Uncertainty Quantification in the Horizontal Projection of Flight Plan Trajectories Using Evidence Theory

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The paper presents an approach based on Dempster-Shafer theory of evidence to quantify uncertainty in flight plan trajectories. The approach is developed for 4D aircraft trajectories. However, as the full data set (latitude, longitude, altitude vs. time) for flight plan trajectories is not publicly available, only uncertainty in the horizontal projection (longitude vs. latitude) of the trajectories is considered in the present paper. Two flight phases – climb and descent – are chosen to demonstrate the proposed approach. The actual dataset for two flights -- Delta DAL1212 from Los Angeles to Atlanta and Airtran Airways TRS841 from Atlanta to Orlando -- are chosen to represent a cross-country flight and a southeastern flight through Florida from the busy airport hub, Atlanta Hartsfield-Jackson International, is used.

Nomenclature

X	=	quantity of interest
$\mathbf{X}$	=	finite set of possible values of X (universal set)
A	=	subset of $\mathbf{X}$
$\overline{A}$	=	compliment of A
m	=	basic probability number, measure of belief
Bel	=	degree of belief
ΔT	=	time deviation
Δ	=	subinterval size (in minutes)
Δ_{total}	=	total range of deviation values
μ,σ	=	mean and standard deviation

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I. Introduction

THE next generation air transportation system (NGATS) requires an increase in air traffic management effectiveness, flexibility, and efficiency, while maintaining safety. The NGATS Vision briefing[1] identified Aircraft Trajectory-Based Operations as the main mechanism for managing traffic in high density or high-complexity airspace. Such operations will require the knowledge of each aircraft's expected four-dimensional trajectory[2] (4DT). A 4DT is an exact flight profile in space and time that includes the "centerline" of a path, the position uncertainty, time information and uncertainty associated with such information. Advanced mathematical and computational tools must be developed for representing uncertainty in 4DTs. This includes uncertainty in the aircraft position at a given time, uncertainty in departure and arrival times, and uncertainty in the arrival time to specific waypoints.

Uncertainties in 4DTs originate from uncertainties in input parameters, model, computational procedure, and atmospheric conditions as well as human behavior in a dynamic situation. It is difficult, if possible at all, to separate and describe the impact of all individual sources of uncertainty. The objective of our research is to develop a rigorous methodology that describes the combined contribution from all uncertainty sources, or the total uncertainty in the aircraft trajectory. The conventional probabilistic metrics, such as the root-sum-square and the sum of the absolute values[3] based on the linear error model, are not suitable for the problem because their use is limited by stochastic uncertainties. Also, they describe the contribution of only recognized uncertainty sources and ignore the interaction of different uncertainty sources. Therefore, these metrics do not accurately reflect the total uncertainty in trajectory predictions.

Measures of the Dempster-Shafer theory of evidence[4] reflect simulation uncertainty more realistically than simple probabilistic metrics because they describe both types of uncertainty: due to stochastic influence and due to the lack of knowledge. In fact, probability theory is a branch of evidence theory. Moreover, evidence theory does not require separation of uncertainty sources and can be directly used to quantify the total uncertainty of predictions. Evidence theory works with limited information and new data can be incorporated as it becomes available. These features make evidence theory perspective in its application to aircraft trajectory predictions. In Poroseva et al.[5], we showed the applicability of tools of the Dempster-Shafer theory of evidence, such as basic probability assignment and belief functions, to quantify the total uncertainty in the flight plan departure and arrival times. The same procedure is applicable to quantify uncertainty in the arrival time to specific waypoints of a 4DT. The current paper seeks to extend the approach to quantify uncertainty in a spatial position of an aircraft. With a full data set (longitude, latitude, altitude vs. time) being available, uncertainty in an expected aircraft position (longitude, latitude, and altitude) at a given time can be quantified with our approach. Unfortunately, such data are not publicly available. Therefore, the paper considers the application of the approach to uncertainty quantification in a given flight phase of the 4DT horizontal projection (longitude and latitude). Here, climb and descent flight phases are analyzed. Available data can also be used to quantify uncertainty in a spatial position of specific flight plan waypoints.

II. Mathematical Background

In evidence theory, the impact of evidence on our belief (confidence) in a given proposition is described by few related functions. Let X denote a quantity and $\mathbf{X}$ the universal set, i.e., the finite set of its possible values. Propositions can be of the form "the true value of X is in A", where A is a subset of $\mathbf{X}$. Whenever A is interpreted as a proposition, its complement $\overline{A}$ (the set of all elements of $\mathbf{X}$ not in A) must be interpreted as the proposition's negation. The set of all subsets of $\mathbf{X}$, the power set, includes the empty set $\varnothing$ (corresponding to a necessarily false proposition, since the true value cannot lie in $\varnothing$) and the entire set $\mathbf{X}$ (corresponding to a necessarily true proposition, since the true value is assumed to be in $\mathbf{X}$). The basic probability assignment function (or simply *m*-function), assigns a number $m(A)$ to each subset A of $\mathbf{X}$ such that $m(\varnothing)=0$ for the empty set $\varnothing$, and the sum of basic probability assignments for all specified subsets A of $\mathbf{X}$ is equal to unity

$$\sum_{A \subseteq \mathbf{X}} m(A) = 1 .$$

The quantity $m(A)$ is a measure of the belief that is committed to A only, but not to any particular subset of A. The belief in A is based on available evidence that supports exactly A. Evidence may be insufficient to commit the belief to a specific subset, but to the entire set $\mathbf{X}$.

As $m(A)$ is a measure of belief committed exactly to A, it does not represent the total belief committed to A. A measure of the total belief (degree of belief) in A is defined in evidence theory as

$$Bel(A) = \sum_{B \subset A} m(B),$$

reflecting the fact that the evidential support committed to one proposition is committed to any subset containing it. Additional properties of *Bel*-functions are:

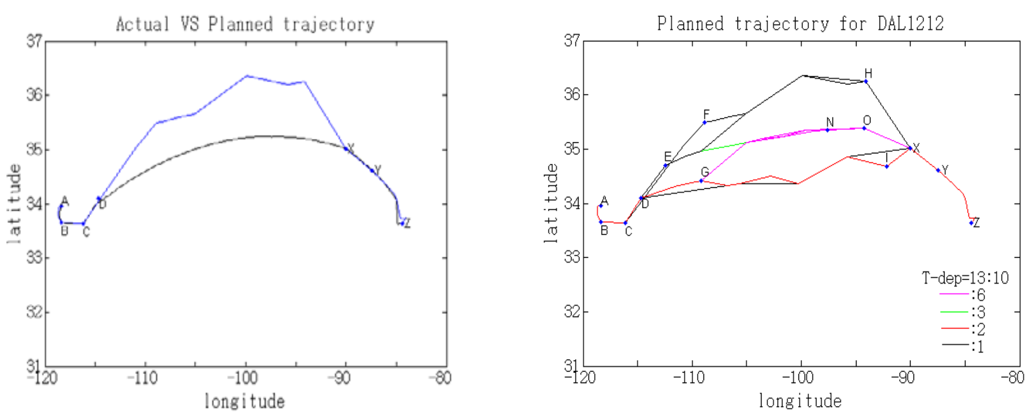
$$Bel(A) + Bel(\overline{A}) \leq 1,$$
$$\text{if } B \subseteq A, \text{ then } Bel(B) \leq Bel(A).$$

A subset A of **X** is called a focal element of a belief function *Bel* over **X** if $m(A) > 0$. The union of all focal elements of a belief function is called its core. If all focal elements are disjoint, $m(A) = Bel(A)$ for all specified subsets A. Although there are other useful functions[1] to describe the impact of evidence on our belief in a given proposition, the current study utilizes only these two.

Notice that the way one defines propositions of interest (subsets A of **X**) and links actual evidence to their basic probability assignments $m(A)$ depends on the problem being considered, one's current knowledge, and available evidence. Additional information can change the set of propositions and how evidence affects our degree of belief in these propositions.

A. Application to Flight Plan Horizontal Projections

A 4DT is defined by a few waypoints, each with specific latitude and longitude. As an example, Figure 1a shows some waypoints marked by letters A, B, C, D, X, Y, and Z for a DAL1212 flight. Even though a new 4DT is issued for each flight each day (Fig. 1b), some waypoints are fixed and can be used to determine flight phases in a given flight with the specified time of departure. In Figure 1, for example, waypoints A and D correspond to the climb phase and waypoints X and Z to the descent phase.



a) b)

Figure 1. Trajectories for the DAL1212 flight with the scheduled departure time at 13:00: a) actual (black) vs. planned (blue), b) planned 4DTs issued from 09/1/2008 to 09/15/2008. Different colors reflect the different number of flights for which the 4DT was issued.

Once a flight phase in the 4DT horizontal projection is identified, the proposition of interest becomes "the shortest distance from an actual trajectory to the 4DT trajectory in a given flight phase deviates inside an interval Δ." Thus, the quantity of interest is the shortest distance S between actual and expected trajectories in a given flight phase.

The shortest distance between actual and planned trajectories is determined for each available actual aircraft position as the distance from the actual aircraft position to a 4DT segment specified by two waypoints closest to the actual aircraft position. In an ideal case, an aircraft follows exactly its expected 4DT, and the shortest distance Δ_{ideal} is equal to zero or is within a certain tolerance determined by the accuracy of trajectory predictions and/or safety requirements. In reality, flight trajectories always deviate from their 4DTs (Fig. 1a), and the shortest distance deviates from its ideal value Δ_{ideal}.

In our algorithm, calculation of the shortest distance is conducted in a standard way[6] with the assumption of the spherical surface of the Earth. For example, the shortest distance between a point ***P*** and an arc formed by a great circle between points A and B (arc AB) is the length of the great circle arc between points P and D (Fig. 2), where the arc PD forms a right angle with the arc AB. Coordinates (latitude and longitude) are known for points A, B, and P, and therefore, the length of arcs AP, AB, and PB can be determined from the following expressions:

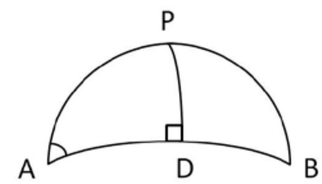


Figure 2. The shortest distance between a point P and an arc AB (arc PD)

$$\cos(\widehat{AB}) = \cos(|90 - Lat_A|)\cdot\cos(|90 - Lat_B|) + \sin(|90 - Lat_A|)\cdot\sin(|90 - Lat_B|)\cdot\cos(|Lon_A - Lon_B|)$$

$$\cos(\widehat{BP}) = \cos(|90 - Lat_B|)\cdot\cos(|90 - Lat_P|) + \sin(|90 - Lat_B|)\cdot\sin(|90 - Lat_P|)\cdot\cos(|Lon_B - Lon_P|)$$

$$\cos(\widehat{AP}) = \cos(|90 - Lat_A|)\cdot\cos(|90 - Lat_P|) + \sin(|90 - Lat_A|)\cdot\sin(|90 - Lat_P|)\cdot\cos(|Lon_A - Lon_P|)$$

where Lat and Lon are the latitude and longitude of a corresponding point (A, B, or P). Knowing the length of each arc, one can determine all three angles in a spherical triangle formed by the three arcs. For example, if we denote as $\widehat{A}$ the angle opposite to the arc BP, its cosine is determined as

$$\cos(\widehat{A}) = \frac{\cos(\widehat{BP}) - \cos(\widehat{AB})\cdot\cos(\widehat{AP})}{\sin(\widehat{AB})\cdot\sin(\widehat{AP})}.$$

Now, in a spherical triangle formed by points A, P, and D, the length of the arc AP and two adjacent angles $\widehat{A}$ and $\widehat{P} = \pi/2 - \widehat{A}$ and are known. Therefore, the length of the arc PD can also be found:

$$\widehat{PD} = \sin^{-1}(\sin(\widehat{A}) * \sin(\widehat{AP})).$$

The assumption of the spherical surface of the Earth is sufficient to demonstrate the approach. More realistic formula can be used in practical applications.

As a rule, the actual flight data contains many radar track points that determine an aircraft position between any two given 4DT waypoints. Therefore, multiple data for the shortest distance are available for a given flight phase during a single flight. The set of the shortest distances from all available actual aircraft positions in a given flight phase to the 4DT segment corresponding to the same flight phase constitutes an extensive database that can be used to describe uncertainty in the past flight plans corresponding to a given flight. The data for several flights with a flight segment in their 4DTs specified by the same waypoints can be combined together to further improve the statistical data representation.

To quantify uncertainty requires constructing belief functions for a quantity of interest. The procedure used in the current paper is the same as developed in Poroseva et al.[5] First, we specify intervals over which the belief functions are to be determined and then, assign the degree of belief to each interval.

As an example, let us consider the procedure for constructing a belief function for the shortest distance during the climb phase. Since the number of past flights and, therefore, of available S - data is finite, the range of deviation values ($\Delta_{total} = max\,S - min\,S$) that includes all S values is finite as well. Therefore, it is always possible to specify at least a single finite interval – the range of deviation values Δ_{total} – as the focal element and to assign the degree of belief $Bel(\Delta_{total}) = m(\Delta_{total}) = 1$ to this interval based on available evidence (values of S). Based on this observation, one can say that all available evidence supports the proposition that S is likely to deviate from its ideal value inside this interval. Obviously, our proposition that evidence supports this specific interval is subjective and corresponds to the available database. More data could possibly increase the size of this interval. Nevertheless, the proposition reflects our current level of knowledge.

A belief function with a single focal element that is the range of all deviation values is not very informative, because the size of such an interval is most likely too large. One would like to find out how uncertainty sources that cause flight plans deviate from reality favor smaller subintervals within the range of all deviation values.

Evidence theory does not impose any limitation on how subsets (in our problem, subintervals Δ_l) should be specified. They can intersect, nest, or be disjoint. They also can be of different size. We recommend, however, that for practical purposes, the universal set **S** been divided into disjoint ordered subintervals of uniform size Δ. An example is shown in Fig. 3 for $\Delta = 0.1^0$. One of the subintervals should always contain the zero value as its endpoint.

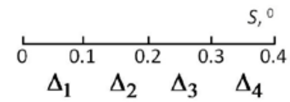


Figure 3. Example of subintervals in the S-universal set.

The subinterval size Δ and the number of subintervals are determined by the following factors: each subinterval should contain at least one *S* value and an *m*-function constructed over the subintervals should be of concave type. Between Δ_{ideal} and Δ_{total}, several values of Δ may produce a concave *m*-function. The smallest of them should be chosen. As previous studies[3-5] show, Δ is typically larger than Δ_{ideal} and almost always less than Δ_{total} even though there is no guarantee for the existence of a concave *m*-function with $\Delta < \Delta_{total}$.

The *m*-value for each subinterval is defined as the ratio of the number *n* of deviation values falling inside the subinterval to the total number *N* of deviation values in the database for a given flight phase of a given flight:

$$m(\Delta_l) = \frac{n_l}{N}, \tag{1}$$

where l is the index over the subintervals. Since, each *S* value unambiguously supports one of the subintervals and subintervals are disjoint, there is no difference between *m*- and belief functions, that is, $Bel(\Delta_l) = m(\Delta_l)$ for any Δ_l.

When belief functions for different flights are compared, one can take into account that databases for different flights contain most certainly different number of data. The same is true when comparing belief functions for different flight phases. Recommended approach in such a case is to choose from all compared belief functions the one with the maximum number of data, N_{max}. For this belief function, the *m*-values are determined as described above and

$$\sum m(\Delta_l) = \frac{\sum n_l}{N_{max}} = \frac{N_{max}}{N_{max}} = 1.$$

For other belief functions, the *m*-values are determined as

$$m(\Delta_l) = \frac{n_l}{N_{max}}, \tag{2}$$

and

$$\sum_l m(\Delta_l) + m(\mathbf{S}) = \frac{\sum n_l}{N_{max}} + \frac{N_{max} - N}{N_{max}} = \frac{N + N_{max} - N}{N_{max}} = 1,$$

Where *N* and *l* vary from one database to another and $m(\mathbf{S}) = (N_{max} - N)/N_{max}$ is our belief committed not to a specific subinterval, but to the entire set **S**. This procedure reflects our belief that conclusions based on larger database are more credible.

A belief function constructed in such a manner completely describes the accuracy and credibility of 4DTs (their horizontal projections in the current paper to be exact) in a given flight phase. Specifically, the credibility is given by the *m*-value assigned to a subinterval. This value reflects the portion of our belief that an actual trajectory deviates from an expected 4DT in this subinterval and thus, represents our confidence (credibility). As previous studies[3-5] show, the maximum degree of belief assigned to a subinterval is always less than one, except when

$\Delta = \Delta_{total}$.

The accuracy is represented by three characteristics of a belief function: Δ, Δ_{total}, and the distance between the point $S = 0$ and the endpoint of the subinterval with the maximum degree of belief closest to this point. The constructed belief functions can be compared with the ideal belief function. The ideal belief function assigns the maximum degree of belief to a single interval $Bel(\Delta_{ideal}) = 1$ that includes the zero deviation between planned and actual trajectories. The ideal belief function corresponds to the proposition that "the actual trajectory deviates from its 4DT in a given flight phase within Δ_{ideal}".

III. Results

The methodology was applied to analyze actual flight data. Two flights between two different city pairs with mostly consistent filed routes and aircraft types over several months time, from 01 Sept 2008 to 17 Jan 2009, are identified using the FAA's Post Operations Evaluations Tool (POET)[7]. The city pairs for the flights were chosen to represent a cross-country flight and a southeastern flight through Florida to a busy airport hub, Atlanta Harsfield-Jackson International (KATL). The two flights chosen are Delta DAL1212 from Los Angeles (KLAX) to Atlanta (KATL) and Airtran Airways TRS841 from Atlanta (KATL) to Orlando (KMCO).

The flight plan and actual flight information is derived from the FAA's Airspace Situation Display to Industry (ASDI) data feed, which is a subsystem of the Enhanced Traffic Management System (ETMS). The ASDI feed provides flight plan and track information from the National Airspace System (NAS) to airlines and other organizations in real-time or near real-time depending on the need.

Flight Delta DAL1212 was scheduled to depart on different days at different times – 13:10, 14:00, and 14:25 – during the analyzed period. Since these scheduled departure times seem to be close, initial impulse is to combine data of the three flights into a single database as it would increase the number of samples. However, it is found that uncertainty in the flight plan trajectories is sensitive to the flight departure time. Therefore, the data for the three departure times are analyzed separately, as they relate to separate flights. The total number of Delta DAL1212 flights analyzed is 130. Among them, 56 flights were scheduled to depart at 13:10, 44 flights at 14:00, and 30 at 14:25.

The scheduled departure time for the Airtran Airways TRS841 flight was 12:55 and 13:45 on different days during the analyzed period. Again, we find that uncertainty in the flight plan trajectories is sensitive to the flight departure time, and therefore, the data for the two departure times are analyzed separately. The total number of Airtran Airways TRS841 flights analyzed is 123. Among them, 56 flights were scheduled to depart at 12:55 and 67 flights at 13:45.

Two flight phases – climb and descent – are considered for each flight. The analysis of the dataset of actual DAL1212 trajectories identified the following waypoints in 4DTs as the beginning and the end of climb and descent flight phases: A and D (climb) and X and Z (descent) (Fig. 1b). For TRS841 trajectories, the corresponding waypoints for the climb flight phase are A and G and for the descent flight phase, they are I and T (Fig. 4). For both flights, specified waypoints identify a corresponding flight phase regardless the flight departure time.

Not all actual trajectories under consideration passed through specified waypoints. Trajectories with different climb and descent waypoints are excluded from the analysis. It reduces the number of analyzed trajectories to 51,

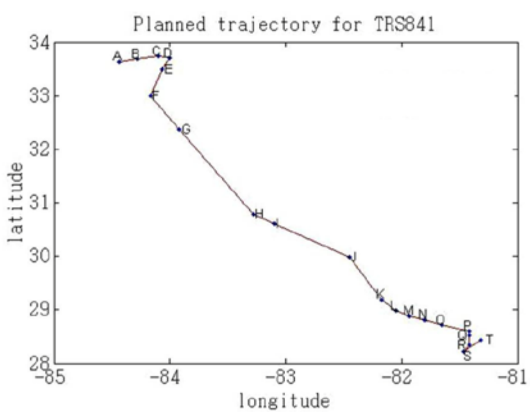


Figure 4. 4DT for the TRS841 flight with the scheduled departure time at 13:45.

36, and 26 for the DAL1212 flight with the scheduled departure time at 13:10, 14:00, and 14:25, respectively, and to 55 and 66 for the TRS841 flight with the scheduled departure time at 12:55 and 13:45, respectively. The number of actual aircraft positions available for each flight in different flight phases to construct belief functions is given in Table 1.

Flight	Flight phase	Departure time	Number of flights	Number of radar track points
DAL1212	climb	13:10	51	1585
		14:00	36	1059
		14:25	26	780
	descent	13:10	51	5643
		14:00	36	4080
		14:25	26	3136
TRS841	climb	12:55	55	2817
		13:40	66	3473
	descent	12:55	55	2273
		13:45	66	2710

Table 1. Statistics available for the two flights

Figure 5 shows four belief functions for the TRS841 flight. Here, the *m*-values are calculated using Eq. (1).

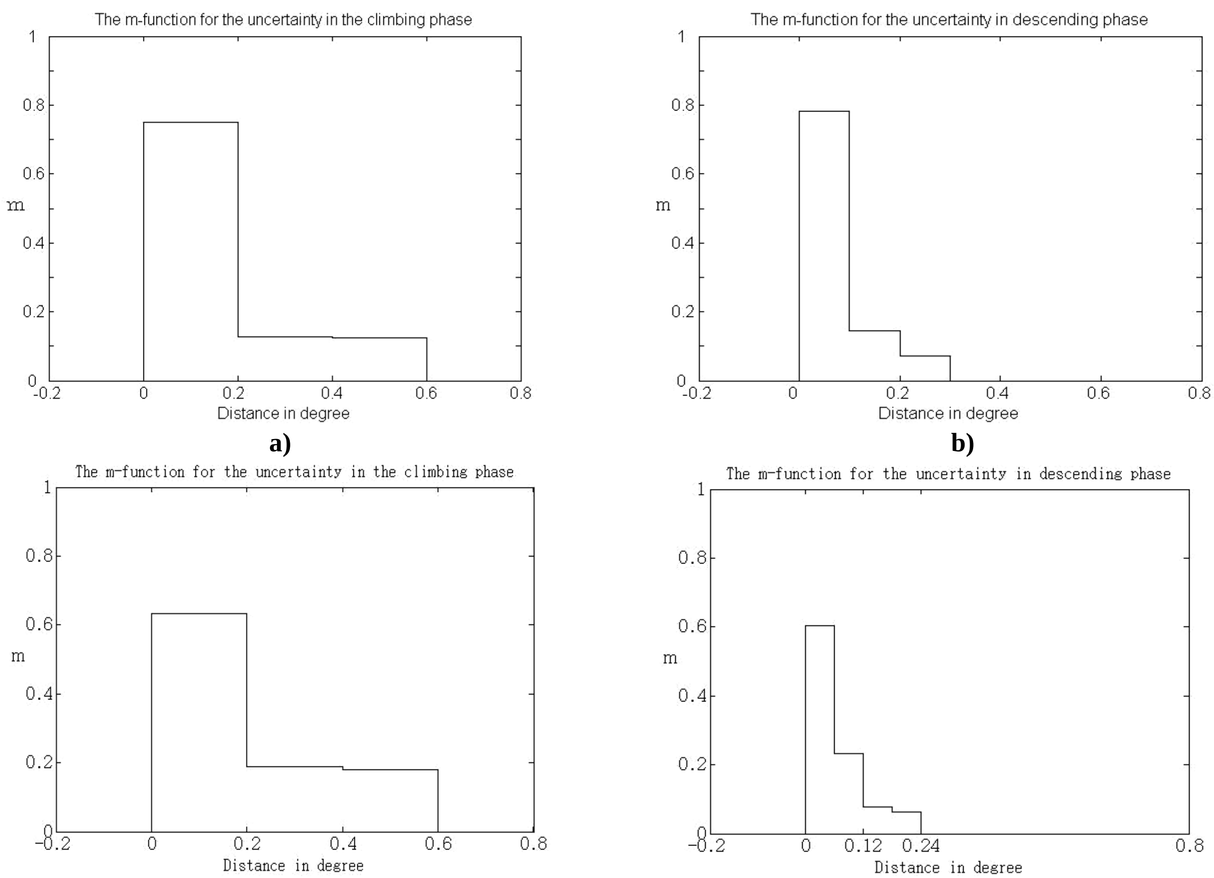


Figure 5. Belief functions for the TRS841 flight with the scheduled departure times 12:55 (climb (a) and descent (b)) and 13:45 (climb (c) and descent (d)).

Since, a procedure chosen to calculate *m*-values affects only the credibility of 4DTs, but not their accuracy, Fig. 5 provides sufficient information for conducting the 4DT accuracy analysis.

Figure 5 demonstrates that the belief functions are different for different flight departure times and different flight phases of the same flight. The accuracy of 4DTs in the climb phase is the same for the two scheduled departure times: same or close to are the total uncertainty ranges, the interval sizes, and the positions of the intervals with the maximum degree of belief. The flight descent phase is predicted more accurately in 4DTs for the scheduled departure time 13:45 (smaller total range of uncertainty and the interval size). For the both departure times, 4DTs in the flight descent phase are almost twice as accurate as in the climb phase (see also Table 2 for exact values of all parameters of belief functions).

For the DAL1212 flight, results of comparing the 4DT accuracy are similar. The interval size and the total range of uncertainty of belief functions representing the accuracy of 4DTs are considerably reduced in the flight descent phase to compare with the flight climb phase. One can also notice (Table 2) that these parameters change from one departure time to another in the same flight phase. 4DTs of the DAL1212 flight with the scheduled departure time 14:00 are the least accurate in the climb phase, but the most accurate in the descent phase.

Also, Table 2 shows that 4DTs of the DAL1212 flight seem to be more accurate than 4DTs of the TRS841 flight. However, how credible is the accuracy evaluation? The last column of Table 2 helps to answer this question. If Eq. (1) is used to evaluate the *m*-values, 4DTs for the DAL1212 are less credible than for the TRS841 flight.

Flight	Flight phase	Departure time	Total range of uncertainty	Interval size	End point of the interval with maximum *m*	Maximum *m*
DAL1212	climb	13:10	0.11	0.011	0	0.44
		14:00	0.14	0.016	0	0.45
		14:25	0.13	0.013	0	0.46
	descent	13:10	0.06	0.007	0	0.41
		14:00	0.01	0.001	0	0.22
		14:25	0.01	0.002	0	0.19
TRS841	climb	12:55	0.6	0.2	0	0.75
		13:45	0.6	0.2	0	0.63
	descent	12:55	0.3	0.1	0	0.78
		13:45	0.24	0.06	0	0.6

Table 2. Uncertainty characteristics for the two flights

However, as previously discussed, the different number of data in different databases should also be taken into account. Table 1 shows that the largest available database is for the descent phase of the DAL1212 flight with the departure time 13:10. That is, maximum *m* will not be changed in this belief function, and $N_{max} = 5643$ in Eq. (2).

Table 3 contains the same data as Table 2 with maximum *m*-values recalculated using Eq. (2) and added data for $m(\mathbf{S})$. The table confirms that conclusions about the accuracy of 4DTs for the TRS841 flight are in general more credible than for the DAL1212 flight. The value of $m(\mathbf{S})$ simply quantifies our lack of knowledge about the distribution of additional data for the shortest distance over intervals if such data would become available to match

Flight	Flight phase	Departure time	Total range of uncertainty	Interval size	End point of the interval with maximum *m*	Maximum *m*	$m(\mathbf{S})$
DAL1212	climb	13:10	0.11	0.011	0	0.12	0.72
		14:00	0.14	0.016	0	0.08	0.81
		14:25	0.13	0.013	0	0.06	0.86
	descent	13:10	0.06	0.007	0	0.41	0
		14:00	0.01	0.001	0	0.16	0.28
		14:25	0.01	0.002	0	0.11	0.44
TRS841	climb	12:55	0.6	0.2	0	0.37	0.5
		13:45	0.6	0.2	0	0.39	0.38
	descent	12:55	0.3	0.1	0	0.31	0.6
		13:45	0.24	0.06	0	0.29	0.52

Table 3. Uncertainty characteristics for the two flights with recalculated *m*-values

the number of data in the database for the descent phase of the DAL1212 flight with the departure time 13:10. More complex formula can be used instead of Eq. (2) if additional information is available to make the evaluation of $m(\mathbf{S})$ more realistic. However, for the purpose of demonstration of the approach, Eq. (2) is sufficient.

Belief functions constructed in such a way can be combined with a new 4DT to form an envelope of uncertainty around the planned trajectory. Such representation can be used for predicting and avoiding possible conflicts. If the full database for 4DTs is available, the same procedure can be used to construct belief functions for an aircraft spatial position variable with time.

IV. Conclusion

The paper demonstrates the applicability of the tools of Dempster-Shafer evidence theory, such as basic probability assignment and belief functions, to quantify the total uncertainty in the flight plan trajectories. A procedure to construct belief functions is described. A computational algorithm to implement the procedure is developed. Application of the approach is demonstrated using data for two actual flights that occurred between 01 Sept 2008 and 17 Jan 2009. The two chosen flights represent a cross-country flight (Delta DAL1212 from Los Angeles to Atlanta) and a southeastern flight through Florida from a busy hub airport (Airtran Airways TRS841 from Atlanta to Orlando). Trajectory predictions accompanied with the uncertainty interval variable in space and time for different trajectory segments and waypoints and with the quantified belief in such predictions is a highly desirable tool for improving conflict resolution and decision-making and, as such, for increasing Air Traffic Management effectiveness and National Airspace System safety.

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