Stat 345 Solutions - Section 3.6

Problem 3-62

Let X = number of defective integrated circuits. Then, $X \sim Bin(n = 40, p = 0.01)$.

$$P(\text{product operates}) = P(X = 0) = \binom{40}{0}(0.01)^0(0.99)^{40} = 0.6690$$

Problem 3-66

Let X = number of parts in the sample that require rework. then $X \sim Bin(n = 20, p = 0.01)$.

$E(X) = \mu = np = 20(0.01) = 0.2$ and $\sigma = \sqrt{np(1-p)} = \sqrt{20(0.01)(0.99)} = 0.4450$

Problem suspected if

$$\begin{aligned} X &> \mu + 3\sigma \\ X &> 0.2 + 3(0.4450) \\ X &> 1.535 \end{aligned}$$

(a)

$$\begin{aligned} P(X > 1.535) &= P(X > 1) \\ &= 1 - P(X \leq 1) \\ &= 1 - [P(X = 0) + P(X = 1)] \\ &= 1 - [\binom{20}{0}(0.01)^0(0.99)^{20} + \binom{20}{1}(0.01)^1(0.99)^{19}] \\ &= 1 - [0.8179 + 0.1652] \\ &= 1 - 0.9831 = 0.0169 \end{aligned}$$

(b) Here, $X \sim Bin(n = 20, p = 0.04)$. Then,

$$\begin{aligned} P(X > 1) &= 1 - P(X \leq 1) \\ &= 1 - [P(X = 0) + P(X = 1)] \\ &= 1 - [\binom{20}{0}(0.04)^0(0.96)^{20} + \binom{20}{1}(0.04)^1(0.96)^{19}] \\ &= 1 - [0.4420 + 0.3683] \\ &= 1 - 0.8103 = 0.1897 \end{aligned}$$

(c) Let Y be the number of samples where $X > 1$. Then $Y \sim Bin(n = 5, p = 0.1897)$.

$$\begin{aligned} P(Y \geq 1) &= 1 - P(Y < 1) \\ &= 1 - P(Y = 0) \\ &= 1 - \binom{5}{0}(0.1897)^0(0.8103)^5 \\ &= 1 - 0.3493 = 0.6507 \end{aligned}$$

<u>Problem 3-70</u>

(a) Let X be the number of mornings the light is green. Then $X \sim Bin(n = 5, p = 0.2)$.

$$P(X = 1) = \binom{5}{1}(0.2)^1(0.8)^4 = 0.4096$$

(b) Let X be the number of mornings the light is green. Then $X \sim Bin(n = 20, p = 0.2)$.

$$P(X = 4) = \binom{20}{4}(0.2)^4(0.8)^{16} = 0.2182$$

(c)

$$\begin{aligned} P(X > 4) &= 1 - P(X \leq 4) \\ &= 1 - [0.0115 + 0.0576 + 0.1369 + 0.2054 + 0.2182] \\ &= 1 - 0.6296 \\ &= 0.3704 \end{aligned}$$