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ON ABILITY OF STANDARD $k-\varepsilon$ MODEL TO SIMULATE AERODYNAMIC TURBULENT FLOWS

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Abstract

New direction to improve the quality of results obtained with two-equation turbulence models is suggested. It is shown that with a new approach to the model coefficients and the standard k and ε equations [1, 2], it is possible to reproduce correctly mean velocity and shear stress profiles in simple flows featuring typical aerodynamic problems. Self-similar free shear flows (plane wake, mixing layer, plane and round jets) and equilibrium boundary layers (with and without pressure gradients) are considered. Nevertheless, the behaviour of the turbulent kinetic energy is described only qualitatively in most test flows except in the round jet. Turbulence structure of the round jet can be predicted perfectly within the framework of the given approach. The analysis can be applied to any model of the two-equation eddy-viscosity type.

Key Words: Turbulent Flows, Modeling

1 INTRODUCTION

Simplicity of two-equation turbulence models is that advantage which makes them a common simulation tool in aerodynamics. The model including the transport equations for the turbulent kinetic energy k and the dissipation rate ε[1, 2] is accepted to be the standard one of this type of turbulence models. At the same time, the failures of such models are well known either.

Various approaches have been developed to solve this problem. As a good review of them, the paper [3] could be cited. The most promising results, in particular for the boundary layer, were obtained by the models in which the equation for another variable than ε was used, that is, for $\omega \sim \varepsilon/k$ [4] or $\varphi = \varepsilon/\sqrt{k}$ [5]. Nevertheless, it is difficult to confirm that crucial improvement has been achieved in this way. It can mean that the ε-equation itself is not, probably, the only reason of the model failures.

It is commonly supposed that the model coefficients in two-equation turbulence models are constants. However, it is nothing else than a simplification. Indeed, it has been found in the modeling of Reynolds stress equations that some model coefficients are necessarily function of several parameters [6, 7]. A similar situation could be expected for two-equation models. The aim of the present paper is to clarify this point. Though analysis will be restricted to the standard $k-\varepsilon$ model [1, 2], it could be applied to any model of the two-equation eddy-viscosity type.

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2 ANALYSIS

2.1 $k-\varepsilon$ Model

The case of an incompressible turbulent flow will be considered. In the high Reynolds number approximation, the standard $k-\varepsilon$ model [1, 2]:

$$\frac{Dk}{Dt} = P - \varepsilon + \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right),$$

$$\frac{D\varepsilon}{Dt} = \frac{\varepsilon}{k}(C_{\varepsilon_1}P - C_{\varepsilon_2}\varepsilon) + \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right),$$

with the Boussinesq hypothesis

$$-<u_iu_j> = \nu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{2}{3}k\delta_{ij},$$

where

$$P = -<u_iu_j>\frac{\partial U_i}{\partial x_j}$$

and

$$\nu_t = C_\mu\frac{k^2}{\varepsilon},$$

involves five model coefficients: C_μ, C_{ε_1}, C_{ε_2}, σ_k, and σ_ε.

All of them were taken as constants in [2]: $C_{\varepsilon_1} = 1.44$; $C_{\varepsilon_2} = 1.92$; $C_\mu = 0.09$; $\sigma_\varepsilon = 1.3$; $\sigma_k = 1$ (hereinafter referred as LS1-coefficients). Now, we assume that at least some of them are not constants.

2.2 Test Flows

To clarify the nature of the coefficients, two-dimensional test flows featuring typical aerodynamic problems, like self-similar free shear flows and equilibrium boundary layers (with and without pressure gradients), have been considered.

Self-similar flow states are observed far away from a flow source. Transport equations describing turbulence structure in such flow area, reduce to a simple form, which enable us to determine easily the influence of the values of the coefficients.

Four free shear flows have been investigated. Self-similar solutions are sought for the plane wake as

$$\frac{U - U_\infty}{U_\infty - U_0} = F'\left(\frac{y}{\delta}\right),$$

where U_∞, U_0, and δ are the external velocity, the centerline velocity, and the wake thickness respectively; for the plane or round jet as

$$\frac{U}{U_0} = F'\left(\frac{y}{\delta}\right),$$

where U_0 and δ are the centerline velocity, and the jet thickness respectively; for the mixing layer as

$$\frac{U - U_2}{U_1 - U_2} = F'\left(\frac{y}{\delta}\right),$$

where U_1, U_2, and δ are the velocities of fast and slow streams, and the mixing layer thickness respectively.

For the outer part of equilibrium boundary layers, the self-similar solution is sought as:

$$\frac{U_e - U}{U_\tau} = F'\left(\frac{y}{\delta}\right),$$

where U_e, U_τ, and δ are the external velocity, the friction velocity, and the boundary layer thickness respectively. Self-similarity is achieved at the high Reynolds number and at the constant pressure-gradient parameter

$$\beta = \frac{\delta_1}{\tau_\omega}\frac{dP'}{dx}.$$

Here, δ_1 is the displacement thickness and τ_ω is the wall shear stress. Self-similarity is assumed for the non-dimensional turbulent shear stress $< uv > /U_\tau^2$ either. The equilibrium boundary layer at four values of β has been tested: -0.211, 0., 5.139, and 19.6. The last value corresponds to a strong adverse pressure-gradient flow.

2.3 Numerical Procedure

The self-similar flow equations reduce simply to ordinary differential equations of the generic form:

$$-A\phi - B\frac{d\phi}{d\eta} - \frac{1}{\eta^j}\frac{d}{d\eta}\left(\eta^j C\frac{d\phi}{d\eta}\right) = S,$$

where $j = 0$ for plane flows and $j = 1$ for axisymmetric flows, ϕ can be the self-similar velocity or one of the turbulent quantities in the self-similar form and η is the self-similar crossflow dimension ($\eta = y/\delta$). In this equation the coefficients A, B, and C are positive. The terms containing A and B are the convection terms, the term containing C is the diffusion term, and S is the source term. In the momentum equation the source term is equal to zero, whereas in the turbulent equations the source term is the sum of two terms: the production, written as S^+, which is positive, and the destruction (or dissipation) of the ϕ quantity, written as S^-, which is negative.

The generic equation is discretized in a finite volume formulation according to the hybrid scheme of Patankar [8]. The term $-A\phi$ is put on the right with the positive part S^+ of the source term and is considered as explicit. The negative part S^- of the source term is put on the left and is considered as implicit. This term is then rewritten as: $S^- = -|S^-|\phi^n/\phi^{n-1}$, where ϕ^{n-1} is the explicit value of ϕ (n is the current iteration).

The discretized equation forms a tridiagonal system for the unknowns ϕ_i at the center of the cells. To reinforce the diagonal dominance of the system, a pseudo-unsteady term $\frac{\partial\phi}{\partial t} = \frac{\phi^n - \phi^{n-1}}{\Delta t}$ is added to the generic equation. The time step Δt is chosen inversely proportional to the maximum of the diagonal coefficients, so that the convergence rate is improved.

The computational domain extends beyond the physical thickness of the flow to ensure that the solution is independent of the boundary conditions at the outer edge. At this edge the quantities are set to zero.

For wake and jet flows, a symmetry condition is prescribed at $\eta = 0$ for the turbulent quantity and the self-similar velocity is set to one.

For equilibrium boundary layers, as the inner part is not computed, the first grid point is taken small enough to be considered in the logarithmic region. The momentum equation is integrated to give an equation for F ($F = \int_0^\eta \frac{U_e - U}{U_\tau} d\eta$), so that natural conditions can be prescribed ($F = 0$ at the first point and $\frac{dF}{d\eta} = 0$ at the edge). For the turbulent quantities, the equilibrium behaviour of the logarithmic law at zero pressure gradient is prescribed, which is not correct in the case of strong adverse pressure gradients.

An adaptive grid procedure is integrated in the numerical process, which tightens the grid points in the region, where the first and second derivative of the variables are important. The computational method has been checked from the point of view of the convergence, the grid point number and the boundary conditions, to ensure that solutions obtained are reliable. Usually computations are performed over 300 grid points and mean residuals fall under 10^{-8} within 2000 iterations.

2.4 Results and Discussion

To describe the decay of isotropic turbulence, the coefficient C_{ε_2} should be equal to 1.92, as it was found experimentally [9, 10]. The value of C_μ can also be estimated experimentally in the logarithmic region of a boundary layer and it gives us 0.09 [11]. Strictly speaking, this does not necessarily mean that the coefficients should have the same values in other turbulence states. However trying to improve the model, we should also keep its simplicity as much as possible. For this purpose, in the present study, C_μ and C_{ε_2} are considered as constants and take the standard values. Thus, only three coefficients need to be determined.

From the study of a large range of the values of the model coefficients, the important conclusions are following.

In fact, it was found that the question on the coefficients σ_ε and σ_k, and the question on the optimal value of the coefficient C_{ε_1} can be considered independently each other.

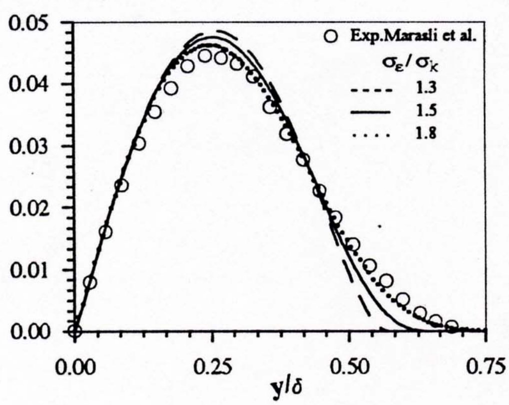

Fig.1: Shear stress profiles in a plane wake

The relation between coefficients σ_k and σ_ε is of crucial importance in describing the slope of the profiles. In different flows, the optimal relation is different (see table 1). However, taking $\sigma_\varepsilon/\sigma_k = 1.5$, one can get appropriate profiles in all cases. Thus, this value is recommended for the practical use instead of the standard value 1.3. As an example, the Reynolds stress

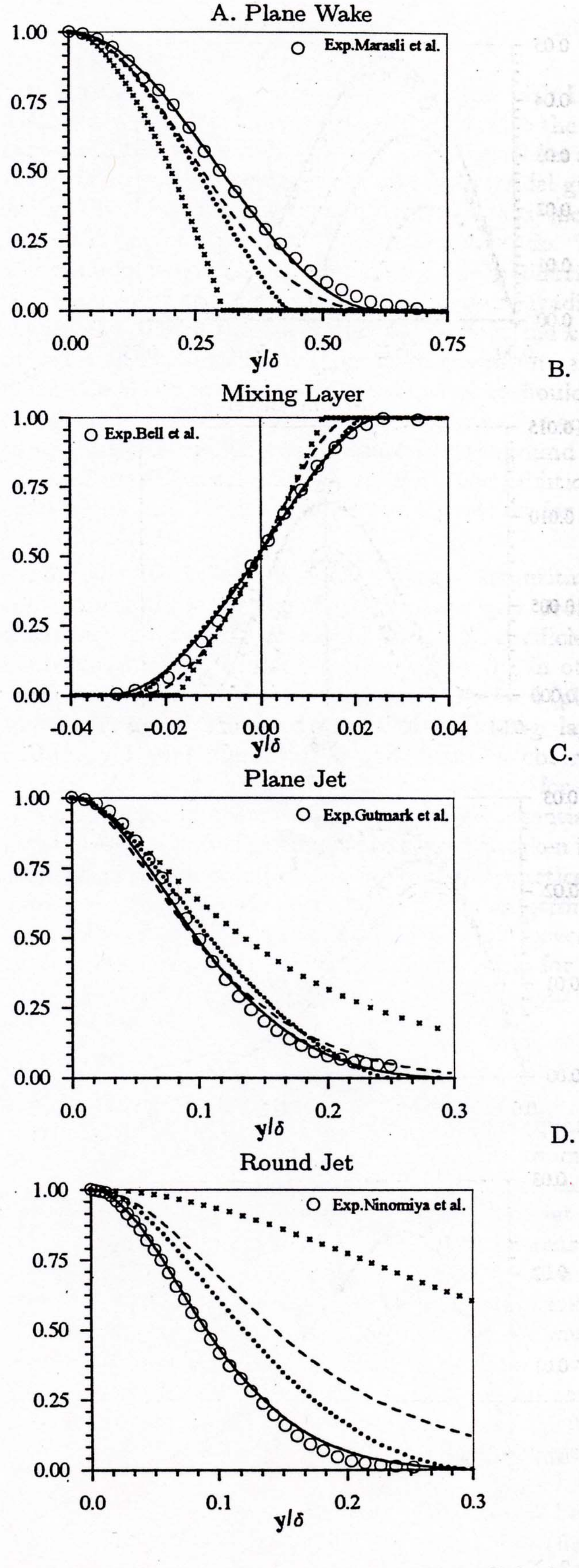

Fig.2: Non-dimensional velocity profiles. Calculated profiles: (—) $k-\varepsilon$ model with LS2-coefficients, (·) $k-\varepsilon$ model with LS1-coefficients, (- -) $k-\varphi$ model, (x) $k-\omega$ model.

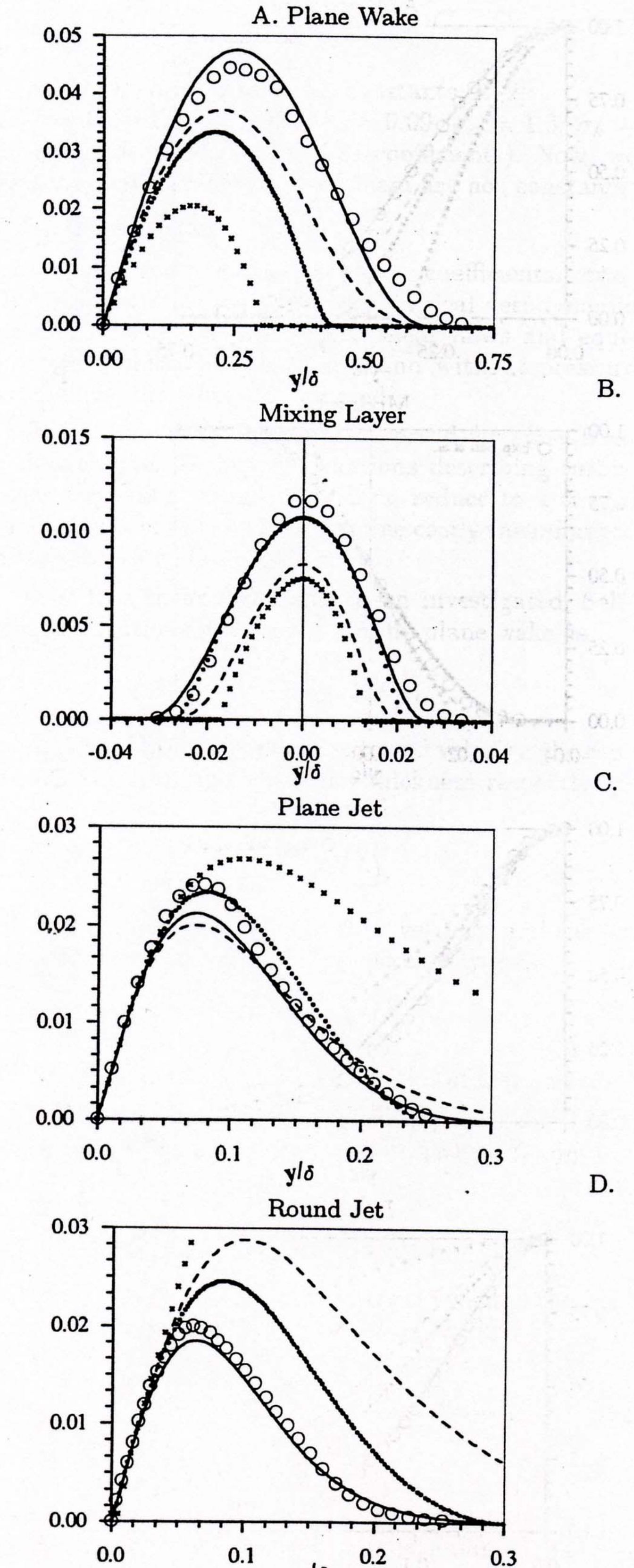

Fig.3: Non-dimensional shear stress profiles (see denotations on Fig.2)

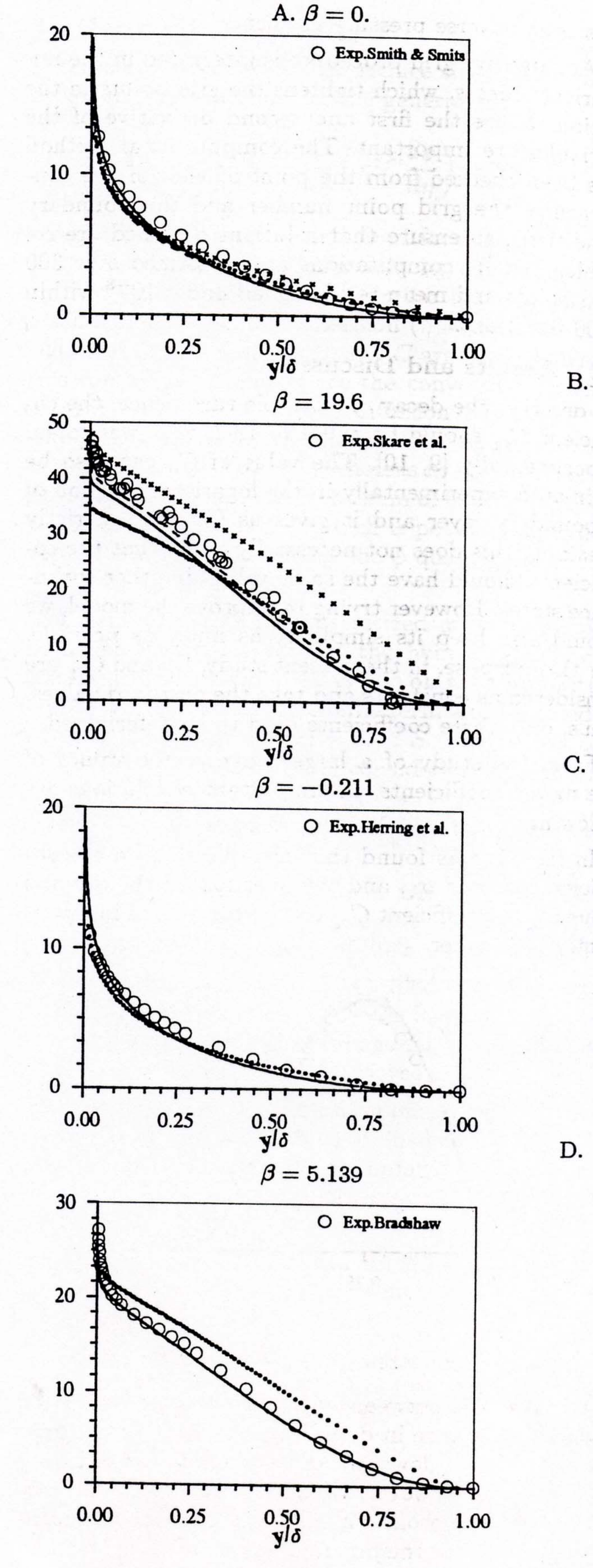

Fig.4: Non-dimensional velocity profiles in the equilibrium boundary layer (see denotations on Fig.2)

Table 1: Optimal relations between σ_ε and σ_k for test flows.

flow	wake	mixing layers	plane jet	round jet
$\sigma_\varepsilon/\sigma_k$	1.8	1.8	1.5	1.3

boundary layers			
$\beta = 19.6$	$\beta = 5.139$	$\beta = 0.$	$\beta = -0.211$
1.5	1.5	1.5	1.5

Table 2: Optimal values of the coefficient C_{ε_1}.

flow	wake	mixing layers	plane jet	round jet
C_{ε_1}	1.2	1.4	1.45	1.5

boundary layers			
$\beta = 19.6$	$\beta = 5.139$	$\beta = 0.$	$\beta = -0.211$
1.5	1.4	1.5	1.5

profiles obtained at different relations of σ_ε and σ_k in the plane wake, is shown in figure 1.

The following values of the coefficients were found to be optimal for all test flows: $\sigma_\varepsilon = 1.$, $\sigma_k = 0.67$ (compare with the standard values of $\sigma_\varepsilon = 1.3$ and $\sigma_k = 1.$). They satisfy the constraints imposed on the coefficients to reproduce the correct behaviour at the edge of a turbulent region [12]:

$$1 < \sigma_\varepsilon/\sigma_k < 2, \qquad \sigma_k < 2, \qquad 2\sigma_k - 1 < \sigma_\varepsilon.$$

The coefficient C_{ε_1} has a strong effect on the calculated results. Even if the optimal values of C_{ε_1} for different flows seem to be close (see table 2), it is not possible to recommend a constant value for C_{ε_1}. It can be argued that its value depends on the type of a flow considered.

Results of simulation of free shear flows, which were calculated by the standard $k-\varepsilon$ model at $C_\mu = 0.09$, $C_{\varepsilon_2} = 1.92$, $\sigma_\varepsilon/\sigma_k = 1./0.67 = 1.5$, and with the values of C_{ε_1} given in table 2 (hereinafter referred as LS2-coefficients), are shown in Figs. 2, 3. For comparison, profiles obtained by $k-\varphi$ [5], $k-\omega$ [4] models, and the standard $k-\varepsilon$ model with LS1-coefficients as well as the experimental data [13 - 16] are presented in the figures.

Mean velocity profiles in the equilibrium boundary layer at the different values of β are given in figure 4. There were used the experimental data [17 - 20] to compare.

It is seen that the standard $k-\varepsilon$ model with LS2-coefficients reproduces in good agreement with the experimental data the behaviour of mean velocities and shear stresses in all considered flows. The model gives essentially better results than the standard model with LS1-coefficients and the $k-\omega$ model do. The $k-\varphi$ model predicts more accurately the equilibrium boundary layer at the strong adverse pressure gradient (figure 4b), but in the outer part of the flow, the $k-\varepsilon$ model with LS2-coefficients describes the mean velocity profile closer to the experimental data. It should be particularly emphasized that the standard $k-\varepsilon$ equations with LS2-coefficients simulate well the round jet, whereas other models do not or they need additional corrections like, for instance, Pope correction for the ε-equation [21].

For the turbulent kinetic energy, quantitative coincidence between experimental data and profiles calculated by the $k-\varepsilon$ model with LS2-coefficients is obtained in the round jet only (figure 5). In other flows, the axis level of k is either overestimated (plane wake) or underestimated (plane jet, mixing layer, boundary layer). Qualitative agreement is observed though. As it is shown in [22], the results for the turbulent kinetic energy can be improved essentially, if the pressure-diffusion effects are correctly taken into account in the k-equation. It is common practice to ignore them in modeling, but they influence strongly turbulence structure in considered flows. Moreover, it is the pressure effects, which are responsible for the variable value of C_{ε_1}. For more details, the paper [22] can be cited.

Plane jet / round jet anomaly

It was shown in [21] that the standard two-equation model cannot solve the plane jet / round jet anomaly. That is, with the same model coefficients, the calculated profile of the mean velocity in the plane jet underlies the similar profile in the round jet, whereas experiments give the opposite result. Calculations made by two-equation models of different types confirm such a conclusion [3]. One of the aims of this work was to show, that different flows should not be described with the same set of the coefficients. Thus, in this sense, the anomaly is solved by different values of C_{ε_1}. Nevertheless, we have done calculations with the same set of coefficients for round and plane jets: $C_{\varepsilon_1} = 1.475$; $\frac{\sigma_\varepsilon}{\sigma_k} = \frac{1.}{0.67} = 1.5$; $C_{\varepsilon_2} = 1.92$; $C_\mu = 0.09$. It was found that the profiles obtained are still satisfactory (figure 6). While the round/plane jet anomaly is still observed in agreement with [21], it is of the negligiblly small value.

3 CONCLUSION

The analysis shows that with the standard k and ε equations and a new approach to the model coefficients which is suggested in the present paper, it is possible to predict correctly mean velocity and shear stress profiles in test flows including the equilibrium boundary layer under strong adverse pressure gradient.

Four from five coefficients can be kept as constants: σ_k, σ_ε, C_μ, and C_{ε_2}, with C_μ and C_{ε_2} taking the standard values: 0.09 and 1.92 respectively. The relation between coefficients σ_k and σ_ε is essential. A constant ratio $\sigma_\varepsilon/\sigma_k = 1./0.67 = 1.5$ can be recommended for practical purposes. This ratio provides a good agreement with experiments for all considered flows and satisfies the necessary conditions for the good behaviour of the numerical solution near the edge of the shear flows.

The value of C_{ε_1} has a strong effect on the calculated results. Its optimal value changes for different flows. The further researches are necessary to understand how its value is determined by the type of a flow considered.

The calculated mean velocity and shear stress profiles coincide quite well with experimental data. However, even if LS2-coefficients are used, the standard $k-\varepsilon$ equations can describe only qualitatively the behaviour of the turbulent kinetic energy in most test flows except in the round jet. Turbulence structure of the round jet can be predicted perfectly within the framework of the given approach.

This work does not completely solve the problem of a correct prediction of aerodynamic flows, but suggests a new promising direction to improve the quality of results obtained with two-equation models. In this sense, the conclusions made could be of interest both for industry and for further researches.

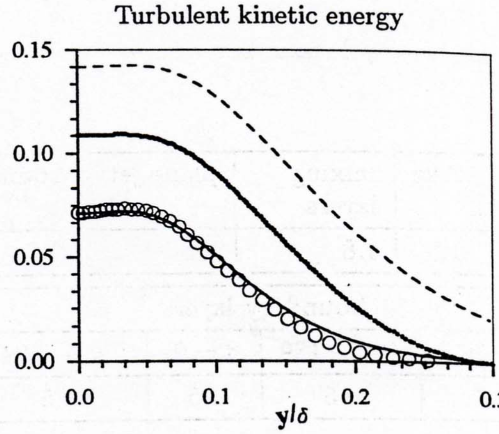

Fig.5: Round pipe (see denotations on Fig. 2)

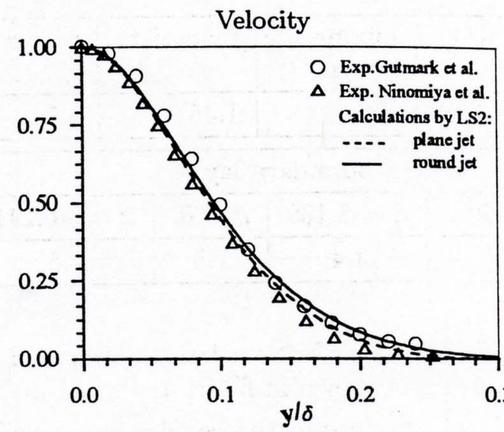

Fig.6: Plane / round jet anomaly

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