Uncertainty Quantification in Flight Plans Using Evidence Theory: Departure and Arrival Times

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The paper presents an approach based on Dempster-Shafer theory of evidence to represent uncertainty in flight plan departure and arrival times. It is shown that the accuracy and credibility of flights plans for a given flight can be completely represented by belief functions. A procedure on how to construct belief functions using the data of actual and scheduled (estimated) departure and arrival times is described. Application of the approach is demonstrated using actual flight data for two flights: Delta DAL1212 from Los Angeles to Atlanta and Airtran Airways TRS841 from Atlanta to Orlando, chosen to represent a cross-country flight and a southeastern flight through Florida from a busy hub airport, Atlanta Harsfield-Jackson Intl.

Nomenclature

X	=	quantity of interest
$\mathbf{X}$	=	finite set of possible values of X (universal set)
A	=	subset of $\mathbf{X}$
$\bar{A}$	=	compliment of A
m	=	basic probability number, measure of belief
Bel	=	degree of belief
ΔT	=	time deviation
Δ	=	subinterval size (in minutes)
Δ_{total}	=	total range of deviation values
μ, σ	=	mean and standard deviation

I. Introduction

THE next generation air transportation system requires an increase in air traffic management effectiveness, flexibility, and efficiency, while maintaining safety. To achieve these goals, advanced mathematical and computational tools must be developed for quantifying and reducing uncertainty in planned aircraft trajectories. This includes uncertainty in the aircraft position at a given time as well as uncertainty in departure and arrival times.

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These uncertainties originate from uncertainties in the input parameters, model, computational procedure, and atmospheric conditions as well as human behavior in a dynamic situation. It is difficult, if possible at all, to separate and describe the impact of all individual sources of uncertainty. The objective of our research is to develop a rigorous methodology that describes the combined contribution from all uncertainty sources, or the total uncertainty in the aircraft trajectory. The conventional probabilistic metrics, such as the root-sum-square and the sum of the absolute values[1] based on the linear error model, are not suitable for the problem because their use is limited by stochastic uncertainties. Also, they describe the contribution of only recognized uncertainty sources and ignore the interaction of different uncertainty sources. Therefore, these metrics do not accurately reflect the total uncertainty in trajectory predictions.

Measures of the Dempster-Shafer theory of evidence[2] reflect simulation uncertainty more realistically than simple probabilistic metrics because they describe both types of uncertainty: due to stochastic influence and due to lack of knowledge. In fact, probability theory may be viewed as a branch of evidence theory. Moreover, evidence theory does not require separation of uncertainty sources and can be directly used to quantify the total uncertainty of predictions. Evidence theory works with limited information and new data can be incorporated as it becomes available. These features make evidence theory perspective in its application to aircraft trajectory predictions. In our previous studies [3-5] we have developed and validated algorithms based on evidence theory to quantify the total uncertainty in turbulent flow simulations and in tropical cyclone track forecasts. Current research seeks to extend the approach to quantify uncertainty in aircraft trajectories issued in flight plans. Aircraft trajectory uncertainty is the uncertainty of aircraft position at a given time and the uncertainty of aircraft arrival time at a fixed trajectory point. As the first step, the present paper reports on quantifying uncertainty in the flight plan departure and arrival times.

II. Mathematical Background

In evidence theory, the impact of evidence on our belief (confidence) in a given proposition is described by a few related functions. Let X denote a quantity and **X** the universal set, i.e., the finite set of its possible values. Propositions can be of the form “the true value of X is in A”, where A is a subset of **X**. Whenever A is interpreted as a proposition, its complement $\overline{A}$ (the set of all elements of **X** not in A) must be interpreted as the proposition's negation. The set of all subsets of **X**, the power set, includes the empty set $\varnothing$ (corresponding to a necessarily false proposition, since the true value cannot lie in $\varnothing$) and the entire set **X** (corresponding to a necessarily true proposition, since the true value is assumed to be in **X**). The basic probability assignment function (or simply *m*-function), assigns a number $m(A)$ to each subset A of **X** such that $m(\varnothing)=0$ for the empty set $\varnothing$, and the sum of basic probability assignments for all specified subsets A of **X** is equal to unity

$$\sum_{A\subseteq \mathbf{X}} m(A)=1\,.$$

The quantity $m(A)$ is a measure of the belief that is committed to A only, but not to any particular subset of A. The belief in A is based on available evidence that supports exactly A. Evidence may be insufficient to commit the belief to a specific subset, but to the entire set **X**.

As $m(A)$ is a measure of belief committed exactly to A, it does not represent the total belief committed to A. A measure of the total belief (degree of belief) in A is defined in evidence theory as

$$Bel(A)=\sum_{B\subset A} m(B)\,,$$

reflecting the fact that the evidential support committed to one proposition is committed to any subset containing it. Additional properties of *Bel*-functions are:

$$Bel(A)+Bel(\overline{A})\le 1\,,$$
$$\text{if } B\subseteq A\text{, then } Bel(B)\le Bel(A)\,.$$

A subset A of **X** is called a focal element of a belief function *Bel* over **X** if $m(A)>0$. The union of all focal elements of a belief function is called its core. If all focal elements are disjoint, $m(A)=Bel(A)$ for all specified subsets A. Although there are other useful functions[1] to describe the impact of evidence on our belief in a given proposition, the current study utilizes only these two.

Notice that the way one defines propositions of interest (subsets A of **X**) and links actual evidence to their basic probability assignments $m(A)$ depends on the problem being considered, one's current knowledge, and available evidence. Additional information can change the set of propositions and how evidence affects our degree of belief in these propositions.

A. Application to Flight Departure/Arrival Times

In application to the flight plan departure and arrival times, the quantities of interest are the Actual Departure Time (ADT) from and the Actual Time of Arrival (ATA) to an airport. For ADT, the proposition of interest is that "the deviation of ADT from the Scheduled Departure Time (SDT) is inside an interval A." For ATA, two pre-planned times are available: the Scheduled Time of Arrival (STA) and the Estimated Time of Arrival (ETA). The former is issued along with SDT in the initial flight plan. The later is issued at the actual departure time. Therefore, there are two relevant propositions of interest: "ATA deviates from STA inside an interval B" and "ATA deviates from ETA inside an interval C."

In mathematical terms, the simplest metric that characterizes how close pre-planned and actual times are is the deviation defined as

$$\varDelta T_D = \mathrm{ADT} - \mathrm{SDT}\,,\ \varDelta T_A = \mathrm{ATA} - \mathrm{STA}\ \text{, and}\ \varDelta T_{EA} = \mathrm{ATA} - \mathrm{ETA}$$

for the two times of interest. As flights occur on a regular basis, there is an extensive database of the three $\varDelta T$ s that can be used to describe the uncertainty in past flight plans corresponding to a given flight.

Describing uncertainty means constructing belief functions for quantities of interest. In regard to departure and arrival times, three belief functions (one for ADT and two for ATA) should be constructed. To do so, one should i) specify intervals over which the belief functions are to be determined and ii) assign the degree of belief to each interval. These belief functions can then be compared with the ideal belief function shown in Fig. 1. The ideal belief function assigns the maximum degree of belief $Bel(A) = 1$ to a single interval including the zero deviation between pre-planned and actual flight time. The size $\varDelta_{ideal}$ of this interval is the minimum desirable, for example, one minute. The ideal belief function corresponds to the proposition that "the actual time deviates from the pre-planned time within one minute".

As an example, let us consider the procedure for constructing a belief function for ADT (belief functions for ATA are built in a similar manner). Since the number of past flights and, therefore, of available $\varDelta T_D$ -data is finite, the range of deviation values ($\varDelta_{Dtotal} = max\,\varDelta T_D - min\,\varDelta T_D$) that includes all $\varDelta T_D$ values is finite as well. Therefore, it is always possible to specify at least a single finite interval A – the range of deviation values – as the focal element and to assign the degree of belief $Bel(A) = m(A) = 1$ to this interval based on available evidence ($\varDelta T_D$ values). Based on this observation, one can say that all available evidence supports the proposition that ADT is likely to deviate from SDT inside this interval. Obviously, our proposition that evidence supports this specific interval is subjective and corresponds to the available database. More data could possibly increase the size of this interval. Nevertheless, the proposition reflects our current level of knowledge.

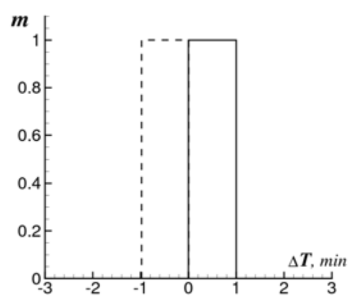


Figure 1. Ideal belief function.

A belief function with a single focal element that is the range of all deviation values is not very informative, because the size of such an interval is most likely too large. One would like to find out how uncertainty sources that cause flight plans deviate from reality favor smaller subintervals within the range of all deviation values.

Evidence theory does not impose any limitation on how subsets (in our problem, subintervals) should be specified. They can intersect, nest, or be disjoint. They also can be of different size. We recommend, however, that for practical purposes, the universal set $\mathbf{\Delta T_D}$ been divided into disjoint ordered subintervals of uniform size $\varDelta_D$. An example is shown in Fig. 2 for $\varDelta_D = 2\ min$. One of the subintervals should always contain the zero $\varDelta T_D$ value as its endpoint.

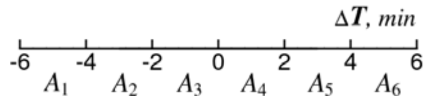


Figure 2. Example of subintervals in the ΔT_D -universal set.

The subinterval size Δ_D and the number of subintervals are determined by the following factors: each subinterval should contain at least one ΔT_D value and an *m*-function constructed over the subintervals should be of concave type. Between Δ_{ideal} and Δ_{Dtotal}, several values of Δ_D may produce a concave *m*-function. The smallest of them should be chosen. As previous studies[3-5] show, Δ is typically larger than Δ_{ideal} and almost always less than Δ_{total} even though there is no guarantee for the existence of a concave *m*-function with $\Delta < \Delta_{total}$.

The *m*-value for each subinterval is defined as the ratio of the number *n* of deviation values falling inside the subinterval to the total number *N* of deviation values in the database: $m\,(\Delta_{Dl}) = n_l / N$, where l is the index over the subintervals Δ_{Dl}. Since, each ΔT_D value unambiguously supports one of the subintervals and subintervals are disjoint, there is no difference between *m*- and belief functions, that is, $Bel(\Delta_{Dl}) = m(\Delta_{Dl})$ for any Δ_{Dl}.

An *m*-function constructed in such a manner completely describes the accuracy and credibility of flight plans in regard to departure and arrival times. The *m*-value assigned to a subinterval reflects the portion of our belief that ADT deviates from SDT in this subinterval and thus, represents our confidence (credibility) in the flight plans for a given flight. As previous studies[3-5] show, the maximum degree of belief assigned to a subinterval is always less than one, except when $\Delta = \Delta_{total}$.

The accuracy is represented by three characteristics of an *m*-function: Δ_D, Δ_{Dtotal}, and the distance between the point $\Delta T_D = 0$ and the endpoint of the subinterval with the maximum degree of belief closest to this point. One can compare these three characteristics with the ideal ones: $\Delta_{ideal} = \Delta_{total} = Min$, where *Min* is the essential minimum desirable for the problem. In the ideal *m*-function, the point $\Delta T = 0$ and an endpoint of the only interval *A* with $m(A) = 1$ coincide. In this study, we assume $\Delta_{ideal} = 1\ min$.

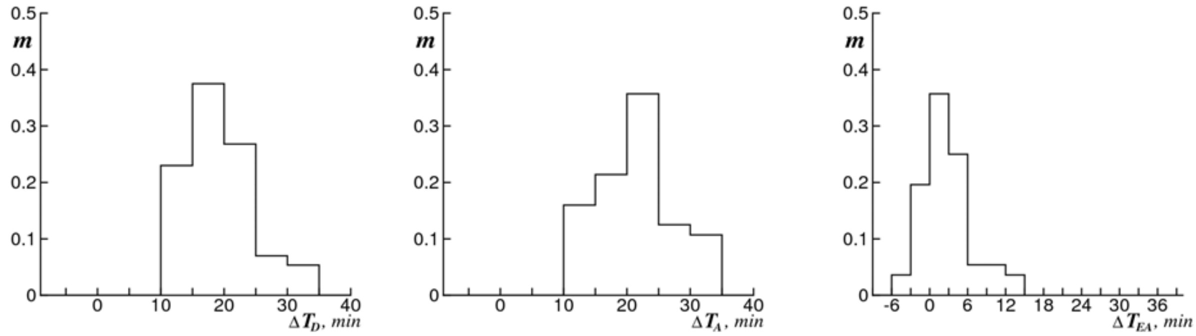
It is worth emphasizing that an *m*-function (or belief function) is not a probability density function. An *m*-function is constructed using the available database and by choosing an appropriate subinterval size. Deviation values support subintervals and not single values. The number of deviation values is finite; additional data can possibly change the subinterval size, the number of subintervals, and the *m*-function itself. The *m*-function represents the total uncertainty (objective and subjective) in flight plans. With more knowledge and data gained, the subjective component of the total uncertainty will be reduced, although never diminish completely.

III. Results

The methodology was applied to analyze actual flight data. Two flights between two different city pairs with mostly consistent filed routes and aircraft types over several months time, from 01 Sept 2008 to 17 Jan 2009, were identified using the FAA's Post Operations Evaluations Tool (POET)[6]. The city pairs for the flights were chosen to represent a cross-country flight and a southeastern flight through Florida to a busy hub airport, Atlanta Harsfield-Jackson Intl (KATL). The two flights chosen are Delta DAL1212 from Los Angeles (KLAX) to Atlanta (KATL) and Airtran Airways TRS841 from Atlanta (KATL) to Orlando (KMCO).

The flight plan and actual flight information was derived from the FAA's Airspace Situation Display to Industry (ASDI) data feed, which is a subsystem of the Enhanced Traffic Management System (ETMS). The ASDI feed provides flight plan and track information from the National Airspace System (NAS) to airlines and other organizations in real-time or near real-time depending on the need.

Flight Delta DAL1212 was scheduled to depart on different days at different times – 13:10, 14:00, and 14:25 – during the analyzed period. Since these scheduled departure times seem to be close, our initial intent was to combine data from the three flights into a single database, as it would result in an increase in the number of samples. However, we found that uncertainty in the actual departure and arrival times is sensitive to the flight departure time. Therefore, the data for the three departure times are analyzed separately. The total number of Delta DAL1212 flights analyzed is 130. Among them, 56 flights were scheduled to depart at 13:10, 44 flights at 14:00, and 30 at 14:25.



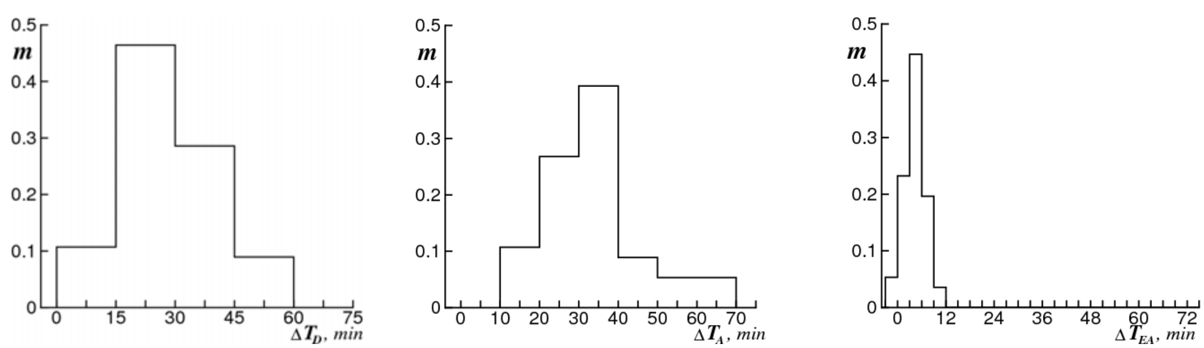
a) b) c)

Figure 3. *m*-functions for the DAL1212 flight with SDT = 13:10: a) uncertainty in predicting the departure time, b) uncertainty in predicting the arrival time as in the flight plan, c) uncertainty in predicting the arrival time at the actual departure time.

The scheduled departure time for the Airtran Airways TRS841 flight was 12:50 and 13:45 on different days during the analyzed period. Again, we found that uncertainty in the actual departure and arrival times is sensitive to the flight departure time, and therefore, the data for the two departure times are analyzed separately. The total number of Airtran Airways TRS841 flights analyzed is 123. Among them, 56 flights were scheduled to depart at 12:50 and 67 flights at 13:45.

Figure 3 shows three *m*-functions: a) *m*-function representing uncertainty in the flight plan departure time (ADT-SDT), b) *m*-function representing uncertainty in the flight plan arrival time as in the initial flight plan (ATA-STA), and c) *m*-function representing uncertainty in the estimated arrival time at the actual departure time (ATA-ETA) for the DAL1212 flight with SDT 13:10. Comparing these three *m*-functions, one can see that the most accurate are the predictions of the arrival time made at the actual departure time (Fig. 3c). The subinterval with the most belief committed to it has the zero ΔT_{EA} value as its endpoint. Also, Δ_{EA} of this *m*-function is the smallest: 3 *min* ($\Delta_D = \Delta_A = 5\,min$). The range of deviation values is also smaller for the *m*-function shown in Fig. 3c: $\Delta_{EAtotal} = 21\,min < \Delta_{Dtotal} = \Delta_{Atotal} = 25\,min$. Overall, the *m*-function in Fig. 3c is closer to the ideal *m*-function than the other two functions in Figs. 3a and 3b. Similar observations are made for the Delta flights with SDT 14:00 and 14:25.

To compare, Fig. 4 shows the same *m*-functions as in Fig. 3 for the TRS841 flight at SDT 12:50. Similar tendencies are observed for this flight. That is, the most accurate are the predictions of EAT (Fig. 4c). Δ_{EA} of this *m*-function is the smallest: 3 *min* ($\Delta_D = 15\,min$, $\Delta_A = 10\,min$). The range of deviation values is also smaller for the *m*-function shown in Fig. 4c: $\Delta_{EAtotal} = 15\,min < \Delta_{Dtotal} = \Delta_{Atotal} = 60\,min$. Overall, the *m*-function in Fig. 4c is closer to the ideal *m*-function than the other two functions in Figs. 4a and 4b. Similar observations are made for flights with SDT 13:45.



a) b) c)

Figure 4. *m*-functions for the TRS841 flight with SDT = 12:50: a) uncertainty in predicting the departure time, b) uncertainty in predicting the arrival time as in the flight plan, c) uncertainty in predicting the arrival time at the actual departure time.

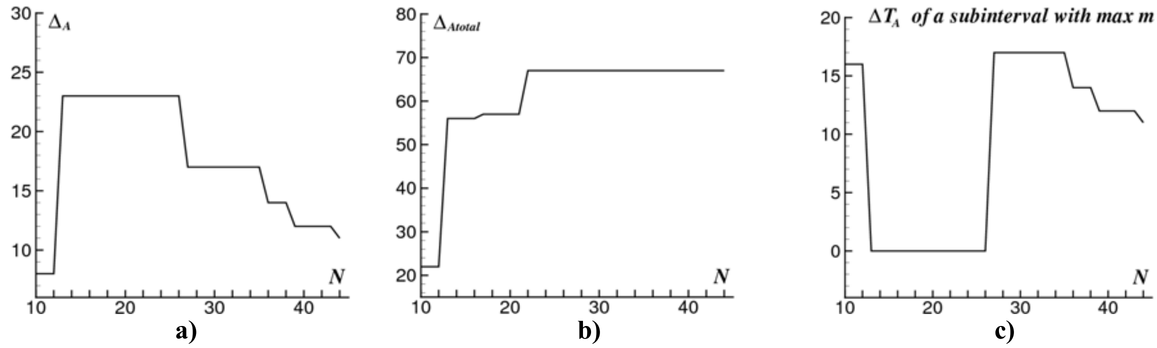


Figure 5. Variation of the *m*-function characteristics with the sample size *N*; *m*-function represents uncertainty in STA at the scheduled DAL1212 flight departure time 14:00: a) the subinterval size Δ_A, b) the total range Δ_{Atotal} of the deviation values ΔT_A, and c) the distance from the subinterval with the maximum degree of belief to the point $\Delta T_A = 0$.

It is interesting to compare the characteristics of *m*-functions in Figs. 3 and 4 for both flights as these flights have closely scheduled departure times (13:10 and 12:50) and the same number of flights (56). Despite the fact that the TRS841 flight is shorter, its actual departure and arrival times are more likely to deviate from the scheduled times with more uncertainty regarding delays. As Atlanta and Orlando are both close to the Atlantic coast, one would expect weather to be a strong factor in these delays. Once in the air, the flight DAL1212 is more likely to arrive on time as the degree of belief assigned to the interval from 0 to 3 *min* is higher for this flight than for the TRS841 flight: 0.36 vs. 0.23. Moreover, this is the interval with the assigned maximum degree of belief for the DAL1212 flight. The estimated arrival time, however, is more predictable for the TRS841 flight than for the DAL1212 flight. Indeed, the total range of deviation values for EAT is smaller for the TRS841 flight (15 *min* vs. 21 *min* for the DAL1212 flight), and more belief is assigned to the interval with the maximum degree of belief (0.45 vs. 0.36 for the DAL1212 flight). In this case, travel distance is most likely the factor.

We also analyzed how the sample size influences the accuracy of the estimation of the *m*-function characteristics. Figure 5 shows, as an example, how the characteristics of the *m*-function representing uncertainty in STA change with the number of data *N* for the flight departure time scheduled at 14.00. Based on the available data for all flights, we can infer that a sample size *N* about or larger than 50 is preferable to achieve convergence of the *m*-function characteristics to constant values.

For comparison, the evolution of the standard statistical characteristics such as the mean and the standard deviation with *N* was also analyzed. As an example, their evolution is shown in Fig. 6 for STA of the flights scheduled to depart at 14:00, that is, for the same case as in Fig. 5. Figure 6c compares the *m*-function representing

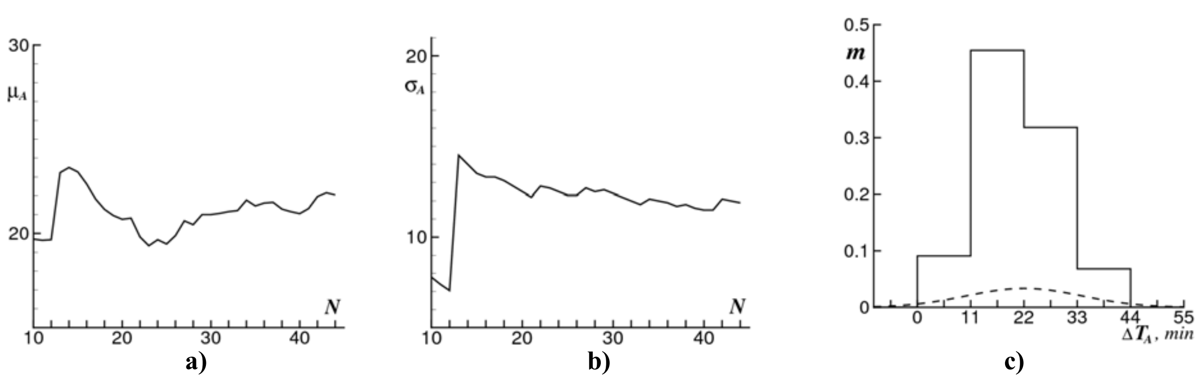


Figure 6. Variation of a) the mean μ_A and b) the standard deviation σ_A of STA in the DAL1212 flight plans at the scheduled flight departure time 14:00; c) comparison of the *m*-function representing uncertainty in STA at the scheduled DAL1212 flight departure time 14:00 and the normal distribution with μ_A and σ_A at *N* = 44.

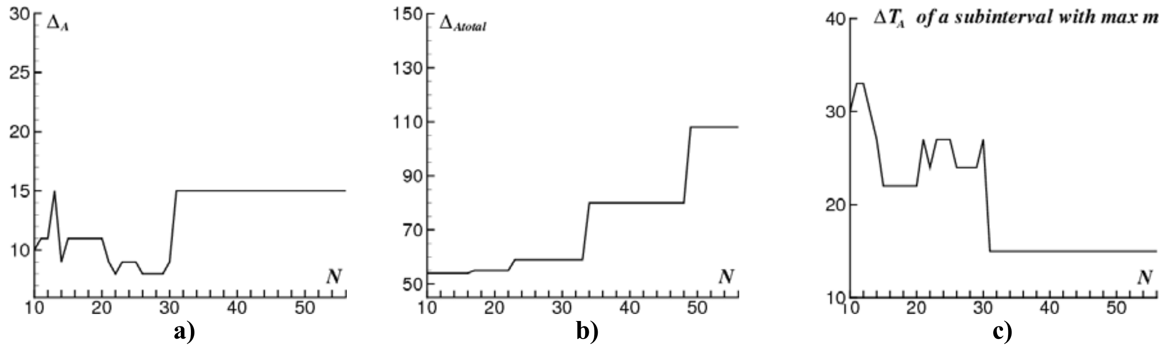


Figure 7. Variation of the *m*-function characteristics with the sample size *N*; *m*-function represents uncertainty in SDT at the scheduled TRS841 flight departure time 12:50: a) the subinterval size Δ_A, b) the total range Δ_{Atotal} of the deviation values ΔT_A, and c) the distance from the subinterval with the maximum degree of belief to the point $\Delta T_A = 0$.

uncertainty in STA at the scheduled flight departure time 14:00 and the normal distribution built with $\mu_A = 22$ and $\sigma_A = 11.9$ at *N* = 44. The normal distribution assigns higher probability of ATA delays in the interval between 22 and 33 minutes: $P(11 \le \Delta T_A < 22) = 0.3158 < P(22 \le \Delta T_A < 33) = 0.3285$ that is in contradiction with what occurred in reality. On contrast, *m*-functions do not rely on any assumption and simply reflect reality.

The results of a similar analysis for the TRS841 flight are presented in Figs. 7 and 8. Figure 7 shows how the characteristics of the *m*-function representing uncertainty in SDT change with the number of data *N* for the flight departure time scheduled at 12:50. In this case again we can infer that a sample size *N* about or larger than 50 is preferable for achieving convergence of the *m*-function characteristics to constant values.

The evolution of the mean and the standard deviation with *N* for the same flight as in Fig. 7 is shown in Fig. 8. In Fig. 8c, the *m*-function representing uncertainty in SDT at the scheduled flight departure time 12:50 is compared with the normal distribution ($\mu_A = 29.3$ and $\sigma_A = 16.4$ at N = 56). The normal distribution assigns similar probabilities of SDT delays in the intervals between 15 and 30 minutes and between 30 and 45 minutes ($P(15 \le \Delta T_A < 30) = 0.3246 > P(30 \le \Delta T_A < 45) = 0.315$). The *m*-function, however, assigns ~1.5 times more belief for delays in the interval from 15 to 30 minutes based on real-life data.

IV. Conclusion

The paper demonstrates the applicability of tools of the Dempster-Shafer theory of evidence, such as basic probability assignment and belief functions, to quantify the total uncertainty in flight plan departure and arrival times. For a given flight, three belief functions should be constructed (one for the departure time and two for the

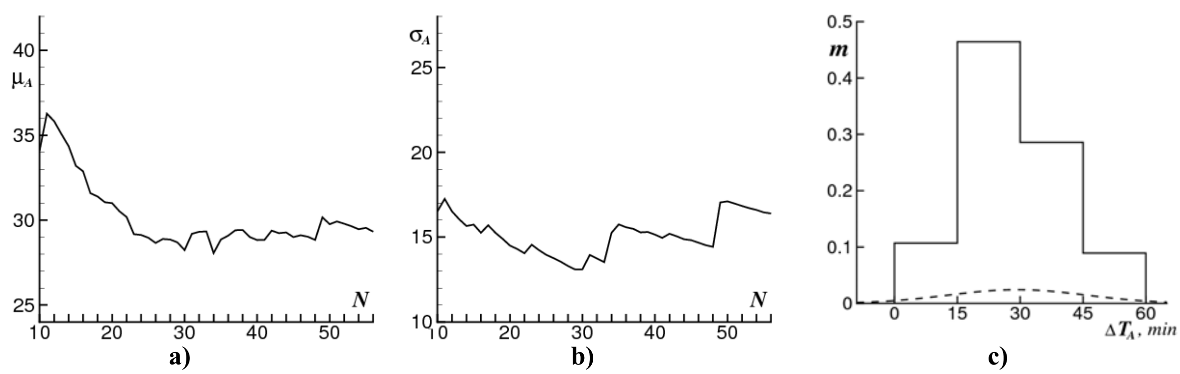


Figure 8. Variation of a) the mean μ_A and b) the standard deviation σ_A of SDT in the TRS841 flight plans at the scheduled flight departure time 12:50; c) comparison of the *m*-function representing uncertainty in SDT at the scheduled TRS841 flight departure time 12:50 and the normal distribution with μ_A and σ_A at *N* = 56.

arrival time). A procedure to construct belief functions is described. It is shown that belief functions constructed in the proposed manner completely describe the accuracy and credibility of flight plans with regard to departure and arrival times.

A computational algorithm to implement the procedure was developed. Application of the approach is demonstrated using data from two actual flights that occurred between 01 Sept 2008 and 17 Jan 2009. The two chosen flights represent a cross-country flight (Delta DAL1212 from Los Angeles to Atlanta) and a southeastern flight through Florida from a busy hub airport (Airtran Airways TRS841 from Atlanta to Orlando). Analysis was conducted on how the sample size influences the accuracy of the estimation of these characteristics. The conclusion is that at least 50 data points are required for most of the *m*-functions constructed so that the majority of characteristics would converge to approximately constant values.

A similar procedure can be used to construct the *m*-functions representing uncertainty in aircraft position (latitude, longitude, and altitude) and arrival time at any fixed trajectory point. Trajectory predictions accompanied with the uncertainty interval variable in spatial and time directions at different trajectory segments and waypoints and with the quantified belief in such predictions is a highly desirable tool for improving conflict resolution and decision-making and, as such, for increasing Air Traffic Management effectiveness and National Airspace System safety. To develop such a tool is one of our future goals.

Acknowledgments

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