```
> pbar<- e/10
> # chi-square statistic
> xtemp<- (o-e)/sqrt((10*pbar*(1-pbar)))
> round(cbind(pbar,o,e,xtemp),3)
       pbar   o       e     xtemp
 [1,] 0.037   0   0.370    -0.620
 [2,] 0.058   0   0.582    -0.786
 [3,] 0.077   0   0.766    -0.911
 [4,] 0.100   1   1.005    -0.005
 [5,] 0.124   1   1.240    -0.231
 [6,] 0.144   3   1.435     1.411
 [7,] 0.168   2   1.685     0.266
 [9,] 0.276   6   2.761     2.291
[10,] 0.393   3   3.933    -0.604
> x2<- sum(xtemp^2)
> pvalue<- 1-pchisq(x2,8)
> round(cbind(x2,pvalue),3)
          x2    pvalue
 [1,]   12.421    0.133.
```

The *xtemp* column contains the 10 contributions to the chi-square statistic showing which deciles exhibit a good fit and which exhibit a poor fit of the model estimated values to the data. The seven-variable model appears to be an adequate but not a great representation of the data (moderate chi-square value).

POISSON MODEL

A linear model involves a continuous outcome and a logistic model a binary outcome. The Poisson model lies somewhere between these two extremes, focused on an outcome variable that is a count. Frequently, counts are expressed as rates classified into a table. A Poisson model is a powerful statistical tool for analyzing a table of rates. The use of a Poisson model to analyze a table of counts follows in Chapter 6.

A Poisson model postulates that the logarithm of a rate is a linear function of k independent variables where the independent variables are frequently derived from categorical classifications. For example, lung cancer deaths (Table 4.9) are classified by a measure of exposure to radiation from the World War II atomic bomb explosions, age at death, gender, and the city of residence (Hiroshima or Nagasaki). These data form a table with dimensions 10 (exposure) $\times 13$ (age) $\times 2$ (sex)$\times 2$ (city). Also recorded are the total person-years at risk for each of the $10 \times 13 \times 2 \times 2 = 520$ categories. The rate (r_i) is the number of lung cancer deaths divided by the person-years at risk for each cell in the table (justified in Chapter 8). A Poisson model relates the logarithm of a rate to k independent variables by a linear function or

$$log(r_i) = a + b_1x_{i1} + b_2x_{i2} + b_3x_{i3} + \cdots + b_kx_{ik} \qquad i = 1, 2, 3, \cdots, n$$

Table 4.9. Data on lung cancer from Hiroshima and Nagaski—deaths and person-years by age and exposure

dose	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	70
HIROSHIMA	MALES												
0	0	1	1	4	0	4	6	16	20	24	11	6	3
	35 465	24 336	35 436	24 152	7 537	12 832	14 428	16 935	16 886	17 186	10 593	6 489	5 582
0.025	0	0	0	3	2	2	3	11	7	9	7	7	1
	31 240	16 275	12 744	13 245	4 404	6 797	7 939	8 560	9 759	8 128	5 957	3 458	2 730
0.075	0	0	0	3	0	0	2	3	5	6	1	2	1
	6 714	4 304	5 836	3 917	1 226	2 150	2 503	2 505	2 665	2 877	1 793	1 063	1 060
0.15	0	0	0	0	0	1	0	2	4	6	2	1	1
	9 500	6 019	4 794	4 009	1 189	2 086	2 446	2 772	3 760	4 089	2 207	1 292	1 122
0.3	0	0	0	1	0	1	0	5	8	6	5	1	1
	10 450	6 314	6 016	6 191	1 492	2 707	4 118	3725	4 443	4 303	2 505	1 601	1 484
0.75	0	0	0	0	1	1	0	2	5	6	5	1	1
	4 289	2 997	3 181	3 408	1 075	161	1 385	2 498	2 844	2 640	1 732	986	804
1.5	0	0	0	1	0	0	1	2	3	0	3	0	0
	2 003	1 324	1 522	2 635	450	1 236	1 123	1 357	1 975	1 229	1 111	516	238
2.5	0	0	0	0	0	0	0	1	1	1	1	0	0
	717	263	708	1 288	270	176	219	289	428	374	173	173	130
3.5	0	0	1	0	0	0	0	0	0	0	0	0	0
	212	141	238	476	48	35	144	149	104	113	141	22	35
5.0	0	0	0	0	0	0	1	0	0	0	0	0	0
	109	95	468	465	90	174	91	130	170	164	98	33	59
HIROSHIMA	FEMALES												
0	0	0	0	1	2	2	7	6	11	8	6	2	0
	39 241	22 963	29 393	46 215	36 919	30 413	30 490	33 660	28 448	23 929	13 366	9 217	8 893
0.025	1	2	0	0	3	6	10	8	12	7	3	3	1
	34 065	18 254	14 738	26 397	23 274	21 404	23 519	22 354	18 172	13 701	8 131	5 763	6 151
0.075	0	0	0	0	1	1	0	2	2	3	1	2	0
	7 586	4 099	585	7 151	5 887	5 649	5 479	5 960	4 679	3 442	2 370	1 441	1 442
0.15	0	0	0	1	1	0	2	3	0	5	6	2	2
	9 817	5 709	4 115	9 460	8 006	7 773	7 541	8 284	7 021	6 024	2 968	1 911	1 987

(continued)

Table 4.9. Data on lung cancer from Hiroshima and Nagaski—deaths and person-years by age and exposure—Continued

dose	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	70
NAGASKI	FEMALES												
0.3	0	0	1	0	0	1	5	5	3	7	5	2	2
	12 206	6 279	7 267	10 133	10 193	8 643	9 251	10 612	8 407	6 798	4 240	2 468	2 547
0.75	0	0	0	1	0	2	0	3	3	1	0	0	0
	4 423	3 068	4 265	6 813	6 219	4 481	4 670	5 380	5 205	3 301	2 146	1 292	1 057
1.5	0	0	0	0	1	0	0	2	2	2	0	0	1
	2 022	1 564	2 039	2 699	2 753	2 259	1 895	2 628	2 243	1 436	777	392	359
2.5	0	0	0	2	0	0	0	1	1	1	0	0	0
	836	367	1 029	963	1 429	719	789	720	768	509	140	108	114
3.5	0	0	0	0	0	0	1	0	0	1	0	0	0
	106	35	1 030	317	209	304	240	125	175	282	4	0	4
5.0	0	0	0	0	0	0	0	0	1	0	0	0	0
	282	131	125	461	363	243	152	193	186	104	98	36	32
NAGASKI	MALES												
0	0	1	0	2	0	4	7	12	17	18	6	7	2
	30 084	28 059	32 930	17 715	4 107	7 371	8 670	8 377	10 351	9 614	5 953	3 409	2 885
0.025	0	0	1	1	0	0	1	4	5	7	2	2	0
	18 448	11 350	11 411	6 014	1 317	1 826	1 956	2 012	2 907	2 729	2 214	1 520	1 121
0.075	0	0	0	0	0	0	1	1	2	0	0	0	0
	2 118	1 767	1 216	752	247	195	292	381	513	435	185	87	168
0.15	0	0	0	0	0	0	0	0	0	0	0	0	0
	1 578	1 586	1 506	969	144	566	291	415	515	429	367	160	175
0.75	0	1	0	0	0	1	0	1	1	1	1	0	0
	2 435	2 342	2 596	1 705	511	379	508	662	538	711	421	338	185
0.3	0	0	0	0	0	1	0	0	0	1	0	1	0
	1 231	1 097	1 129	1 225	326	197	423	349	342	399	128	120	105
1.5	0	0	0	0	0	1	0	1	1	1	1	0	0
	1 175	927	846	772	35	508	254	240	326	279	362	71	68
2.5	0	0	0	0	0	0	0	0	0	0	0	1	0
	338	317	361	406	71	71	71	43	66	27	0	34	3
3.5	0	0	0	0	0	0	0	0	0	0	0	0	0
	176	65	42	176	35	71	0	7	1	0	11	0	19
5.0	0	1	0	0	0	0	0	0	0	0	0	0	0
	71	176	291	176	0	55	91	98	58	116	31	15	0
NAGASKI	FEMALES												
0	1	0	0	1	7	3	7	3	2	3	6	2	1
	30 411	27 722	28 272	29 145	24 405	16 958	16 243	16 681	15 593	11 387	7 664	4 621	4 642
0.025	0	0	0	0	4	0	1	3	5	1	4	1	1
	17 685	13 598	15 486	11 620	12 088	96 930	9 486	8 965	7 759	6 767	4 479	2 244	2 256
0.075	0	0	0	1	0	0	1	0	1	0	0	0	0
	2 268	1 908	2 195	1 335	1 142	987	844	1 076	909	767	417	215	189
0.15	0	0	0	0	0	0	0	0	0	1	0	0	0
	1 913	1 909	2 344	1 595	1 371	1 212	1 316	1 064	1 067	618	168	260	235
0.3	0	0	1	0	0	0	0	0	2	0	0	0	0
	2 374	248	2 565	3 364	1 838	1 564	1 913	1 397	1 024	871	613	297	299
0.75	0	0	0	1	0	1	1	0	0	0	0	0	0
	1 223	1 551	2 504	2 098	1 695	1 049	606	938	750	726	333	194	193
1.5	0	0	0	0	2	0	0	2	1	0	0	1	0
	860	701	1 289	1 760	1 571	799	494	837	374	448	266	154	60
2.5	0	0	0	0	0	0	0	0	1	0	0	0	0
	215	353	492	388	247	335	242	196	130	143	6	36	25
3.5	0	0	0	0	0	0	0	0	0	0	0	0	0
	109	247	0	0	139	71	35	21	0	6	0	0	5
5.0	0	0	0	0	0	0	0	0	0	0	0	0	0
	216	106	141	141	142	35	104	159	43	50	35	10	1