Perspectives of turbulence second–order modelling in a swirling flow

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Abstract. Reynolds stress transport models (RSTM) used usually in the practice, can not reproduce correctly behaviour of statistical characteristics along the rotating pipe axis, in particular. Because of strong inhomogeneity of the flow, one of the reasons of such shortcoming of RSTMs could be unsufficient accuracy of turbulent diffusion models (TDM) applied. Verification of several TDMs showed that the main condition on the TDM used in practical calculations is its tensor – invarience at present.

1 Introduction

In the present work, a turbulent incompressible isothermal flow in a rotating cylindrical pipe is considered. Such flows are of interest because they occur in various engineering systems and possess by all features of boundary – layer flows of practical interest, whose rotation is caused by a rotating surface. Also the core of a pipe flow at a moderate swirl models turbulent transport in natural large – scale concentrated vortex formations.

As it is known from experiments, e.g. [Kikuyama (83)], [Zaets (85)] the structure of turbulence changes essentially under swirl in dependence on both pipe length and degree of the flow swirl, i.e., behaviour of the second – and higher – order moments is non – monotonic with increasing both these parameters. In the saturation region ($z/D = 68 \div 168$) all statistical characteristics achieve their limit profiles. Components of the mean flow velocity change monotonic. The longitudinal component tends to reach the parabolic profile, like one in a laminar flow, but does not achieve that, and the circumferential mean velocity component has the parabolic limit profile instead of the linear one as it would be in the case of solid – body rotation.

A good model of turbulence must reflect these and other features of such flow. Note also, that testing the new numerical method should be carried out with the use of the correct turbulence model.

2 Reynolds Stress Transport Models

At present, it is well known that simple models of turbulence, like $k - \epsilon$ ones, can not describe swirl flows. RSTM also has problems. For instance, in a swirling pipe flow, no known RSTM, as we found out, is able to reproduce behaviour of statistical characteristics along the pipe axis completely: from the initial pipe section ($z/D \leq 30$) to the saturation flow region.

The best results were obtained by models [Launder (89)], [Kurbatskii (95)]. The model with the modified expression for the pressure – strain correlation [Launder (89)] reproduces correctly profiles of the mean velocity components only in the saturation regime, whereas the use in the standard RSTM of the modified ϵ – equation [Kurbatskii (95)] gives satisfactory results on behaviour of mean velocity components as well as the second – order moments only in a short pipe ($z/D = 25$) . Both models give results only under the mild swirl although they were specially developed for a swirling flow. Some calculated data are shown on Fig: 1.

There are different approaches to improve results obtained by RSTMs. Because of strong inhomogeneity of the flow, one of the reasons of shortcomings of the RSTM could be unsufficient accuracy of the turbulent diffusion model applied.

2.1 Governing Equations

The set of exact transport equations for the mean velocity vector and the turbulent stress tensor in the case of a stationary incompressible flow is written in the general tensor form as:

$$U^j U_{i;j} = \nu g^{jk} U_{i;jk} - < u_i u^j >_{;j} - \tilde{P}_{;i}/\rho$$
$$U^k < u_i u_j >_k = D_{ij} + P_{ij} + \Pi ij - \epsilon_{ij} \qquad (1)$$
$$U^i_{;i} = 0$$

where $\epsilon_{ij} = 2\nu g^{km} < u_{i;m} u_{j;k} >$ is the dissipation tensor; $D_{ij} = - < u_i u_j u^m >_{;m} - (< pu_i >_{;j} + < pu_j >_{;i})/\rho + \nu(g^{km} < u_i u_j >_{;k})_{;m}$ is the diffusion term; $P_{ij} = - < u_j u^k > U_{i;k} - < u_i u^k > U_{j;k}$ is the production term; $\Pi_{ij} =< p(u_{i;j} + u_{j;i}) > /\rho$ is the pressure – strain correlation.

To close set (1), terms D_{ij}, Π_{ij}, and ϵ_{ij} should be modelled.

2.2 Model Approximations

The dissipation tensor ϵ_{ij} is modelled by the isotropic expression with correction on low Reynolds numbers near a solid wall [So (86)]. The equation for ϵ is taken as

$$U^k \epsilon_{;k} = [g^{km}(\nu\epsilon_{;k} + C_e \frac{k}{\epsilon} < u_k u^l > \epsilon_{;l})]_{;m} + (C_{e1} P - C^*_{e2}\epsilon)\frac{\epsilon}{k} - \frac{2\nu\epsilon}{x_n^2} f_1. \quad (2)$$

Notations: $P = 1/2 \cdot P_{ii}$; C_α are model constants, and $C^*_{e2} = max(1.4, C_{e2} f_2(1 - C_{e3} Ri))$ [Kurbatskii (95)]; Ri is Richardson number; f_1 and f_2

are damping functions [So (86)]. Equation (2) at $Ri = 0$ transforms into "standard" one for dissipation of turbulent kinetic energy (k).

The pressure – strain correlation is modelled with the wall correction [Gibson (78)].

For the turbulent diffusion term D_{ij} we sequentially introduced in (1) several TDMs. The best results have been gotten with ones suggested in [Hanjalic (72)], [Kurbatskii (97)]. The former model (*HL*) is of the gradient type and the later one (*KP*) is derived from the exact transport equation for triple velocity correlations and includes ten model equations for correlations which can be solved in sequence, one after another. Both models are tensor – invariant.

2.3 Numerical Procedure

The set of the closed equations for required functions (U, W, ϵ, $< u^2 >$, $< v^2 >$, $< w^2 >$, $< uv >$, $< uw >$, $< vw >$) was written in the axisymmetric cylindrical frame of references in the boundary layer approximation. The calculation conditions were the same as that of the experiments, where a swirling flow was obtained by conveying a fully-developed isothermal incompressible turbulent flow from the stationary pipe section of the 100-diameter length into a rotating pipe of the same diameter. For solving the differential equations, the control volume technique was used [Spalding (77)]. Boundary conditions were following.

$r = 0$:

$$\frac{\partial U}{\partial r} = \frac{\partial \epsilon}{\partial r} = \frac{\partial < u^2 >}{\partial r} = \frac{\partial < v^2 >}{\partial r} = \frac{\partial < w^2 >}{\partial r} = 0,$$

$$< uv > = < vw > = < uw > = W = 0;$$

$r = R$:

$$U = \epsilon = < u^2 > = < v^2 > = < w^2 > = < uv = < vw > = < uw > = 0, W = W_o$$

The grid was not uniform along r, with total number of nodes in this direction being 128. The forward – step δz size was $0.001R$.

2.4 Results and Discussion

In a stationary pipe flow *KP* – model gives good results for triple velocity correlations in the whole flow area, whereas *HL* – model does not describe correctly behaviour of skewness factor in the axial direction near a pipe axis as well as in a wall area (Fig: 2). The simple model of the gradient type [Daly (70)], which is usually used in practice does not describe behaviour of triple velocity correlations at all.

However, it was found that for reproducing behaviour of second-order moments as well as mean velocity components, rather only tensor – invariance of the applied TDM is important. Note also, that in any case damping functions as wall corrections should be kept in the ϵ – equation and the pressure-strain correlation model.

Under swirl , more accurate TDM gives better results. On Fig: 3 some profiles obtained by the RSTM with the use of HL – model are shown at $z/D = 25$. They are compared with the data of [Kurbatskii (95)], where DH – model was applied to model turbulent diffusion. Difference is strong especially near a wall and insreaces with the swirl. However, to describe short pipe flow, modification of the ϵ – equation by Richardson number is necessary anyway.

Profiles obtained with the use of KP – and HL – models are close and with increasing the swirl and Reynolds number, difference between them becomes less. Also, HL – model gives more stable results at different values of the swirl number. So, we come to paradox conclusion that less correct TDM is more preferable at present for practical calculations. The reason lies in uncorrect model expressions for other terms in set (1).

In a long pipe, application of more accurate TDMs lets describe behaviour of the second-order moments in the saturation regime (Fig: 3). However, mean velocity profiles are influenced, in the main, by the model expression for the pressure –strain correlation.

3 Conclusion

To calculate the first – and second – order velocity moments in a swirling flow of practical interest, the main condition for the TDM is its tensor – invariance and the Hanjalic – Launder model is the best choice at present. Application of more correct TDM results in better description of transformation of statistical characteristics profiles under swirl, not cardinally though. Even if improved TDMs are used, we should keep the damping functions as wall corrections and the Richardson number to describe the swirling flow in a short pipe.

The main reason of shortcomings of RSTM lies rather in modelling pressure – strain correlations and dissipation tensor. The use of more accurate TDMs, such as suggested in [Kurbatskii (97)], gives possibility to develop more physically correct models for these terms and, finally, get the RSTM which will be possible to apply to more wide range of flows.

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