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REYNOLDS-STRESS SIMULATIONS OF A FULLY-DEVELOPED CHANNEL FLOW USING A NEW VELOCITY/PRESSURE-GRADIENT MODEL

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ABSTRACT

Novel models for the velocity/pressure-gradient correlations through the fourth order have been developed by the authors and successfully compared against Direct Numerical Simulations (DNS) in *a priori* testing in planar wall-bounded flows. In the current work, the models for second-order velocity/pressure-gradient correlations are tested using a *posteriori* simulations in a planar fully-developed channel flow.

In this initial Reynolds stress transport (RST) model testing, the turbulent diffusion and dissipation terms are represented by their DNS profiles to isolate the effects of velocity/pressure-gradient modelling. In such a formulation, only the equations for mean velocity, shear stress, and Reynolds stress in the normal-to-the-wall direction are coupled. Simulations are also conducted with the RST equations, where all terms but those corresponding to molecular diffusion are represented by DNS data. Two solvers were used to independently confirm the computational results: a second-order accuracy numerical code for fully-developed two-dimensional axisymmetric flows and the open-source OpenFOAM software.

Results reveal a significant sensitivity of the RST equations to the accuracy of DNS data used to represent terms in the equations. This issue can be addressed by incorporating the DNS “balance” error as a source term in the RST equations. No balance terms are required in the coupled equations for the mean velocity and the shear stress. Successful testing of the velocity/pressure-gradient model has currently been achieved with these two equations. Coupling with the transport equations for normal Reynolds stresses is currently in a process, with an effort being directed towards overcoming a sensitivity of these equations to other factors including minor variations in velocity/pressure-gradient correlations.

INTRODUCTION

Recently, there has been renewed interest in using RST models for high-Reynolds-number external aerodynamics (Gerolymos et al., 2011; Cécora et al., 2012; Rodio et al., 2014). A major impetus behind this interest is the difficulty of existing one- and two-equation models to consistently predict separated flows.

Among terms requiring modelling in the RST equations are velocity/pressure-gradient correlations. A traditional approach to modelling these correlations is to separate them into the pressure diffusion and the pressure/velocity-gradient (pressure-strain) correlations (Rotta, 1951). Although the approach simplifies the mathematical formulation of the problem, it introduces difficulties in the modelling of the separate terms. For example, the majority of models for the pressure-strain correlations are based on the assumption that only strictly valid in homogeneous flows (Rotta, 1951). There is also little support for the gradient diffusion model (Daly and Harlow, 1970) typically used to represent diffusion-type terms including the pressure-diffusion correlations.

An integrated approach for modeling the velocity/pressure-gradient correlations through the fourth-order correlations was recently proposed in Poroseva and Murman (2014a,b). The models for these correlations were successfully verified in *a priori* testing using DNS data for a fully-developed planar channel flow (Jeyapaul et al., 2014) and a zero-pressure-gradient boundary layer over a flat plate (Spalart, 1988; Sillero et al., 2013). This integrated approach (theoretically) removes the need for wall damping or similar treatments to account for the behavior in the buffer and viscous layers near the wall. The goal of current work is to validate the models for the second-order velocity/pressure-gradient correlations through simulations conducted with an RST model.

MODEL FORMULATION

Here a brief review is provided, and the reader is referred to the original references (Poroseva and Murman, 2014a,b) for full details.

A tensor-invariant model for the „slow“ part of the velocity/pressure-gradient correlations Π_{ij}

$$\Pi_{ij}^{(s)} = -B_1 D_{ij}^{(T)} \tag{1}$$

was obtained by analyzing tensor properties of the second term in the exact integral-differential expression (without the surface integral):

$$-\frac{1}{\rho}< p_{,j}u_i > = -\frac{1}{2\pi}\iiint\left[U'_{m,n} < u'_n u_i >'_{,m}\right]'_{,j}\frac{1}{r}dV' - \frac{1}{4\pi}\iiint < u'_m u'_n u_i >'_{,mnj}\frac{1}{r}dV', \qquad (2)$$

(Chou, 1945). Hereafter, Cartesian notations are used, In (1), B_1 is a model coefficient and $D_{ij}^{(T)}$ is the turbulent diffusion term in the corresponding RST equation. In (2), u and p are turbulent velocity and pressure fluctuations; ρ is the density; U is the mean velocity; $f_{i,j} = \partial f_i / \partial x_j$; " ' " above a flow variable indicates that it should be evaluated at a point Y' with coordinates x'_i, which ranges over the region of the flow; r is the distance from Y' to the point Y with coordinates x_i; and dV' is the volume element. The velocity/pressure-gradient correlations on the left side of (2) are evaluated at point Y, whereas all derivatives on the right side are taken at Y'.

Models for the "rapid" terms (the term with the mean velocity gradients in (2)) were developed based on the analysis of the DNS budgets of transport equations in planar wall-bounded flows (Spalart, 1988; Sillero et al., 2013; Jeyapaul et al., 2014). An integrated model for "slow" and "rapid" terms is the following:

$$\Pi_{xx} = -0.1D_{xx}^{(T)} + 0.02P_{xx} + P_{xy},$$
$$\Pi_{xy} = -0.5D_{xy}^{(T)} - 0.02P_{xx} - P_{xy},$$
$$\Pi_{yy} = -0.5D_{yy}^{(T)} - 0.025P_{xx} - 0.45P_{xy},$$
$$\Pi_{zz} = -0.5D_{zz}^{(T)} + 0.025P_{xx} - 0.55P_{xy}. \qquad (3)$$

Here, x, y, and z indices correspond to the streamwise, normal-to-the-wall, and spanwise flow directions, respectively. The positive y-direction is towards outside the wall with $y = 0$ at the wall. Notation P is used to the production terms in the RST equations. Currently, no tensor-invariant form is available for (3) due to the lack of necessary DNS data for the budgets of transport equations in free shear flows and wall-bounded flows in the spanwise direction to validate such a form.

Figure 1compares DNS profiles (symbols) with the model profiles (solid lines) obtained with (3) using DNS data for production and turbulent diffusion terms in a channel flow and a zero pressure-gradient boundary layer over a flat plate (Sillero et al., 2013; Jeyapaul et al., 2014). The agreement observed is good up to the wall in both flows without using any wall function. More comparison for the boundary layer at various Reynolds numbers is given in Poroseva and Murman (2014a). The values of model coefficients did not vary for considered flow geometries and the Reynolds numbers, but can be adjusted if more accurate near-wall behaviour of the *Π*-components is required. Similar good agreement between (3) and DNS data was observed for all higher-order correlations (Poroseva and Murman, 2014a,b). This is certainly an advantage of the proposed models.

NUMERICAL APPROACH

A fully-developed planar channel flow was chosen for initial model testing due to availability of DNS data for

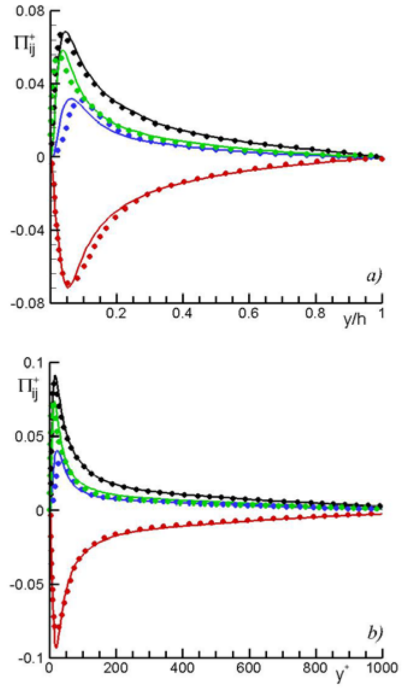

Figure 1. Π-profiles obtained from (3) (solid lines) using DNS data for $D^{(T)}$ and P-terms and from DNS (symbols): a) channel flow (Jeyapaul et al., 2014), b) zero-pressure gradient boundary layer over a flat plate (Sillero et al., 2013). Colors: red Π_{xx}, green Π_{zz}, blue Π_{yy}, black Π_{xy}.

velocity moments up through the fourth order and terms in their transport equations (Jeyapaul et al., 2014).

Two solvers were utilized: open-source OpenFOAM software (http://www.openfoam.com/) and a second-order accuracy code for fully-developed axisymmetric flows. In the latter, the control volume technique (Spalding, 1977) is implemented with a pseudo-time marching scheme to solve parabolic equations. Both solvers were previously verified. The code-to-code comparison allowed for more accurate separation of modelling effects from those due to a numerical procedure. Also, OpenFOAM can be used for more general flow configurations. Since similar trends were observed in simulations with the two solvers, only results obtained with the one-dimensional solver are reported here.

A grid used in simulations is non-uniform in the direction normal to the channel wall with the total number of nodes in this direction being 100 (98-node grid was used for DNS in Jeyapaul et al., 2014). The grid sensitivity analysis demonstrated that this is sufficient for obtaining grid-independent results. Time step is 0.1 s. Flow conditions: $Re_\tau = u_\tau h/\nu = 392$, friction velocity $u_\tau = 0.03798$ *m/s*, and the half-channel width h = 1 *m*, match those used in Jeyapaul et al. (2014) to generate DNS data. DNS profiles are used as initial conditions for the mean velocity component U along the channel axis and Reynolds stresses $< u_i u_j >$ to accelerate the results convergence. At the channel wall, no-slip boundary conditions are applied. At the channel axis,

$$\frac{\partial U}{\partial y} = \frac{\partial < u_\alpha u_\alpha >}{\partial y} = < uv > = 0\,, \qquad (4)$$

where $\alpha = x, y, z$ (no summation over α). In OpenFOAM, wall-to-wall simulations are conducted with no-slip conditions on both walls.

RST EQUATIONS

DNS data are considered to be the most accurate representation of a turbulent flow field obtained through computations. Therefore, to separate the effects of modelling velocity/pressure-gradient correlations from other terms, the turbulent diffusion and dissipation terms (shown in blue) in the RST equations

$$\frac{D < u_i u_j >}{Dt} = D_{ij}^{(M)} + D_{ij}^{(T)} + P_{ij} + \Pi_{ij} - \varepsilon_{ij} \qquad (5)$$

are represented by their DNS profiles. Other terms in (5) are production P_{ij} and molecular diffusion $D_{ij}^{(M)}$. In a fully-developed planar channel flow, this leads to coupling the transport equations for the axial mean velocity U, shear stress $< uv >$, and Reynolds stress $< v^2 >$ in the normal-to-the-wall direction.

Computations are also conducted with the RST equations, where all terms but those corresponding to molecular diffusion

$$\frac{D < u_i u_j >}{Dt} = D_{ij}^{(M)} + D_{ij}^{(T)} + P_{ij} + \Pi_{ij} - \varepsilon_{ij} \qquad (6)$$

are represented by DNS data to quantify uncertainty in such simulations. In (6), terms from DNS are shown in blue. This set of equations is uncoupled.

Other formulations of the RST equations solved are discussed in the following section.

RESULTS

Calculations with uncoupled equations (6) reveal a significant sensitivity of the RST equations to the accuracy of DNS data used to represent terms in the equations. Figure 2 shows the Reynolds stresses obtained in such computations. The mean velocity profile (not shown here) was affected as well. Similar results were obtained with both solvers.

Inaccuracy in the U- and $< uv >$-profiles is self-mitigated when the equations for these two parameters are coupled. That is, in the production term of the $< uv >$-equation

$$\frac{D < uv >}{Dt} = D_{xy}^{(M)} + D_{xy}^{(T)} - < v^2 > \frac{\partial U}{\partial y} + \Pi_{xy} - \varepsilon_{xy}\,, \qquad (7)$$

$\partial U / \partial y$ is computed along with the term $\partial < uv > / \partial y$ in the U-equation. The results (dashed lines in Fig. 3) are in agreement with the DNS data (shown by symbols).

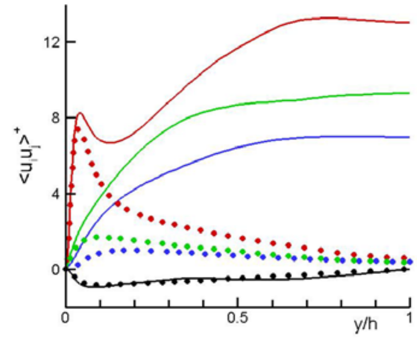

Figure 2. Reynolds stresses obtained using (6). Simulations: solid lines; DNS data: symbols. Colors: red $< u^2 >$, blue $< v^2 >$, green $< w^2 >$, black $< uv >$.

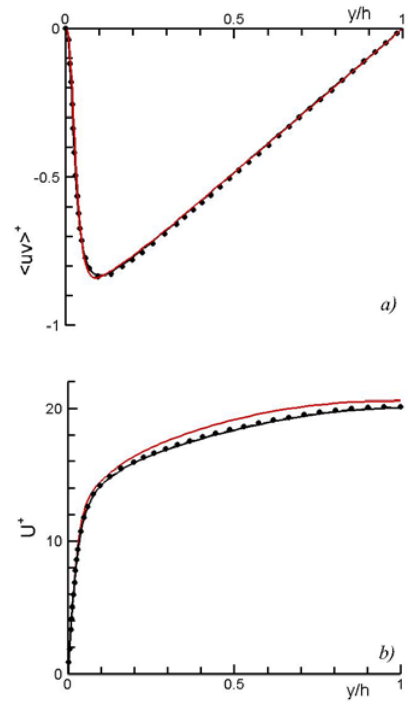

Figure 3. Profiles a) $< uv >$ and b) U. Symbols: DNS data; - - Eq. (7); — Eq. (8); — Eqs. (5) and (9).

The problem with calculating the normal Reynolds stresses can be solved by incorporating the DNS balance error as a source term in the transport equations. Results are shown in Fig. 4. Notice that the deviation of the computed $< u^2 >$ and $< w^2 >$ from their DNS profiles observed in Fig. 4 is due to errors associated with the interpolation of the DNS balance terms on the grid used with the one-dimensional solver. In OpenFOAM simulations, a grid similar to the one from DNS is used, and DNS and calculated profiles are in agreement. Adding the balance term to the $< uv >$-equation also improves the result when the equation is solved independently from others. However, the effect is negligible when the equations for U and $< uv >$ are solved together.

Simulations were conducted with the coupled equations for U, $< v^2 >$, and $< uv >$, that is, with $\partial U/\partial y$ and $< v^2 >$ in the $< uv >$-equation

$$\frac{D < uv >}{Dt} = D_{xy}^{(M)} + D_{xy}^{(T)} - < v^2 > \frac{\partial U}{\partial y} + \Pi_{xy} - \varepsilon_{xy}\,, \quad (8)$$

being computed. The balance term was added in the $< v^2 >$–equation. No coupling of the equations is possible otherwise. Results shown by black lines in Fig. 3, overlap with those of the coupled U-$< uv >$ computations.

The testing of model (3) for the Π_{xy}-correlations was performed using equation (5) for $< uv >$ coupled with the U- and $< v^2 >$-equations (with no modelling in the latter). Good aggrement was obtained with DNS data, which was further improved with the following expression :

$$\Pi_{xy} = -0.9 D_{xy}^{(T)} - 0.928 P_{xy} \quad (9)$$

(red lines in Fig. 3). The value of U^+ is slightly over predicted due to the under predicted value of friction velocity used to define U^+. The improved performance of expression (9) was also confirmed in *a priori* testing with DNS data used for the production and turbulent diffusion terms.

Coupling with the model transport equations for the normal-to-the-wall Reynolds stresses is currently in a process, with an effort being directed towards overcoming a sensitivity of these equations to other factors including minor variations in velocity/pressure-gradient correlations.

CONCLUSIONS

Simulations with the RST equations were conducted in a fully-developed planar channel flow. Results reveal a significant sensitivity of the RST equations to the accuracy of DNS data used to represent terms in the equations. This issue can be addressed by incorporating the DNS balance error as a source term in the RST equations. No balance terms are required in the coupled equations for the mean velocity and the shear stress.

Successful testing of the velocity/pressure-gradient model has currently been achieved with the coupled U-$< uv >$ equations. Coupling with the transport equations for normal Reynolds stresses is currently in a process, with an effort being directed towards overcoming a sensitivity of these equations to other factors including minor variations in velocity/pressure-gradient correlations.

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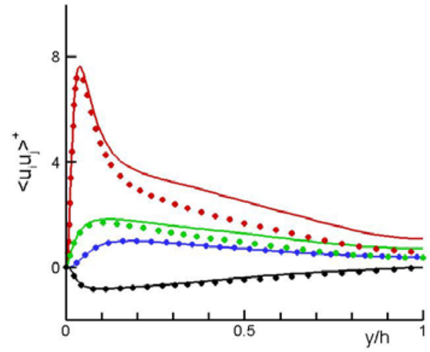

Figure 4. Reynolds stresses obtained using (6) with added DNS balance terms. Notations as in Fig. 2

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