Stat 345 Solutions - Practice Exam II

1. (a) $X \sim$ Geometric($p = .98$) so $p(x) = (0.02)^{x-1}0.98,\ x = 1, 2, \ldots$
   (b) $p(1) = 1(0.98) = .98;\ p(2) = 0.02(0.98) = 0.0196;$
   $p(2) = (0.02)^2(0.98) = 0.000392$
   (c) $E(X) = \frac{1}{p} = \frac{1}{0.98} = 1.02$
   (d) Now $X \sim$ Negative Binomial($r = 2, p = 0.98$) and $E(X) = \frac{r}{p} = \frac{2}{0.98} = 2.04$

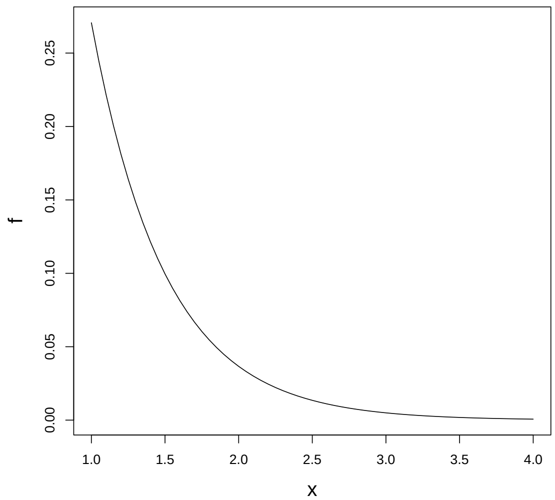
2. (a) $X \sim$ Binomial($n = 6, p = 0.98$) so $p(x) = \binom{6}{x}(0.02)^{6-x}(0.98)^x,\ x = 0, 1, \ldots, 6$
   (b) $E(X) = np = 6(0.98) = 5.88$
   (c) $P(X = 0) = \binom{6}{0}(0.02)^6(0.98)^0 = (0.02)^6 = 0.000000000064$
   (d) $p(X < 2) = p(0) + p(1) = 0.000000000064 + 0.000000018816 = 0.00000001888$

3. (a) $X \sim$ Hypergeometric($N = 20, K = 5, n = 10$) so $p(x) = \frac{\binom{5}{x}\binom{15}{10-x}}{\binom{20}{10}};\ x = 0, 1, 2, 3, 4, 5$
   (b) $E(X) = 10\frac{5}{20} = \frac{5}{2}$
   (c) After cancelling, $p(0) = \frac{7\cdot 3}{2\cdot 19\cdot 2\cdot 17} = 0.0162539;$
   $P(X \le 5) = 1$ since there are only 5 white balls.
   (d) $x = 0, 1, 2, 3, 4, 5$

4. (a) $X \sim$ Poisson($\lambda = 2$), so $p(x) = \frac{e^{-2}2^x}{x!};\ x = 0, 1, 2, \ldots$
   (b) $p(0) = e^{-2} = 0.1353$
   (c) $E(X) = 2$
   (d) $P(X > 0) = 1 - p(0 = 1 - 0.1353 = 0.8647$

5. (a) $P(X > 3) = \int_3^\infty 2e^{-2x}dx = -e^{-2x}|_3^\infty = 0 + e^{-6} = 0.00248$
   (b) $P(3 < X < 4) = \int_3^4 2e^{-2x}dx = -e^{-2x}|_3^4 = -e^{-8} + e^{-6} = 0.00248 - 0.00034 = 0.00214$
   (c) $P(X < x) = \int_0^x 2e^{-2w}dw = -e^{-2w}|_0^x = 1 - e^{-2x} = .1 \Rightarrow e^{-2x} = .9 \Rightarrow x = -\ln(.9)/2 = 0.0527$

(d)



(e)

$$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-2x} & 0 \le x \end{cases}$$

6. (a) $f(x) = \begin{cases} \frac{1}{4} & -2 < x < 2 \\ 0 & \text{o.w.} \end{cases}$

   (b) $E(X) = \frac{-2+2}{2} = 0$, and $V(X) = \frac{4^2}{12} = \frac{4}{3}$

   (c) $P(-x < X < x) = \int_{-x}^{x} \frac{1}{4} dw = \frac{w}{4}|_{-x}^{x} = \frac{x}{2} = 0.8 \Rightarrow x = 1.6$

7. (a) $E(X) = \int_0^1 3x^3 dx = \frac{3}{4}$, $E(X^2) = \int_0^1 3x^4 dx = \frac{3}{5}$, $V(X) = \frac{3}{5} - (\frac{3}{4})^2 = \frac{48-45}{80} = \frac{3}{80}$

   (b) $P(X > 0.5) = \int_{0.5}^{1} 3x^2 dx = x^3|_{0.5}^{1} = 1 - \frac{1}{8} = \frac{7}{8}$
   $P(0.25 < X < 0.75) = \int_{0.25}^{0.75} 3x^2 dx = x^3|_{0.25}^{0.75} = \frac{27}{64} - \frac{1}{64} = \frac{26}{64} = \frac{13}{32}$

   (c) $F(x) = \begin{cases} 0 & x < 0 \\ x^3 & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$

8. (a) $P(X > 31.7) = P(\frac{X-30}{2} > \frac{31.7-30}{2}) = P(Z > 0.85) = 0.197662$

   (b) $P(29.3 < X < 33.5) = P(\frac{29.3-30}{2} < \frac{X-30}{2} < \frac{33.5-30}{2}) = P(-0.35 < Z < 1.75) = 0.959941 - 0.363169 = 0.596772$

   (c) $P(X < 25.5) = P(\frac{X-30}{2} < \frac{25.5-30}{2}) = P(Z < -2.25) = 0.012224$

   (d) $P(X < x) = .99 \Leftrightarrow P(\frac{X-30}{2} < \frac{x-30}{2}) \Leftrightarrow P(Z < \frac{x-30}{2}) = .99 \Leftrightarrow \frac{x-30}{2} = 2.33 \Leftrightarrow x = 34.66$

9. Let T = time for one individual to be served. Then $f(t) = \frac{e^{-t/4}}{4}$; $t > 0$. $P(T < 3) = 1 - e^{3/4} = 0.528$ If X = number of days served under 3 minutes in the next 6 days, then $X \sim \text{binomial}(n = 6, p = 0.528)$, and $P(X = 4) = \binom{6}{4}(0.472)^2(0.528)^4 = 0.259723$

10. (a) $X \sim \text{Geometric}(p = 0.2)$ so $p(x) = 0.8^{x-1} \cdot 0.2;\ x = 1, 2, \ldots$.
    (b) $E(X) = \frac{1}{0.2} = 5$
    (c) $p(3) = 0.8^2 \cdot 0.2 = 0.128$

11. (a) $X \sim \text{binomial}(n = 20, p = 0.2)$, so $p(x) = \binom{20}{x} 0.8^{20-x} \cdot 0.2^x;\ x = 0, 1, 2, \ldots, 20$
    (b) $E(X) = 20(.2) = 4$
    (c) $P(X = 20) = 0.2^{20} = 0.0000000000000001048576$
    (d) $P(X = 0) = 0.8^{20} = 0.01153$
    (e) $P(X \leq 2) = p(0) + p(1) + p(2) = 0.206085$