Stat 345-001 Name: ____________________

Spring 2001

Practice Problems for Test 2

1. A random variable X has the following pmf:

x	-5	-3	3	5
$p(x)$	0.40	0.15	0.30	0.15

   (a) Find the expected value of the random variable X.

   (b) Find the variance of X.

   (c) Find $E(1/X)$.

2. Four of a company's 24 trucks emit excessive amounts of pollutants and the other 20 do not. As part of an air-pollution survey, an inspector decides to examine the exhaust of six of the company's trucks. Let the random variable Y be the number of trucks in the inspector's sample that contain pollutants.

   (a) Write the pmf of Y. (Hints: Be sure to include the possible values that Y can take on and note that the sampling is done without replacement).

   (b) What is the probability that none of the trucks that emit excessive pollutants will be included in the inspector's sample?

3. Let the random variable W be the number of UFO sightings in New Mexico within a month. Suppose that, on average, there are 3 sightings every month. What is the probability that there are 2 or more sightings in July?

4. The pdf of a continuous random variable is given by

$$f(x) = \begin{cases} cx^2, & -1 \leq x \leq 1 \\ 0, & otherwise \end{cases}$$

   (a) Find c so that $f(x)$ is a density function.

   (b) Find $P(0.25 \leq X \leq 0.75)$.

   (c) Find $E(X)$ and $Var(X)$.

   (d) Find the cdf of X.

5. A five-sided fair die is to be tossed 20 times. Let Y denote the number of times a 1 occurs.

   (a) What is the distribution of Y? (Be able to give the pmf or to give the name of the distribution, specifying the values of any parameters involved.)

   (b) What is the expected value of Y?

   (c) What is the SD of Y?

   (d) What is the probability that exactly ten 1's are obtained in the 20 tosses?

6. The pdf of a continuous random variable is given by

$$f(x) = \begin{cases} c(x+1), & 2 \leq x \leq 4 \\ 0, & otherwise \end{cases}$$

   (a) Find c so that $f(x)$ is a density function.

   (b) Find $P(1.5 \leq X \leq 3.5)$.

   (c) Find $E(X)$ and $Var(X)$.

   (d) Find the cdf of X.

7. Suppose that the probability of winning an instant lottery is $\frac{1}{9}$, and that each lottery ticket purchased is independent of the other tickets purchased.

   (a) Allison buys 10 instant lottery tickets. Let X be the number of winning tickets she purchases. What is the distribution of X? (Give the name of the distribution and the values of the parameters of the distribution).

   (b) Find the probability that Allison has purchased exactly one winning ticket.

   (c) Find the probability that Allison has purchased at least one winning ticket.

   (d) Find the mean and the variance of the number of winning tickets Allison purchases.

   (e) Let Y be the number of tickets Allison must purchase until she purchases a winning ticket. What is the distribution of Y? (Give the name of the distribution and the values of the parameters of the distribution).

   (f) What is the probability that Allison must buy 5 tickets to obtain the first winning ticket?

   (g) Find the mean and the variance of the number of tickets that Allison must purchase to obtain the first winning ticket.