Stat 345 Solutions - Section 3.7

Problem 3-72

$X \sim$ geometric(p=0.4) (p=0.4 since $\frac{1}{p} = 2.5$)

(a) $P(X = 1) = (1-p)^{1-1}p = p = 0.4$
(b) $P(X = 4) = (1-p)^{4-1}p = (0.6)^3 0.4 = 0.0864$
(c) $P(X = 5) = (1-p)^{5-1}p = (0.6)^4 0.4 = 0.0518$
(d)

$$\begin{aligned} P(X \leq 3) &= P(X = 1) + P(X = 2) + P(X = 3) \\ &= (1-p)^0 p + (1-p)^1 p + (1-p)^2 p \\ &= 0.4 + (0.6)0.4 + (0.6)^2 0.4 \\ &= 0.784 \end{aligned}$$

(e) $(P(X > 3) = 1 - P(X \leq 3) = 1 - 0.784 = 0.216$

Problem 3-75

Let X = the number of calls until connected. Then $X \sim$geometric(p=0.02).

(a) $P(X = 10) = (0.98)^{10-1}(0.02) = 0.0167$
(b)

$$\begin{aligned} P(X > 5) &= 1 - P(X \leq 5) \\ &= 1 - [P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)] \\ &= 1 - [0.02 + (0.98)(0.02) + (0.98)^2(0.02) + (0.98)^3(0.02) + (0.98)^4(0.02)] \\ &= 1 - 0.0961 = 0.9039 \end{aligned}$$

or,

$$P(X > 5) = \sum_{x=6}^{\infty} f(x) = \sum_{x=6}^{\infty} (.98)^{x-1}(.02) = (.02)\frac{.98^5 - 0}{1 - .98} = .98^5 = .9039$$

(c) $E(X) = \frac{1}{p} = \frac{1}{0.02} = 50$

Problem 3-80

The pmf for a negative binomial RV is

$$f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, x = r, r+1, r+2, ...$$

If $r = 1$ then

$$f(x) = \binom{x-1}{0} p(1-p)^{x-1} = (1-p)^{x-1}p, x = 1, 2, 3, ...$$

which is the geometric pmf.

For the negative binomial distribution, $E(X) = \frac{r}{p}$ and $Var(X) = \frac{r(1-p)}{p^2}$. If $r = 1$, then $E(X) = \frac{1}{p}$ and $Var(X) = \frac{(1-p)}{p^2}$, which are the same as the geometric distribution.

<u>Problem 3-81</u>

$X \sim$ neg bin$(r = 4, p = 0.2)$
(a) $E(X) = \frac{r}{p} = \frac{4}{0.2} = 20$
(b) $P(X = 20) = \binom{20-1}{4-1}(0.2)^4(0.8)^{20-4} = 0.0436$
(c) $P(X = 19) = \binom{19-1}{4-1}(0.2)^4(0.8)^{19-4} = 0.0459$
(d) $P(X = 21) = \binom{21-1}{4-1}(0.2)^4(0.8)^{21-4} = 0.0411$

<u>Problem 3-84</u>

Let $X =$ the number of transactions until all 3 computers fail. Then $X \sim$ neg bin$(r = 3, p = 10^{-8})$.

(a) $E(X) = \frac{r}{p} = \frac{3}{10^{-8}} = 3 \times 10^8$
(b) $Var(X) = \frac{r(1-p)}{p^2} = \frac{3(1-10^{-8})}{(10^{-8})^2} = 3 \times 10^{16}$