Section 2.1

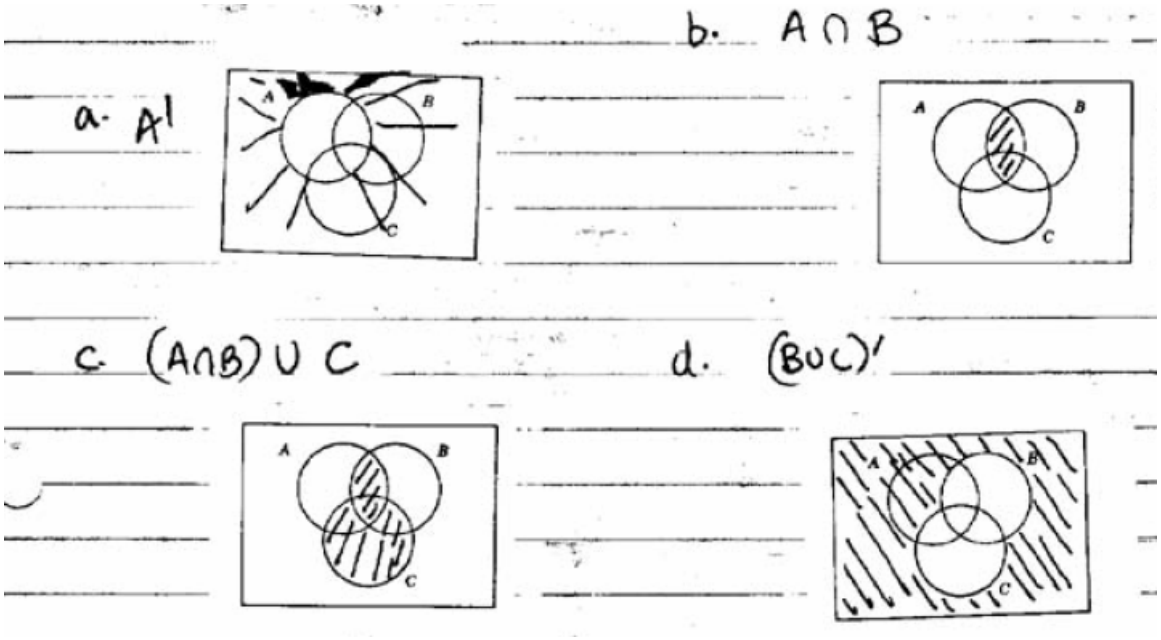
2.2 Let 1 represent an error, 0 represent a correct transmission. There are $2^4 = 16$ outcomes in S in the form (4^{th} bit, 3^{rd} bit, 2^{nd} bit, 1^{st} bit).
S={(0000), (0001), (0010), (0011), (0100), (0101), (0110), (0111), (1000), (1001), (1010), (1011), (1100), (1101), (1110), (1111)}

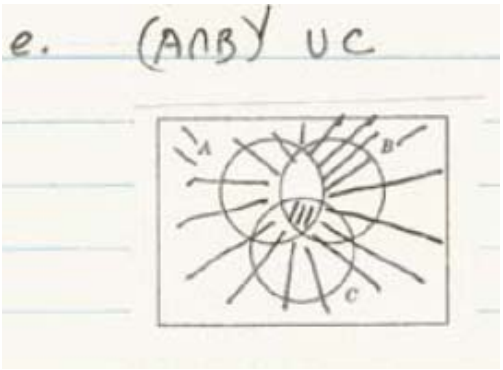
2.6 Each of three spaces can be filled by any of 10 digits (0,1,2,3,4,5,6,7,8,9)

_____ _____ _____

So there are 10 x 10 x 10 = 10^3 elements in S (the three digit numbers from 000 to 999).

2.19





2.23 This is the same setup as 2.2. Let 1 represent an error, 0 represent a correct transmission. There are $2^4 = 16$ outcomes in S in the form (4^{th} bit, 3^{rd} bit, 2^{nd} bit, 1^{st} bit).

a. S={(0000), (0001), (0010), (0011), (0100), (0101), (0110), (0111), (1000), (1001), (1010), (1011), (1100), (1101), (1110), (1111)}

b. Clearly not mutually exclusive since two or more bits may be distorted.

c. A_1 = {(0001), (0011), (0101), (0111), (1001), (1011), (1101), (1111)} (all the odd numbers in binary representation).

d. A_1' = {(0000), (0010), (0100), (0110), (1000), (1010), (1100), (1110)} (all the even numbers in binary representation)

e. $A_1 \cap A_2 \cap A_3 \cap A_4$={(1111)} all bits distorted

f. $(A_1 \cap A_2) \cup (A_3 \cap A_4)$ = {(0011), (0111), (1011), (1111), (1100), (1101), (1110)}. Either 1^{st} two or last two (or all) distorted

Section 2.2

2.34 S={a,b,c,d,e}, A={a,b,c}, B={c,d,e}, P(a) = P(b) = .1, P(c) = P(e) = .2, P(d) = .4

a) P(A) = P(a) + P(b) + P(c) = .1+.1+.2 = .4

b) P(B) = P(c) + P(d) + P(e) = .2+.4+.2 = .8

c) P(A′) = 1 – P(A) = 1-.4 = .6

d) P(A ∪ B) = P(S) = 1

e) P(A ∩ B) = P(c) = .2

2.37 Let C_1, …, C_8 be the 8 cavities.

a) $S = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8\}$

b) $P(\{C_1, C_2\}) = P(C_1) + P(C_2) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

c) $P(\{C_3, C_4\}') = 1 - P(\{C_3, C_4\}) = 1 - \frac{1}{4} = \frac{3}{4}$

2.41 S = {Correct, Minor, Major}

a) P(Correct) = .25

b) P({Minor, Major}) = P(Minor) + P(Major) = .7 + .05 = .75 (=1 – P(Correct))

Section 2.3

2.50 A, B, C mutually exclusive with P(A) = .2, P(B) = .3, P(C) = .4

a) $P(A \cup B \cup C) = P(A) + P(B) + P(C) = .2 + .3 + .4 = .9$
(The only reason you can add is because they are m.e. !)

b) $P(A \cap B \cap C) = P(\varnothing) = 0$ (m.e.!)

c) $P(A \cap B) = P(\varnothing) = 0$

d) $P[(A \cup B) \cap C) = P(\varnothing) = 0$ (C does not share anything with A or B)

e) De Morgan's laws state that $(A \cup B)' = A' \cap B'$
$(A \cap B)' = A' \cup B'$

so

$$P(A' \cap B' \cap C') = P[(A \cup B \cup C)'] = 1 - P(A \cup B \cup C) = 1 - .9 = .1$$

$A' \cap B' \cap C'$ is just whatever is not in any of them, so this is easy to get directly, too.

2.52

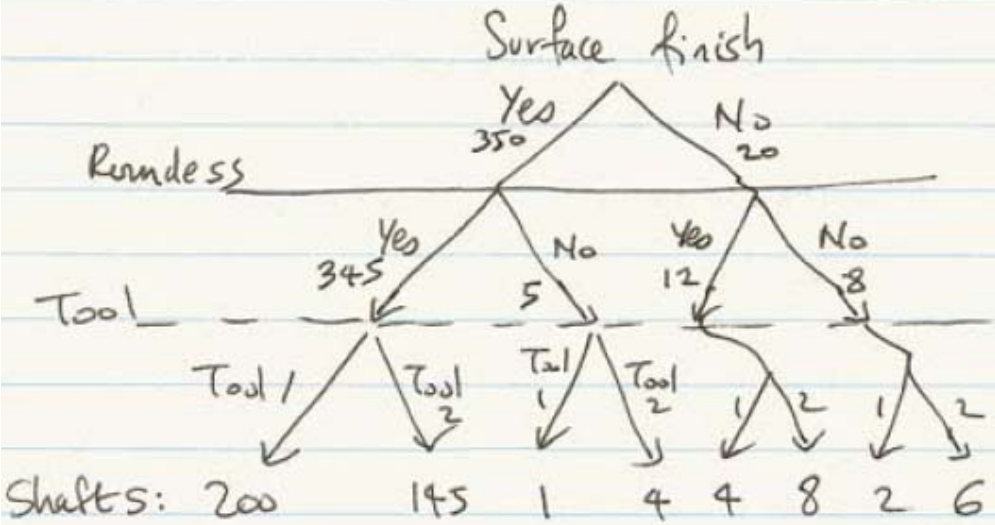
		Shock Resistance H	Shock Resistance L	
Scratch Resistance	H	80	9	89
	L	6	5	11
		86	14	100

a) $P(ScrH \cap ShkH) = \frac{80}{100} = .8$

b) $P(ScrH \cup ShkH) = \frac{89}{100} + \frac{86}{100} - \frac{80}{100} = \frac{95}{100} = .95$

c) No, a disk can be both highly shock resistant and highly scratch resistant at the same time.

2.56



a) Surface Yes ∪ Roundess Yes ∪ Tool 1

$$\frac{350 + 12 + 2}{370} = \frac{364}{370}$$

b) Surface Yes ∪ Roundness No ∪ Tool 2

$$\frac{345 + 8 + 5 + 8}{370} = \frac{366}{370}$$

c) (Surf Yes ∩ Round Yes) ∪ Tool 2

$$\frac{345 + 4 + 8 + 6}{370} = \frac{363}{370}$$

d) (Surf Yes ∪ Tool 2)

$$\frac{350 + 8 + 6}{370} = \frac{364}{370}$$

An alternative method is:

a) $P(\text{Surf Yes} \cup \text{Round Yes} \cup \text{Tool 1})$

$= P(\text{Surf Yes}) + P(\text{Round Yes}) + P(\text{Tool 1}) - P(\text{Surf Yes} \cap \text{Round Yes})$
$- P(\text{Surf Yes} \cap \text{Round Yes}) - P(\text{Surf Yes} \cap \text{Tool}) - P(\text{Round Yes} \cap \text{Tool})$
$+ P(\text{surf Yes} \cap \text{Round Yes} \cap \text{Tool 1})$

$= \frac{350 + 357 + 207 - 345 + 201 - 204 + 200}{370} = \frac{364}{370}$

b) $P(\text{Surf Yes} \cup \text{Round No} \cup \text{Tool 2}) =$

$P(\text{Surf Yes}) + P(\text{Round No}) + P(\text{Tool 2}) - P(\text{Surf Yes} \cap \text{Round No})$
$- P(\text{Surf Yes} \cap \text{Tool 2}) - P(\text{Round No} \cap \text{Tool 2}) + P(SY \cap RN \cap T2)$

$= \frac{350 + 13 + 163 - 5 - 149 - 10 + 4}{370} = \frac{366}{370}$

c) $P[(SY \cap RY) \cup T2] = P[SY \cap RY] + P[T_2] - P[SY \cap RY \cap T_2]$

$= \frac{345 + 163 - 145}{370} = \frac{363}{370}$

d) $P[SY \cup T2] = P[SY] + P[T_2] - P(SY \cap T_2)$

$= \frac{350 + 163 - 149}{370} = \frac{364}{370}$