MODELING OF THE LIMITING REGIME OF STABILIZATION OF THE AVERAGE VELOCITY OF A TURBULENT FLOW IN A ROTATING STRAIGHT CIRCULAR TUBE

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Results of calculations of the limiting regime of stabilization of the average velocity of a flow and an investigation of the behavior of statistical characteristics of turbulence in a straight circular tube that rotates about the longitudinal axis based on a differential model of transfer of Reynolds stresses are presented. It is shown that, in a correct description of transformation of the structure of swirling-flow turbulence (Reynolds stresses) with simultaneous attainment of a regime of stabilization by the profiles of the axial and circumferential components of the average flow velocity, we need to allow for the action of the rotation on the spectral expenditure of kinetic energy of turbulence.

1. Introduction. The structure of a developed turbulent flow directed from a nonrotating circular tube to a section of a tube of the same diameter that rotates about the longitudinal axis is altered substantially. The main effect of the action of the flow's swirl manifests itself [1-7] in a decrease in the turbulence intensities in both the radial and axial directions and a decrease in the turbulent friction as the swirl parameter N increases. The profiles of the longitudinal U and circumferential W components of the average flow velocity attain a limiting regime of stabilization in a long rotating tube: in the range $x/D = 68-168$, the longitudinal-velocity profiles become quite similar but (for $N = 1$) are still far from the parabolic profile of a laminar regime of flow, while the circumferential-velocity profiles have a limiting parabolic shape ($W/W_0 = (r/R)^2$).

Mathematical modeling of the structure of the turbulence of a swirling flow in a circular tube that rotates about the longitudinal axis using the $E-\varepsilon$ model turns out to be unsatisfactory: for a developed swirling flow the model precalculates a profile of the circumferential velocity W that is linear with respect to the radial coordinate, which corresponds to solid-body rotation, which is completely inconsistent with experimental data. The model of transfer of Reynolds stresses (MTRS) [8, 9] enables us to obtain, for moderate swirls of the flow ($N \leq 1$), profiles of the longitudinal and circumferential components of the average velocity that are in satisfactory agreement with experimental data.

Employment of a "standard" equation for the dissipation of the kinetic energy of turbulence ε that fails to allow for the effect of rotation in the MTRS of [8] and neglect of the damping influence of the wall on the turbulent motion make it impossible to reproduce suppression of turbulence in a short rotating tube ($x/D \leq 25$) or on the initial segment of a long ($x/D \leq 168$) rotating tube that fits the experimental data adequately. In [8], no results of calculation of second-order moments (Reynolds stresses) are given.

The MTRS proposed in [9] describes behavior of turbulent stresses (tangential and normal stresses) of a developed turbulent flow in both a nonrotating tube and a short circular tube ($x/D \leq 25$) that rotates about the longitudinal axis in accordance with data of measurements [1-3]. This is achieved by a correction to the destruction term of the ε-equation in the form of an explicit dependence on the rotational Richardson number Ri. The MTRS also involves needed modifications of the transport equations that allow for the effect of low Reynolds numbers near the tube wall.

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Presented below are results of calculating statistical characteristics of a developed turbulent flow in a long rotating circular tube by the model of transfer of Reynolds stresses of [9] with the aim of elucidating the possibilities of describing the limiting stabilization regime for the profiles of the longitudinal and circumferential components of the average flow velocity and the transformation of turbulent stresses along the tube.

2. Model of Turbulent Transfer of Reynolds Stresses. The system of exact transport equations for the average-velocity vector and the tensor of turbulent stresses for a steady-state incompressible flow is written in general tensor form as

$$U^i_{,i} = 0\,; \quad U^j\, U_{i,j} = \nu g^{jk}\, U_{i,jk} - \langle u_i\, u^j\rangle_{,j} - \hat{P}_{,j}/\rho\,; \tag{1}$$

$$U^k\,\langle u_i\, u_j\rangle_{,k} = D_{ij} + P_{ij} + \Pi_{ij} - \varepsilon_{ij}\,, \tag{2}$$

where $\varepsilon_{ij} = 2\nu g^{km}\langle u_{i,m}u_{j,k}\rangle$ is the dissipation tensor; $D_{ij} = \langle u_iu_ju^m\rangle_{,m} - (\langle pu_i\rangle_{,j} + \langle pu_j\rangle_{,i})/\rho + \nu(g^{km}\langle u_iu_j\rangle_{,k})_{,m}$ is the diffusion; $P_{ij} = -\langle u_ju^k\rangle U_{i,k} - \langle u_iu^k\rangle U_{j,k}$ is the generation; $\Pi_{ij} = \langle p(u_{i,j} + u_{j,i})\rangle/\rho$ is the correlation of pressure–velocity shift.

To obtain a closed form of system (1)-(2), we need model representations for the terms D_{ij}, Π_{ij}, and ε_{ij} [9].

The model for the diffusion terms in (2) is written as

$$D_{ij} = (\nu g^{km}\,\langle u_i\, u_j\rangle_{,k} + C_s\,\frac{E}{\varepsilon}\,\langle u_k\, u^l\rangle\,\langle u_i\, u_j\rangle_{,l})_{,m}\,, \tag{3}$$

where $C_s = 0.22$ is a numerical coefficient.

The dissipation tensor is taken in the "standard" isotropic form with a correction for low Reynolds numbers near a solid wall:

$$\varepsilon_{ij} = \frac{2}{3}\,\delta_{ij}\,\varepsilon + 2\nu\,\frac{\langle u_i\, u_j\rangle}{x_{\text{n}}^2}\,. \tag{4}$$

The equation for the rate of dissipation of the kinetic energy of turbulence has the form

$$U^k\,\varepsilon_{,k} = \left[g^{km}\left(\nu\,\varepsilon_{,k} + C_e\,\frac{E}{\varepsilon}\,\langle u_k\, u^l\rangle\,\varepsilon_{,l}\right)\right]_{,m} + (C_{el}P - C^*_{e2}\,\varepsilon)\,\frac{\varepsilon}{E} - \frac{2\nu\varepsilon}{x_{\text{n}}^2}\,f_1\,, \tag{5}$$

where $P = P_{ii}/2 = -\langle u_iu^k\rangle U_{i,k}$ is the generation of the turbulence energy; $f_1 = \exp\,(-C_{f1}x_{\text{n}}u_{*0}/\nu)$; $C_{f1} = 0.5$, $C_{e1} = 1.35$; $C^*_{e2} = \max\,(1.4C_{e2}f_2(1 - C_{e3}\text{Ri}))$ [9]; $C_{e2} = 1.8$; $C_{e3} = 2.0$; $f_2 = 1 - (2/9)\exp\,(-E^4/(6\nu\varepsilon)^3)$; $C_e = C_s$. The Richardson number Ri describes the action of the curvature of the current lines on the turbulence by analogy with the influence of stratification of the medium on turbulent transfer.

The balance equation for the kinetic energy of turbulence is obtained as a result of convolution of Eq. (2) by the subscripts i and j with allowance for the model expression (3) for the term of turbulent transfer:

$$U^k\,E_{,k} = \left[g^{km}\left(\nu\,E_{,k} + C_s\,\frac{E}{\varepsilon}\,\langle u_k\, u^l\rangle\,E_{,l}\right)\right]_{,m} + P - \varepsilon - \frac{2\nu\varepsilon}{x_{\text{n}}^2}\,. \tag{6}$$

The correlation of pressure–velocity shift Π_{ij} is represented as a sum of three terms [10]:

$$\Pi_{ij} = \Pi^{(1)}_{ij} + \Pi^{(2)}_{ij} + (\Pi'^{(1)}_{ij} + \Pi'^{(2)}_{ij})\,f(x_{\text{n}})\,, \tag{7}$$

where

$$\Pi^{(1)}_{ij} = -\,C_{s1}\,\frac{\varepsilon}{E}\left(\langle u_i\, u_j\rangle - \frac{2}{3}\,\delta_{ij}\,E\right) \quad (C_{s1} = 1.5)\,; \tag{8}$$

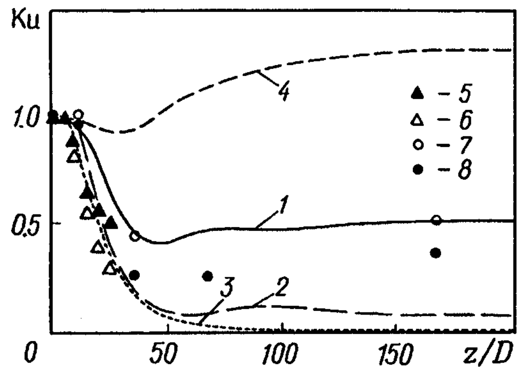

Fig. 1. Behavior of the coefficient of suppression of the longitudinal component of the kinetic energy of turbulence along the tube axis ($r/R = 0$): 1) $N = 0.34$, 2) 0.62, 3) 1; for Ri = 0 and expression (12) instead of (9): 4) $N = 0.34$ [1-4) results of calculations by the MTRS of [9], $\mathrm{Re}_0 = 37{,}000$]; 5) $N = 0.34$, 6) 0.62 ([1-3], $\mathrm{Re}_0 = 37{,}000$): 7) $N = 0.5$; 8) 1 ([6], $\mathrm{Re}_0 = 30{,}000$) [5-8) experimental data].

$$\Pi_{ij}^{(2)} = -C_{s2}\left(P_{ij} - \frac{2}{3}\delta_{ij}P\right) \quad (C_{s2} = 0.6)\,; \tag{9}$$

$$\Pi_{ij}^{\prime(1)} = C_{s1}^{\prime}\frac{\varepsilon}{E}\left[\langle u_n^2\rangle g_{nn}\delta_{ij} - \frac{3}{2}\left(\langle u_n u_j\rangle g_{in} + \langle u_n u_i\rangle g_{jn}\right)\right]; \tag{10}$$

$$\Pi_{ij}^{\prime(2)} = C_{s2}^{\prime}\frac{\varepsilon}{E}\left[\Pi_{nn}^{(2)} g_{nn}\delta_{ij} - \frac{3}{2}\left(\Pi_{nj}^{(2)} g_{in} + \Pi_{ni}^{(2)} g_{jn}\right)\right], \tag{11}$$

with the damping function $f = (1/5)E^{3/2}/(\varepsilon x_n)$, $C_{s1}^{\prime} = C_{s2}^{\prime} = 0.3$.

The system of closed equations for the sought functions of the average-velocity vector, turbulent stresses, and rate of dissipation of the kinetic energy of turbulence (1)-(11) is written [9] in an axisymmetric cylindrical coordinate system in the boundary-layer approximation and is solved by the reference-volume finite-difference method. Details of the numerical procedure and the boundary conditions used for the sought functions can be found in [9].

3. Results of Numerical Modeling. MTRS (1)-(11), in which ε-equation (6) is modified by introduction of the rotational Richardson number in the destruction term, reproduces to a satisfactory degree of accuracy [9] the transformation of the profiles of the longitudinal U and circumferential W components of the average flow velocity and the Reynolds stresses under the action of a moderate swirl of the flow ($N \le 0.6$) in a short rotating tube ($x/D = 25$). The efficiency of this model for the case of a long tube is discussed below.

The results of the calculations were compared with experimental data of [1-6]. In the experiments of [1-3], statistical characteristics of a turbulent flow (the first- and second-order moments of the velocity field) were measured in the outlet cross section of a short rotating tube for $x/D = 25$, $\mathrm{Re}_0 = 37{,}000$, and $N \le 0.6$. In the experiments of [4-6], the behavior of the components of the average flow velocity (circumferential and longitudinal) and the longitudinal intensity of the turbulence u' in a long rotating section of a tube ($x/D \le 168$) was investigated for $\mathrm{Re}_0 = 0.5{-}3\cdot 10^4$ and $N = 0.5{-}3$. Because of different experimental conditions it is difficult to compare quantitatively the data of the measurements.

Figure 1 shows results of calculating the coefficient of suppression of the longitudinal component of the turbulence energy Ku for three values of the swirl parameter ($N = 0.34$, 0.62, 1.0) as a function of the distance along the tube axis. In [1-3] most of the experimental data was obtained precisely for these N. For qualitative comparison, the figure also shows experimental points of [6] for $N = 0.5$, 1.0 in the cross sections $x/D = 11$, 36, 68, and 168. It can be seen that for $N = 1$ the model precalculates almost complete suppression of the turbulence intensity in the cross section $x/D = 168$. We should note that when the Reynolds numbers are close the experimental

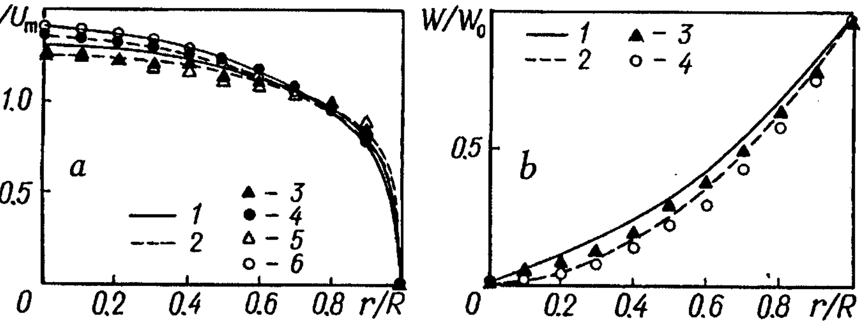

Fig. 2. Profiles of the longitudinal (a) and circumferential (b) components of the average flow velocity: a: 1) $Re_0 = 10^4$, 2) 37,000 (results of calculations), 3) $N = 0$, 4) 0.5 ([5], $Re_0 = 10^4$), 5) $N = 0$, 6) 0.5 ([7], $Re_0 = 2 \cdot 10^4$) [3-6) experimental data]; b: 1) by MTRS, 2) parabolic profile [1-2) results of calculations], 3) [5], $Re_0 = 3 \cdot 10^4$, 4) [7], $Re_0 = 2 \cdot 10^4$ [3, 4) experimental data].

data of [1-3] point to stronger suppression of the turbulence in the initial segment of the rotating tube ($x/D = 25$) than the data of [6]. It can be inferred that the MTRS of [9] reproduces (for Ri $\neq 0$) the transformation of the longitudinal component of the turbulence energy in a short rotating tube and describes qualitatively the behavior of Ku as the distance along the axis of a long rotating tube increases in the case of a moderate swirl ($N = 0.34$). For $N = 1$ the model reproduces the flow structure incorrectly.

For the profiles of the longitudinal and circumferential components of the average flow velocity, the model describes correctly their approach to the limiting state of stabilization [4-7]. However, quantitatively, these profiles disagree with the profiles measured in the experiment. Furthermore, their dependence on both the Reynolds number and the swirl parameter is observed, and the stabilization state itself is recorded, at much larger distances from the start of the rotating section of the tube than is observed in the experiments of [4-7]. The model reproduces qualitatively the decrease in the friction factor on the tube wall due to "laminarization" of the longitudinal-velocity profile (more precisely, its "parabolization") and the anisotropy of the components of the kinetic energy of turbulence.

If we set Ri = 0 in the ε-equation of the model of [9], then, as results of calculations (not given here) show, this "standard" MTRS, which is similar to the model of [8], describes correctly neither the behavior of the statistical characteristics of the flow in the initial segment of a rotating tube nor the approach of the profiles of the axial and circumferential velocities to the regime of stabilization in a long tube, although the increase in turbulence intensity in the segment of a rotating channel $68 \leq x/D \leq 168$ observed in [6] is qualitatively reproduced.

In [11], there is evidence that for swirling flows it is appropriate to modify the part $\Pi_{ij}^{(2)}$ of the correlations with pressure pulsations by including in the model expression for it the tensor of convective transfer $A_{ij} = (\partial/\partial x_k)(U_k\langle u_i u_j\rangle)$:

$$\Pi_{ij}^{(2)} = -C_{s2}\left(P_{ij} - A_{ij} - \frac{1}{3}\delta_{ij}(P_{kk} - A_{kk})\right). \quad (12)$$

Substitution of (12) for (9) did not lead, for Ri $\neq 0$, to better results than those presented above for a swirling flow in a long tube ($x/D \leq 168$).

Results of a calculation by the model of [9] for Ri = 0 with the modified expression (12) for $\Pi_{ij}^{(2)}$ are presented in Fig. 2 together with data of measurements [5, 7] obtained at $Re_0 = 10^4$. The profiles of the axial and circumferential components of the average flow velocity are shown in the cross section $x/D = 160$ [5] and 120 [7] for different swirl parameters. For moderate swirls, the model reproduces correctly the transformation of the profile of the longitudinal velocity U along the tube under the influence of the superimposed rotation on the flow and its approach to the limiting regime (Fig. 2a). It can be seen that the relationship of U with the Reynolds number in the stabilization regime is weak – a result that is in agreement with experiment [4-7]. The profile of the circumferential velocity W (Fig. 2b) for $N \leq 1$ attains a limiting parabolic dependence on the radial coordinate that is independent of Re. However the behavior of the longitudinal intensity of the turbulence is reproduced by the model only qualitatively (see Fig. 1, curve 4).

Thus, the structure of a developed turbulent flow in a straight circular tube experiences a complex transformation over the entire length of the rotating section of a long tube. A correct description of the action of rotation on the turbulence characteristics calls for adequate modeling of the spectral expenditure of turbulence energy in the rotating flow. For this purpose, the ε-equation based on the concept of equilibrium turbulence is used in the majority of modern turbulence models [12, 13]. Simple corrections like incorporation of the rotational Richardson number in the destruction term of the ε-equation turn out to be insufficient for a complete description of the action of rotation on turbulence. It is essential that model representations for a physically correct description of the process of turbulence-energy dissipation be developed further.

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NOTATION

$N = W_0/U_0$, swirl parameter of the flow; W_0, rotational speed of the tube wall; U_0, flow-rate-mean velocity of the flow; U and W, longitudinal and circumferential components of the average flow velocity; x and r, coordinates in the longitudinal and radial directions; R and D, radius and diameter of the tube; U_i and u_i, i-components of the vectors of the average velocity and the flow-velocity pulsations, respectively; $E = \langle u_i^2 \rangle/2$, kinetic energy of turbulence; ε, rate of dissipation of the kinetic energy of turbulence; $\mathrm{Ri} = (W/r)(\partial W/\partial r)/((\partial U/\partial r)^2 + (\partial W/\partial r)^2)$, rotational Richardson number; g^{ij}, metric tensor; δ_{ij}, Kronecker symbol; $\langle ... \rangle$, time averaging; $\hat{P}$, average pressure; p, pressure pulsations; ρ, density; ν, kinematic-viscosity factor; x_n, distance along the normal to the tube wall; $\mathrm{Re}_0 = U_0 D/\nu$, Reynolds number; $u' = \sqrt{\langle u^2 \rangle}$, longitudinal intensity of the turbulence; C, numerical coefficient; f, function; u_{*0}, friction velocity on the wall. Subscripts and superscripts: i, j, k, l, and m, subscripts and superscripts in the equations in the tensor form of representation; $,i$, covariant differentiation with respect to the coordinate x^i; s, index of the coefficient in the equation for the Reynolds stresses; e, index of the coefficient in the ε-equation; n, index of the direction along the normal to the wall.

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