,” Badiou presents the whole argument of *Logics of Worlds* as part of an attempt to theorize what escapes the assumptions of contemporary “natural belief,” what he sees as the confining dogmas of postmodern relativism and conventionalism.<sup>45</sup> According to Badiou, the most central axiom of this order of “natural belief” that, having demystified all theological categories, can claim to disbelieve in the existence of anything beyond the material world of contemporary scientific rationality, is the reductive claim that “there are only bodies and languages.” Such views, Badiou thinks, can only ultimately yield a monotonous regime of “democratic materialism” that, in seeing all cultures and their claims as on a level, forecloses both any possibility of real development and any effective intervention to produce fundamental change.<sup>46</sup> Badiou proposes to replace this axiomatic of contemporary conviction with the one of what he calls, following Althusser, a “materialist dialectic.” The central difference here is Badiou’s unhesitating affirmation of what he calls Truths, which are, according to him, generally denied or suppressed in the contemporary orthodoxy of belief.<sup>47</sup>

Badiou’s notion of truth, however, is, as we’ve seen, a heterodox one, not to be understood in terms of any familiar (e.g., correspondence or coherence) notion. For Badiou, the central mark of a Truth is its capacity to *break with* (or “subtract itself from”) an existing regime of knowledge, and so to define a direction of radical transformation which, if followed out, will lead to the substantial re-ordering of basic possibilities of presentation and representation within the existing order.<sup>48</sup> This vector of transformation is, for Badiou, always infinite; and thus the punctual articulation of a Truth by means of an eventual break with a given situation is always partial, and liable to be taken up again, even after a lapse of centuries or

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<sup>45</sup> Badiou (2006) (henceforth: *LofW*), pp. 2-3.

<sup>46</sup> *LofW*, pp. 2-8.

<sup>47</sup> *LofW*, p. 4

<sup>48</sup> *LofW*, pp. 9-10.

millennia, through the renewal of a faithful tracing, by the agency of what Badiou terms the “subject,” of the consequences of a subsequent event.<sup>49</sup>

Much of this terminology is familiar from *Being and Event*'s theory of radical, eventual change, and Badiou's aim here is not so much to alter that theory in any fundamental way as to remedy certain deficiencies he now sees in it. In particular, *Being and Event* described the *ontological* structure and conditions for the event to come about, but it failed to consider in detail the conditions governing the *appearance* of events in determinate, structured situations, what Badiou now terms “worlds.” Worlds, like the “situations” of *Being and Event*, are irreducibly multiple, and the potentially radical implications of the event are again to be traced in its capacity to transform these existing situations. However, Badiou now gives a much more detailed theory of the structure of situations or worlds, and is thereby able to describe in much more specificity the various possibilities of their change or transformation.

This supplemental task of understanding the structuration of appearances and how they change and develop does lead Badiou to various innovations and modifications of his earlier theory. Most significant here is the consideration that, whereas it is plausible that ontology is static and non-relational, the realm of appearances is inherently relational, dynamic, and variable. Accordingly, in the realm of appearances, there are *degrees* of existence and of “identity” between two objects, and even greater or lesser degrees of identity between an object and itself.<sup>50</sup> These relations of identity and existence determine degrees of intensity of appearance, ranging from a minimal (effectively zero) degree, corresponding to complete invisibility or failure to appear, to a maximal degree of appearance, corresponding to maximal presence or effectiveness within the structured world.<sup>51</sup> Badiou demonstrates these relationships *in concreto* by working out in detail several different examples of “worlds” and their phenomenal elements or objects in their degrees of intra-world existence: a country road at sunset, a painting by Hubert Robert, a Parisian political demonstration, and the city of Brasilia.

Whereas *Being and Event* theorized the overarching structure of Being (at least insofar as it is speakable) as modeled by the axioms of standard set theory, *Logics of Worlds* turns instead to category theory to model the domain of appearing. In general, a category can be understood as a structure of relations; the identity of the objects thus structured is irrelevant, as long as this structure of relations is preserved. Just as it is possible to found much or even all of mathematics on sets and set theory, it is also possible to give

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<sup>49</sup> *LofW*, pp. 33-35.

<sup>50</sup> *LofW*, pp. 118-140.

<sup>51</sup> *LofW*, pp. 138-140.



a wide variety of mathematical objects a foundation in categories; and such a practice of foundation has interesting consequences for how we can think of the structure of these various domains and relations.<sup>52</sup> Most significantly for Badiou's project, however, it is also possible to use a special kind of categorical structures, known as *topoi*, to model *logical* ones; for instance, we can use topos theory to model algebraically all of the axioms and relations of standard, classical propositional logic. In fact, it is a consequence of this categorical method that the logics modeled *need not* be classical ones; indeed, we can use topos to model any number of non-classical logics, including intuitionist and many-valued ones. These non-classical logics can uniformly be understood as determined by total algebraic structures called Heyting algebras, and their various determinate parameters can be taken to determine, for each world, the possible relations of compatibility and inclusion between any of its objects.<sup>53</sup>

Using this category-theoretical framework, Badiou can thus define the underlying structures determining the "logic" or relations of appearance which in turn determine what is treated as existent in each world. He terms the specific structure determining these logical relationships and intensities of existence for a particular world its "transcendental." And although the terminology echoes idealist theories from Kant to Husserl, Badiou emphasizes that in speaking of such a structuring "transcendental" he does not in any sense intend to give a theory of the transcendental *subject*. Instead, Badiou's relationships of structuration of appearance are explicitly *objective*, determining without exception what can be understood to "exist" in a particular world, and what remains "inexistent" or invisible within its own particular way of structuring its phenomena.<sup>54</sup>

Similarly, although the employment of Heyting algebras and the multiplicity of differently structured logics suggest intuitionist or constructivist motivations, at least with respect to the realm of appearances, Badiou emphasizes that the "transcendentals" that structure worlds are not, for him, in any important sense *linguistic* structures. In fact, although the *multiple* logics that define the plurality of the transcendental structures of worlds are subsumed to a larger, single *mathematical* (i.e. category-theoretical) structure that Badiou terms a "Greater Logic," he emphasizes that this Greater Logic has nothing to do with the ordinary logic of propositions or language, which is itself to be subsumed simply as a special case:

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<sup>52</sup> For a perspicuous presentation, see, e.g., Goldblatt (2006)

<sup>53</sup> *LofW*, pp. 173-190.

<sup>54</sup> *LofW*, pp. 231-241.

To think the ‘worldly’ multiple according to its appearing, or its localization, is the task of logic, the general theory of objects and relations. It is conceived here as a Greater Logic, which entirely subsumes the lesser linguistic and grammatical logic.

To wrest logic away from the constraint of language, propositions and predication, which is merely a derivative envelope, is no doubt one of the stakes of *Logics of Worlds*. Section 4 of Book II scrupulously demonstrates that ordinary formal logic, with its syntax and semantics, is only a special case of the (transcendental) Greater Logic which is set out herein. Having said that, this demonstration is not the principal aim of the Greater Logic. Of course, its polemical advantage lies in ruining the positive claims of the entirety of so-called ‘analytic’ philosophies.<sup>55</sup>

Additionally, by steadfastly avoiding describing the structuration of appearances as in any way dependent on structures within us or created by us, Badiou aims to break entirely with all forms of idealism. Any such position, he avers, will fail to grasp the objectivity of what is phenomenal within a world, the capability of objects to appear and take on their distinctive degrees of existence without any contribution whatsoever from the “human animal.”<sup>56</sup>

Badiou is thus able to theorize (using category theory) the phenomenal structure within appearance of what are *also* (already according to *Being and Event*) thinkable (using set theory) as sets or multiplicities within being itself. More specifically, the worldly existence of a phenomenal “object” is wholly determined by the transcendental indexing of the world in which it appears, and this transcendental indexing reciprocally determines the possibility of decomposing such an object into its (phenomenally) simple parts, what Badiou calls “atoms.” On the other hand, however, any atom is also, Badiou asserts, identical to an *ontological* element of the larger set which (ontologically) “supports” the phenomenal appearance of any object in which it appears; in other words, every (phenomenal) atom is simultaneously a “real atom” of ontology. This assertion, which has the unargued status of an axiom, also guarantees that there can indeed be some relationship between the otherwise disjoint realms of being and appearance. Badiou terms it the “postulate of materialism.”

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<sup>55</sup> *LofW*, p. 93. Badiou makes a similar argument in chapter 8 of *Briefings on Existence* (Badiou (1998b)), defending there (p. 111) the theses that “we have to break with the linguistic turn...” due to its penchant to “[end] up with the pure and simple dislocation of philosophical desire as such” in a “pragmatism of cultures” or a “Heideggerian dependency,” and that this break requires the production of a “new thought” of the relationship of mathematics and logic whereby logic is mathematized, rather than mathematics being logicized.

<sup>56</sup> To support the necessity of affirming objective degrees of existence in a world, Badiou cites (pp. 118-19) an argument against all idealist or “correlationist” philosophies that relate subject to object given by Quentin Meillassoux (2006). For more on Meillassoux, see endnote – to chapter 10, below.

A real atom is a phenomenal component ... which, on one hand, is an atomic component (it is simple, or non-decomposable), and on the other, is strictly determined by an underlying element  $a \in A$ , which is its ontological structure. At the point of a real atom, being and appearing conjoin under the sign of the One.

It only remains to formulate our ‘postulate of materialism’, which authorizes a definition of the object. As we know, this postulate says: every atom is real. It is directly opposed to the Bergsonian or Deleuzian presupposition of the primacy of the virtual. In effect, it stipulates that the virtuality of an apparent’s appearing in such and such a world is always rooted in its actual ontological composition.<sup>57</sup>

The axiomatic postulate of materialism thus ensures the ontological consistency of any world by means of the isolation of its atomic elements, thereby also blocking, as Badiou makes clear, the hypothesis of any “virtual” structuring of existence by language or the phenomenal dimension of appearance in excess of its ontological substrate. Any such structuring will thus, for Badiou, always be the outcome of processes that are *simultaneously* phenomenological *and* ontological; in fact, the underlying identity of phenomenal objects and ontological multiplicities that is assured by the “postulate of materialism” leads to the primary innovation of *Logics of Worlds*’ new theorization of evental change.

This is the idea of a specific “retroaction” of appearance on being, whereby the fact of the phenomenal appearance of a particular (ontological) set within a structured world brings about a train of changes which will ultimately transform the transcendental structure of the world itself.<sup>58</sup> This strange retroactive effect of appearing on being, which defines the structure of the event, is possible only through a paradoxical effect of self-reference or self-belonging. In particular, for an event to occur, it is necessary not only that a certain set (a multiple in being) be a member of itself, but *also* that this particular being appear, in a world, as an *element* of a *transcendental indexing* that again indexes that very being. More specifically:

...it can happen that multiple-being, which is ordinarily the support for objects, rises ‘in person’ to the surface of objectivity. A mixture of pure being and appearing may take place. For this to happen, it is enough that the multiple lays claim to appearing in such a way that it refers to itself, to its own transcendental indexing. In short, it is enough that a multiple comes to play a double role in a world where it appears. First, it is objectivated by the transcendental indexing of its

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<sup>57</sup> *LofW*, pp. 250-51.

<sup>58</sup> *LofW*, p. 94, pp. 221-3.

elements. Second, it (self-)objectivates, by figuring among its own elements and by thus being caught up in the transcendental indexing of which it is the ontological support. Worldly objectivation turns this multiple into a synthesis between the objectivating (the multiple support and referent of a phenomenality) and the objectivated (belonging to the phenomenon). We call such a paradoxical being a ‘site.’<sup>59</sup>

Here, as in *Being and Event*, we should think of the precarious way in which a historical event, for instance, can bring into existence the very terms and signifiers that will subsequently provide for its own evaluation, and thus play an essential role in *constituting* it as the event it (given the new terms and degrees of evaluation they imply) will subsequently be visible as having been.<sup>60</sup> The particular underlying structure of an event as such is therefore again, as in *Being and Event*, to be understood as a matter of the event’s “self-inclusion” or “self-reference,” which as Badiou again suggests gives it an inherently “paradoxical” being that is responsible for the very possibility of the radical changes it can bring about.

However, there are two important differences from *Being and Event*’s account of the event’s underlying formal structure. First, since Badiou now theorizes the structure of the event (on the side of appearances and existence, at least) as residing entirely in the “worldly” phenomena of its structural placement or localization, including importantly its relationship to its own transcendental indexing, he can now assimilate the structure of an event entirely to that of its “site,” which is defined as just this localization. Thus, rather than distinguishing (as Badiou now suggests was the case in *Being and Event*, unclearly) between the worldly “site” and the partially extra-worldly “event,” Badiou can now simply assimilate the two and deal directly with the structure of “sites” themselves. However, this also yields a more complex taxonomy of possible change, since Badiou can also now distinguish *among* self-referring sites between those that are structurally capable, and those that are not capable, of supporting an “event” in the full sense.

Second, and perhaps more importantly, the “self-referential” or “self-inclusive” structure of the event is now somewhat more complicated than the simple schema of *Being and Event* would suggest. There, the event’s capacity for self-reference was understood simply as a matter of its auto-designation, or its ability to provide a name for itself. Here, the structure of self-reference is more complex: it is not simply the event’s provision of a name for itself, but rather its relation to its own transcendental indexing, that

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<sup>59</sup> *LofW*, p. 360.

<sup>60</sup> *LofW*, pp. 364-66.

provides its basis. An evental “site” – the “place” from which radical, transformative change is alone possible – is thus definable as a multiple that contains within itself an element which determines the “transcendental indexing” of the whole world in which that very multiple appears as a phenomenon, and which can thus alter this fundamental structure. Thus, while *ontologically* speaking, an evental site is simply (as in *Being and Event*) a “reflexive set” or “a multiple to which it happens that it is an element of itself” (p. 366), in terms of appearance and phenomenology the structure is a bit more complex: “A multiple which is an object of [a] world – whose elements are indexed on the transcendental of the world – is a ‘site’ if it happens to count itself in the referential field of its own indexing. Or: a site is a multiple which happens to behave in the world in the same way with regard to itself as it does with regard to its elements, so that it is the ontological support of its own appearance.” (p. 363) The role of auto-nomination in *Being and Event* is thus replaced, to a certain extent at least, with a auto-nomothesis that is not a positing simply of names, but of laws (of appearance and “existence”) as well.

These further details and specifications lead Badiou, in *Logics of Worlds*, also to develop a more specific and subtle theory of the varieties of change than was present in *Being and Event*. When the quasi-paradoxical structure of the “evental site” occurs, according to Badiou, it is immediately possible for it to be taken up in a variety of different ways, corresponding to different degrees and intensities of change in the world. In the most dramatic case, a subject’s faithful tracing of the implications of the structure of the evental site results in the element which was formerly *minimal* in its degree of existence – what had earlier literally “in-existed” in that particular world, being present in its being but completely invisible to the world’s logic – suddenly attaining a *maximal* degree of existence, bringing with it all the changes in the existing structure that this implies.<sup>61</sup> The analogy here is to the sort of political revolution in which (as an old Marxist motto runs) “we who are nothing shall be all!”; but Badiou also thinks of this kind of evental change as possible in other domains, for instance in the kind of “paradigm shift” in science that not only elicits new objects and makes visible phenomena that previously escaped attention and thus “lacked existence,” but even fundamentally re-organizes the large-scale structure of what counts as existing in the (newly transformed) world.

Once the possibility of such a radical change in the structuration of worlds is demonstrated, the remainder of Badiou’s analysis focuses on the ways in which the implications of the event can be brought out and eventually used to transform the structure of the world by the action of a faithful subject. It is essential to this tracing, Badiou thinks, that the consequences of the initial event be sequentially filtered through a

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<sup>61</sup> *LofW*, pp. 374-79.

series of either/or decisions, what Badiou calls “points.”<sup>62</sup> Fidelity to the event is then the “organization” of such points, and the ultimate appropriateness of the binary decisions made at each one to the infinite constitution of the Truth that – if the binary decisions are made “correctly” -- will thus have been brought progressively into being.<sup>63</sup>

This allows Badiou to give a full taxonomy of the kinds of change that are possible according to the various structures of worlds and the types of sites that are present in them. First, there are worlds in which there are no sites: in which no object or set of objects effectively indexes its own transcendental and so becomes the basis of the possibility of fundamental change. Here, in what Badiou suggests we may term “atonic” worlds, there are gradual “modifications” that simply extend the existing structural logic of the situation, but no fundamental, structural change is possible. Second, there are worlds which do indeed possess sites, but whose sites are nevertheless not characterized, according to the transcendental logic of the world, by a “very strong” or “maximal” degree of existence; Badiou calls such sites “facts.” “Facts” may be capable of producing a kind of upsurge of novelty, but because the site does not possess “maximal” existence, the novelty will be re-assimilated into the (stronger) existing logic of the situation and so will vanish from the world without, in this case, producing any lasting or enduring change.<sup>64</sup> By contrast with these, a “singularity” is a site that indeed has a “maximal” degree of existence, and will therefore be able to “compensate” by way of its “force of existence” for its tendency to vanish from the world.<sup>65</sup> However, it still can occur that although a site is a singularity, and its *own* degree of existence is “maximal,” its *consequences* do not have a maximal degree of existence and so, even if they are faithfully drawn out, again do not have the potential to bring about fundamental structural change. In this case the “singularity” is termed “weak,” and distinguished from a “strong singularity” in which both the degree of existence of the site and of its consequences are existentially maximal. Only in this last case do we have an “event” in the full sense, and the possibility of radical, structural change of a fundamental sort.

Finally – and here, at the end of *Logics of Worlds*, Badiou recapitulates on a schematic basis a typology that is already proposed in Book I – the structural possibility of an event and the points related to it

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<sup>62</sup> *LofW*, pp. 403-411.

<sup>63</sup> Thus: “A point concentrates the degrees of existence, the intensities measured by the transcendental, into only two possibilities. Of these two possibilities, only one is the ‘good one’ for a truth-procedure that must pass through this point. Only one authorizes the continuation, and therefore the reinforcement, of the actions of the subject-body in the world. All of a sudden, the transcendental degrees are in fact distributed into two classes by a given point that treats the becoming of a truth: the degrees associated with the ‘good’ value and those associated with the bad one.” (*LofW*, p. 416).

<sup>64</sup> *LofW*, p. 371.

<sup>65</sup> *LofW*, p. 372. This is a somewhat different sense of “singularity” from that in *Being and Event*.

defines the various possibilities for the form and structure of a “subject,” which are to be understood in terms of their varying capacities for unfolding the event’s fateful trace. First, as is already familiar from *Being and Event*, there is the *faithful* subject. The faithful subject, retaining fidelity to the event, draws out its consequences “point for point,” inscribing them in the present by means of its creation of a “body” that acts as a kind of “active unconscious” of the event’s unfolding trace.<sup>66</sup> However, given any event, there will also be any number of subjects that respond to it in the mode of *reaction*; although these subjects indeed claim to produce novelty in the wake of the event, the consequences actually produced by such a “reactive subject” are in fact limited to the production of a “measured” present that assimilates the event by way of limiting and appropriating it. Such a present will be “extinguished” in that, although in a certain sense it recognizes the event, it resists the potential “catastrophe” for the existing order that the event represents, and so has saved this order from its radically transformative implications.<sup>67</sup> By contrast with this, a third figure of the subject is the “obscure” subject that simply “abolishes” the event, consigning its “present” to the “night of non-exposition.”<sup>68</sup> The obscure subject practices, in authoritarian fashion, an active “obscurantism” that imposes a fictional and anti-evental unifying order under the figure of a “full Tradition or Law.” This always involves the invocation of a full and unified “ahistorical or anti-evental Body,” for instance “City, God,” or “Race...” which actively suppresses the traces of the event by forbidding the very possibility of its taking place and “reducing to silence” all that affirms it or draws out its consequences.

### **The Many Worlds of Badiou**

On the basis of *Logics of Worlds*’ complex formal and structural analysis of the possibilities of change and transformation, we can now add one more distinction to those summarized in the table of contrasts above (p. ---). For since the theory of *Logics of Worlds* formalizes the possibilities of evental change as inherently situated and always operative within particular, *local* worlds of appearance, it maintains a fundamental commitment to the *multiplicity of worlds* and the *diversity of their structures*. This commitment is intimately connected to Badiou’s *subordination* of logic to mathematics, which allows him, by means of the device of Heyting algebras, to present the specific structures of the various worlds – up to what is linguistically expressible in each -- as each characterized by a specific “logic” grounded in

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<sup>66</sup> *LofW*, p. 53.

<sup>67</sup> *LofW*, p. 55.

<sup>68</sup> *LofW*, p. 59.

the larger category-theoretical (i.e. mathematical) structure. Thus Badiou's theory in *Logics of Worlds* makes all the more explicit the commitment already present in *Being and Event*. This is a fundamental commitment to the *unity* of consistent mathematics and the *plurality* of worlds and logics.

By stark contrast, as we may already suspect and will have the occasion to verify in the following, *for the formal thought underlying paradoxico-criticism, whatever may be the diversity of existing languages and cultures, there is but one world and but one logic*. The "ontological" attitude of paradoxico-criticism is thus not that (as Badiou holds) "ontology is mathematics," but rather that "ontology is logic," and that mathematics is to be *reduced* to logic rather than the other way around.<sup>69</sup> Accordingly, the critical consequences of formalism are here to be traced in the implications of the problematic reflection of the totality of the (one) world into itself by means of the (single) structure of logic, or language as such. This is what Lacan called *lalangue*, and for paradoxico-criticism, its "virtual" or "ideal" status does not at all prevent the consequences of its existence from being structurally and indelibly inscribed in every moment of ordinary reflexive linguistic *praxis*. These consequences may indeed yield paradoxes and contradictions but they do not require or suggest, as Badiou consistently does, the fundamental formal splitting of the world of appearance and effective change into an irreducible plurality.

The irreducible multiplicity of worlds in *Logics of Worlds* again follows *directly* from Badiou's interpretation of the implications of Russell's paradox, which we have already investigated above. In Book II of *Logics of Worlds*, Badiou again considers the inconsistency of the Russell set – what he here calls the "Chimera" – to demand the non-existence of a Whole, or a set of all sets. For, he says:

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<sup>69</sup> For paradoxico-criticism, as we have seen above, the paradoxical results of Russell and especially Gödel therefore do not establish (as they have often been taken to) a specific irreducibility of mathematics to logic. The claim of such an irreducibility is itself the basis for Badiou's employment, in *Logics of Worlds*, of the Heyting device of rooting the (purported) plurality of classical and non-classical "logics" in the more foundational structure of category theory. From the perspective of paradoxico-criticism, this is backwards: the paradoxes of Russell and Carnap indeed establish the presence of limit-paradoxes and contradictions in any logical/ontological thinking of the reflexive One, but they do not at all demand or suggest the irreducibility of mathematics or its purported distinct "objects". Paradoxico-criticism thus remains faithful (even despite the radical implications of the paradoxes, which are here again assimilated as positive critical results) to the original "event" of logicism in its classic (Fregean) formulation, and even to the profoundly motivating intuition that set theory, as an "allegory of predication," is to be understood as an organic extension of the formal logic of concepts and terms. (see note – to chapter 1 above). Nor would it be right to say that the original logicist program is, even today, dead in the context of analytic philosophy; recently, a number of philosophers have attempted a revival of Frege's original project of reducing mathematics to logic by employing techniques in second-order logic. For a useful overview of this contemporary "neo-logicism" or "neo-Fregeanism," see MacBride (2003).



Since the Chimera can be neither reflexive nor non-reflexive, and since this partition admits of no remainder, we must conclude that the Chimera is not. But its being followed necessarily from the being that was ascribed to the Whole. Therefore, the Whole has no being.<sup>70</sup>

With this decision, Badiou unilaterally excludes the possibility, decisive for the paradoxico-critical orientation, that the Russell set instead is to be treated as a set that, *inconsistently* and *paradoxically*, both is and is not a member of itself, and so can serve as a model for the paradoxical being of language, which is, (as, for instance, Agamben says) both included and not included in itself by means of its own problematic self-reflection. But Badiou's decision here also has, almost immediately, the consequence of introducing a formal plurality, to which the whole apparatus of *Logics of Worlds'* discussion of the multiple structures of worlds will be submitted. For:

If there was a being of the Whole, we could undoubtedly separate out any multiple from it by taking into account the properties that singularize it. Moreover, there would be a universal place of multiple-beings, on the basis of which both the existence of what is and the relations between beings would be set out. In particular, predicative separation would uniformly determine multiplicities through their identification and differentiation within the Whole. But, as we have just seen, there is no Whole...

From the fact [*sic*] that there is no Whole it follows that every multiple-being enters into the composition of other multiples, without this plural (the others) ever being able to fold back upon a singular (the Other). For if all multiples were elements of one Other, that would be the Whole. But since the concept of Universe inconsistencies, as vast as the multiple in which a singular multiple is inscribed may be, there exist others, not enveloped by the first, in which this multiple is also inscribed.

In the end, there is no possible uniformity among the derivations of the thinkability of multiples, nor a place of the Other in which they could all be situated. The identifications and relations of multiples are always local.<sup>71</sup>

It is thus on the basis of his interpretation of Russell's paradox that Badiou affirms the irreducible and essential *localization* of all identity and relations of objects. The (stipulated) non-existence of the Whole means that it is not, contrary to first appearances, possible to determine objects wholly by means of their

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<sup>70</sup> *LofW*, p. 110.

<sup>71</sup> *LofW*, p. 112.

properties and predicates (this was, recall, the assumption of Frege’s original “universal” comprehension principle) but that any such predicative determination is always inherently *relative* and hence can be considered to be possible only within the specifically determined structure of a particular world. Thus, it follows as a direct consequence that *languages are themselves plural, and claims and expressions are relative to them*: the possibility and meaning of any positive assertion whatsoever is only to be understood in terms of its value within a particular, structurally determined situation. Indeed, this relativity of linguistic predication is the very basis for Badiou’s first definition of a “world” itself. According to this definition, “a multiple, related to a localization of its identity and of its relations with other multiples, is a being...” and “a local site of the identification of beings” is what we may term, “in what is still a rather vague sense, a world.”<sup>72</sup> And just as every being thus has its identity, and hence its “existence,” only relative to the world and its transcendental structure, there is no such thing as an *absolute* nonexistent. That is, what appears “as” an empty set or void within any particular world may very well be a positive object in another. The (stipulated) non-existence of the Whole thus establishes, according to Badiou, that *within* any particular world its beings may be thought as compositions out of the element that is void *for that world*. This means that we may indeed measure their degree of complexity or “rank” by means of an identification of this (localized) void.

Badiou’s interpretation of the Russell paradox and his choice to preserve consistency while sacrificing totality (or wholeness) thus underlies all of the following: the definition of a world, the relativity of languages and logics, the subordination of logic to mathematics, and (insofar as there is thus accordingly no possible way of thinking or speaking of any object as it actually *is*, outside its context in a particular world of appearance) the division between being and appearance itself. But insofar as all of this is justified by the interpretation, it follows from a demonstration that proceeds by means of the very terms (“the Russell set;” “the Chimera;” “the Whole”) that it will itself purportedly show to be incapable of referential meaning, and thus yields an enjoinder of Being (“there is no Whole”; “the concept of Universe ... inconsistencies”) that undermines itself in its very statement. And all of this is, again, starkly divergent from the hypothesis of a totality of signification whose concept is not ruined by the paradoxes to which it leads, or (in Lacanian terms) of a big Other whose inconsistency is no bar to the systematic and structural efficacy of the consequences of its inscription.

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<sup>72</sup> *LofW*, pp. 112-113.