# Application of the "Selfish" Algorithm for the Survivability Analysis of Systems with Multiple Loads

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The goal of our research is the development of analytical and computational tools for quantifying the survivability of engineering systems with sources and sinks due to the system's topology. An example of such a system is a satellite power subsystem with multiple power sources and loads. In our previous work, we developed and validated a probabilistic approach for evaluating the topological survivability of systems with multiple sources and a single sink. We also proposed the "selfish" algorithm for reducing the computational cost of survivability analysis of systems with multiple sinks. The current paper reports on a computational implementation of the "selfish" algorithm and its verification.

## Nomenclature

VT	=	link adjacent to a generator
VB	=	link adjacent to a load
Η	=	link between two interconnections
М	=	number of total elements
т	=	number of faulted elements
S	=	number of "no response" outcomes
R	=	number of "reconfiguration" outcomes
F	=	number of "failure" outcomes
P(S)	=	probability of a "no response" outcome for m faulted elements
P(R)	=	probability of a "reconfiguration" outcome for m faulted elements
P(F)	=	probability of a "failure" outcome for m faulted elements

## I. Introduction

The success of any space exploration mission depends on the survivability of spacecrafts involved in the mission. The survivability of a spacecraft is determined by many factors. In our research, we analyze the survivability of spacecraft subsystems due to the subsystem's topology. Of particular interest are systems with sources and sinks such as, for example, the electric power subsystem (EPS) in a satellite.

The purpose of EPS is to generate and distribute power from sources such as power generators and batteries to sinks that are various power-consuming loads. Therefore, the EPS survivability can be defined as its ability to deliver power to loads in the amount sufficient to satisfy the loads' demand in the presence of multiple faults in the EPS components. Faults can either be accumulated with time due to natural causes or occur suddenly due to hostility of the space environment. Faults due to natural causes are expected and predictable. Faults caused by adverse events are sudden and unpredictable. In reality, any combination of expected and sudden faults may occur. Therefore, a number of faults and their location within EPS at any given time are generally unpredictable. As a result, the EPS lifetime is also unpredictable, but expected to be less than estimated for normal operational conditions.

The EPS lifetime can be increased by mitigating at least some of the factors that influence the EPS survivability at the early stage of the EPS design. In particular, the EPS ability to survive depends on how a given number of power sources and loads are connected with one another (or on the EPS topology). Some topologies are more survivable than others. By choosing the EPS topology with the most potential for surviving, one can increase the EPS lifetime.

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Comparing the survivability of alternative topologies requires mathematical and computational tools. Development of such tools is the goal of our research. A probabilistic framework for the survivability analysis of systems with sources and sinks due to the system's topology was proposed in Ref. 2. The analysis considers all combinations of faults (or fault scenarios) possible in a given system and system's responses to the fault scenarios. A computational implementation of the approach was developed in Ref. 3 for the analysis of small- and medium-sized systems with multiple sources and a single sink. The approach was applied to various engineering systems including the all-electric ship power system<sup>4</sup> and the mini-satellite UoSAT-12 EPS<sup>5</sup>.

The survivability analysis of systems with complex topologies and of larger size may become computationally intractable. In a system with *M* components, the total number of possible combinations of faults is  $2^{M}$ . In this regard, the survivability analysis may be categorized, at the very least, as an exponential time problem. To expand the applicability range of the analysis, the "selfish" algorithm was proposed in Poroseva<sup>6</sup>. In the algorithm, the computational time is reduced by the substitution of a search problem with a decision problem (see Goldreich<sup>7</sup> for more discussion on search and decision problems).

In the search problem, the system's topology is represented by a graph. Faults modify the initial graph. To determine the availability and connectivity of sources and sinks in a modified graph, a graph search algorithm is applied. The search procedure on a large graph is computationally expensive. It also should be repeated for every fault scenario possible in the initial graph.

In the decision problem, the system's response to a given combination of faults is determined by requesting and analyzing information from a previously created database. That is, no search procedure is applied to the initial system's topology. The database is created by mapping the initial system's topology with multiple sources and sinks onto a set of smaller sub-topologies. Each sub-topology contains the same number of sources as the initial system, but only one sink connected to the whole system by a single link. Other sinks and links associated with them are removed from the sub-topology along with the relevant nodes. Thus, the size and complexity of sub-topologies are reduced to compare with the initial topology. The search procedure is conducted on the sub-topologies to determine their responses to fault scenarios. The information about fault scenarios and the sub-topologies' responses to them is stored in the database. When a fault scenario in the whole system is generated, it is projected on the sub-topologies' database. Based on the individual sub-topologies' responses to the fault scenario in the system's response to the fault scenario. As demonstrated in Ref. 5, the "selfish" algorithm substantially reduces the computational time of the survivability analysis.

In the current study, we have developed a computational implementation of the "selfish" algorithm. The code's verification stage has been completed and its results are presented below for a test topology. The algorithm's application to the mini-satellite UoSAT-12 EPS is in progress. Results are expected to be presented at the conference.

#### **II.** Problem Formulation

In the survivability analysis, a fault is assumed to be a failure of a EPS component. A faulty component becomes unavailable to the system. Origin of faults and their development in time are not considered. Multiple faults are viewed as simultaneous events. Fault scenarios are considered as unpredictable; any fault scenario is possible. Therefore, EPS responses to all fault scenarios are analyzed. The approach described below is applicable to any system with sources and sinks. Therefore, "system" substitutes EPS everywhere in this and following sections.

For a given fault scenario, identification of the system's response means establishing a number of power sources connected to loads after faults have occurred and a degree in which the loads' demand for power is satisfied by the survived sources. In this regard, a fault scenario can be one of three types: "no response", reconfiguration, and complete failure. In the "no response" scenario, faults do not reduce power supply to loads. A fault scenario, in which power supply is reduced but not interrupted, requires the system's reconfiguration or loads shedding. Such a scenario is called the reconfiguration scenario. A fault scenario in which faults completely interrupt power supply to all loads is the complete system's failure. For systems with one load, there are only two types of fault scenarios: no response or complete failure.

Currently, we assume that power generated by all power sources is necessary to fully satisfy the loads' demand for power. If any single power source is unavailable, a fault scenario is of the reconfiguration or complete failure type depending on the numbers of survived sources and loads. We also assume that any single power source can satisfy the demand for power of any single load. That is, if a power source is connected to a load, a fault scenario is of the "no response" type for a system with one load. For a system with multiple loads, this is "no response" or reconfiguration fault scenario depending on the numbers of survived sources and loads. The current assumptions can easily be adjusted to specifications of a real system. If *N* is the total number of all fault scenarios possible in a system, and *S*, *R*, and *F* are the numbers of "no response", reconfiguration, and complete failure scenarios, respectively, then, N = S + R + F. In a system with a single load, N = S + F. These numbers can be used to determine probabilities of the system's responses to faults: P(S) = S / N (probability of "no response" scenarios), P(R) = R / N (probability of reconfiguration scenarios), and P(F) = F / N (probability of complete failure)<sup>2</sup>. These probabilities are hereafter called survival probabilities. In a similar manner, one can define survival probabilities at a given number of faults *m*:

$$P_m(S) = S(m) / N(m)$$
,  $P_m(R) = R(m) / N(m)$ ,  $P_m(F) = F(m) / N(m)$ ,

where  $\sum_{m=1,\dots,M} N(m) = N$ ,  $\sum_{m=1,\dots,M} R(m) = R$ ,  $\sum_{m=1,\dots,M} S(m) = S$ ,  $\sum_{m=1,\dots,M} F(m) = F$ . The survival probabilities sum to

unit. These definitions of the survival probabilities are valid under the assumption that all fault scenarios are equally likely. This is a justified assumption for unpredictable fault scenarios. If information is available, probabilities of individual fault scenarios can be incorporated into the analysis.

The three response probabilities completely characterize the system's topological survivability. The system's topology with the highest probability P(S) and ratio P(S)/P(R) and the lowest probability P(F) has the most potential to survive to compare with alternative topologies with the same numbers of sources and loads. Also, survivability of individual loads in a system can be evaluated and compared<sup>2-6</sup> in a similar manner. Our current research effort is focused on combining information about survival probabilities of individual loads in a system with multiple loads for evaluating the system's survivability as a whole. The "selfish" algorithm<sup>6</sup> is used for this purpose. Figure 1 shows the algorithm's general structure.



Figure 1. The structure of the "selfish" algorithm.

#### III. Test System

Due to the exponential growth of the total number of faults scenarios with the number of system's components, the survivability analysis can only be conducted computationally for most engineering systems. In the current study, a computational implementation of the "selfish" algorithm has been developed and verified in a few test problems for which survival probabilities can be obtained manually. Figure 2a shows the largest topology, which was used as a test problem for the code verification. The topology includes two power sources that supply power to two loads. In Figure 2b, the same topology is represented by links. Such system's representation was found<sup>4,6</sup> to be the most beneficial for the survivability analysis. Three different types of links are: links adjacent to power sources ("VT" links), links adjacent to loads ( "VB links), and links between interconnections ("H" links). In the topology, an interconnection corresponds to a physical connection of wires in parallel. Several wires connected in series are represented by a single link in the survivability analysis<sup>4,6</sup>. To recognize the three types of links mathematically, the VB- and VT-links are assigned weights. The weight sign corresponds to the flow direction through a link: VT-links have positive weight and VB-links have negative weight. No sign is assigned to H-links, because the flow direction through such links may vary depending on the system's topology after faults. A value of the weight assigned to a

link can be based, for example, on its capacity or a maximum amount of power it can transport in relation to the total demand. Currently, all links are equally weighted. Weights can be adjusted for a real system.

The number of system's components in the topology shown in Fig. 2b is 11. The total number of fault scenarios in this topology is  $2^{11} = 2048$ .

The further analysis requires disintegrating the topology shown in Fig. 2b into sub-topologies for each individual VB link. Since there are two VB-links in Fig. 2b, there are two sub-topologies to analyze (Figs. 3a and b). In the



Figure 2. Test system: a) node-link representation, b) representation by links only. Notations: circles with "+" are power sources; circles with "-" are loads; VT are links adjacent to power sources; VB are links adjacent to loads; H are any other link.

sub-topology shown in Fig. 3a, two links, H2 and H3, are connected in series after the VB2 link has been removed. Therefore, they can be represented by a single link. Similarly, links H4 and H5 can be represented by a single link in the topology shown in Fig. 3b.

One can notice that the two sub-topologies in Figs. 3a and b are equivalent. Thus, only one sub-topology (Fig. 3c) has to be analyzed instead of two. Table 1 shows the correspondence of links of the sub-topology shown in Fig. 3c to those in Figs. 3a and b.



Figure 3. Sub-topologies of the test system: a) sub-topology for VB1 link, b) sub-topology for VB2 link, c) final sub-topology to analyze.

The sub-topology in Fig. 3c has nine components, that is, 512 fault scenarios have to be generated and 512 modified sub-topologies have to be applied the search algorithm to. This number can further be reduced by noticing that any fault scenario with fault in link A3 is the complete failure of the sub-topology. Thus, a sub-topology's response to only 256 fault scenarios needs to be analyzed with the search algorithm. This is to compare with 2048 fault scenarios in the initial topology in Fig. 2b. Since the sub-topology contains one load connected to the system by the single link, only two responses are possible: "no response" and complete failure.

Table 1.	Correspondence	of links in	Fig. 3c to	links in Fig	s. 3a and b.
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Sub-topology for	A1	A2	A3	A4	A5	A6	A7	A8	A9
VB1	VT2	VT12	VB1	H4	H1	H5	H6	H7	H2,H3
VB2	VT1	VT2	VB2	H3	H1	H2	H7	H6	H4,H5

# **IV.** Results

Table 2 shows the survival probabilities for the sub-topology in Fig. 3c without considering faults in link A3.

m	P(S)	P(F)
0	1	0
1	1	0
2	0.93	0.07
3	0.79	0.21
4	0.53	0.47
5	0.18	0.82
6	0.04	0.96
7	0	1
8	0	1

Table 2. Survival probabilities of the sub-topology in Fig. 3c with non-faulty link A3.

The data stored in the database are not those in Table 2, but in a table with complete information about faulty components in each fault scenario and the sub-topology's response to each fault scenario. When a fault scenario in the whole system is generated, it is projected on the table with complete information. Then, responses of each sub-topologies to a given fault scenario are merged and the decision is made about the response of the system as a whole to the fault scenario.

In the analysis of the system's survivability as a whole, faults in loads and links adjacent to them are not considered. That is, all VB-links are assumed to be non-faulty. This is the assumption of the worst case scenario: power supply may be reduced, but the demand for power stays the same. Table 3 provides survival probabilities of the system in Fig. 2b with non-faulty VB links.

## Table 3. Survival probabilities of the system in Fig. 2b with non-faulty VB links.

m	P(S)	P(R)	P(F)
0	1	0	0
1	0.78	0.2	0
2	0.53	0.44	0.03
3	0.27	0.65	0.08
4	0.05	0.75	0.2
5	0.01	0.57	0.42
6	0	0.24	0.76
7	0	0.06	0.94
8	0	0	1
9	0	0	1

Figure 4 shows survival probabilities of the sub-topology in Fig. 3c and the test system in Fig. 2b, both with non-faulty links adjacent to loads.



Figure 4. Survival probabilities of a) sub-topology in Fig. 3c, b) test system from Fig. 2b.

## V. Conclusions

In the current study, a computational implementation of the "selfish" algorithm for the analysis of the topological survivability of systems with multiple sources and sinks such as, for example, an electric power subsystem in a satellite has been developed. The code was successfully verified in test problems, solutions for which can be obtained manually. Results for the largest test system considered by now are shown. The analysis of the mini-satellite UoSAT-12 EPS survivability due to its topology is in progress. Results are expected to be presented at the conference.

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