Stat 345 Solutions - Section 2.6

Problem 2-84

$P(A \cap B) = \frac{80}{100}, P(A) = \frac{82}{100}, P(B) = \frac{90}{100}$
Since $P(A \cap B) \neq P(A)P(B)$ then A and B are not independent.

Problem 2-87

Let $H_i$ denote the event that the $i^{th}$ specimen contains high levels of contamination.

(a) $P(H_1' \cap H_2' \cap H_3' \cap H_4' \cap H_5') = P(H_1')P(H_2')P(H_3')P(H_4')P(H_5')$ by independence. $P(H_i') = 0.9$, so the answer is $(0.9)^5 = 0.590$.

(b) $A_1 = H_1' \cap H_2 \cap H_3 \cap H_4 \cap H_5$
$A_2 = H_1 \cap H_2' \cap H_3 \cap H_4 \cap H_5$
$A_3 = H_1 \cap H_2 \cap H_3' \cap H_4 \cap H_5$
$A_4 = H_1 \cap H_2 \cap H_3 \cap H_4' \cap H_5$
$A_5 = H_1 \cap H_2 \cap H_3 \cap H_4 \cap H_5'$
We want $P(A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5)$. By independence $P(A_i) = (0.9)^4(0.1) = .0656$ for each of the $A_i$. The $A_i$ also are mutually exclusive, so
$P(A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5) = \sum_{i=1}^{5} P(A_i) = 5(.0656) = 0.328$.

(c) If B is the event in part (a), i.e. B is the event no sample contains high levels of contamination, then $P(B) = .059$. We want $P(B') = 1 - P(B) = 1 - 0.59 = 0.41$.

Problem 2-90

Let A = event all upper devices function, B = event all lower devices function.
Everything operates independently, so
$P(A) = (0.9)(0.8)(0.7) = 0.504$, $P(B) = (0.95)(0.95)(0.95) = 0.8574$, and
$P(A \cap B) = (0.504)(0.8574) = 0.4321$.
The circuit operates is the event $A \cup B$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.504 + 0.8574 - 0.4321 = 0.9293$